Physics 4D Winter 2012 Homework 2 Solutions

Chapter 31

22. The average energy transferred across unit area per unit time is the average magnitude of the Poynting vector, and is given by Eq. 31-19a.

$$\overline{S} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(0.0265\,\mathrm{V/m}\right) = \boxed{9.32\times10^{-7}\,\mathrm{W/m^2}}$$

23. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let ΔU represent the energy that crosses area A in a time ΔT.

$$S = \frac{cB_{\mathrm{rms}}^2}{\mu_0} = \frac{\Delta U}{A\Delta t} \quad \rightarrow$$

$$\Delta t = \frac{\mu_0 \Delta U}{AcB_{\mathrm{rms}}^2} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(335\,\mathrm{J}\right)}{\left(1.00\times10^{-4}\,\mathrm{m^2}\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(22.5\times10^{-9}\,\mathrm{T}\right)^2} = 0.194\,\mathrm{W/m^2}$$

$$= \boxed{2.77\times10^{7}\,\mathrm{s}} \approx 321 \text{ days}$$

24. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let ΔU represent the energy that crosses area A in a time Δt.

$$S = c\varepsilon_0 E_{\mathrm{rms}}^2 = \frac{\Delta U}{A\Delta t} \quad \rightarrow$$

$$\frac{\Delta U}{\Delta t} = c\varepsilon_0 E_{\mathrm{rms}}^2 A$$

$$= \left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(0.0328\,\mathrm{V/m}\right)^2\left(1.00\times10^{-4}\,\mathrm{m^2}\right)\left(3600\,\mathrm{s/h}\right)$$

$$= \boxed{1.03\times10^{-6}\,\mathrm{J/h}}$$

$\boxed{25.}$ The intensity is the power per unit area, and also is the time averaged value of the Poynting vector. The area is the surface area of a sphere, since the wave is spreading spherically.

$$\overline{S} = \frac{P}{A} = \frac{(1500\,\mathrm{W})}{\left[4\pi(5.0\,\mathrm{m})^2\right]} = 4.775\,\mathrm{W/m^2} \approx \boxed{4.8\,\mathrm{W/m^2}}$$

$$\overline{S} = c\varepsilon_0 E_{\mathrm{rms}}^2 \quad \rightarrow \quad E_{\mathrm{rms}} = \sqrt{\frac{\overline{S}}{c\varepsilon_0}} = \sqrt{\frac{4.775\,\mathrm{W/m^2}}{\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}} = \boxed{42\,\mathrm{V/m}}$$

28. The power output per unit area is the intensity, and also is the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{0.0158\,\text{W}}{\pi\left(1.00\times10^{-3}\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}}$$

$$= 1376.3\,\text{V/m} \approx \boxed{1380\,\text{V/m}}$$

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{1376.3\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{4.59\times10^{-6}\,\text{T}}$$

31. In each case, the required area is the power requirement of the device divided by 10% of the intensity of the sunlight.

(*a*) $A = \frac{P}{I} = \frac{50\times10^{-3}\,\text{W}}{100\,\text{W/m}^2} = 5\times10^{-4}\,\text{m}^2 = \boxed{5\,\text{cm}^2}$

A typical calculator is about 17 cm x 8 cm, which is about $140\,\text{cm}^2$. So yes, the solar panel can be mounted directly on the calculator.

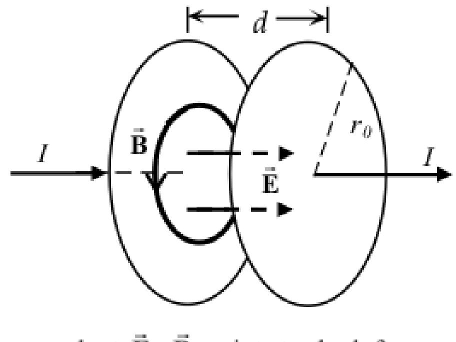
(*b*) $A = \frac{P}{I} = \frac{1500\,\text{W}}{100\,\text{W/m}^2} = 15\,\text{m}^2 \approx \boxed{20\,\text{m}^2}$ (to one sig. fig.)

A house of floor area $1000\,\text{ft}^2$ would have on the order of $100\,\text{m}^2$ of roof area. So yes, a solar panel on the roof should be able to power the hair dryer.

(*c*) $A = \frac{P}{I} = \frac{20\,\text{hp}\left(746\,\text{W/hp}\right)}{100\,\text{W/m}^2} = 149\,\text{m}^2 \approx \boxed{100\,\text{m}^2}$ (to one sig. fig.)

This would require a square panel of side length about 12 m. So no, this panel could not be mounted on a car and used for real-time power.

32. (*a*) Example 31-1 refers back to Example 21-13 and Figure 21-31. In that figure, and the figure included here, the electric field between the plates is to the right. The magnetic field is shown as counterclockwise circles. Take any point between the capacitor plates, and find the direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$. For instance, at the top of the circle shown in Figure 31-4, $\vec{\mathbf{E}}$ is toward the viewer, and $\vec{\mathbf{B}}$ is to the left. The cross product $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ points down, directly to the line connecting the center of the plates. Or take the rightmost point on the circle. $\vec{\mathbf{E}}$ is again toward the viewer, and $\vec{\mathbf{B}}$ is upwards. The cross product $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ points to the left, again directly to the line connecting the center of the plates. In cylindrical coordinates, $\vec{\mathbf{E}} = E\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B\hat{\boldsymbol{\varphi}}$. The cross product $\hat{\mathbf{k}} \times \hat{\boldsymbol{\varphi}} = -\hat{\mathbf{r}}$.



(*b*) We evaluate the Poynting vector, and then integrate it over the curved cylindrical surface between the capacitor plates. The magnetic field (from Example 31-1) is $B = \frac{1}{2}\mu_0\varepsilon_0 r_0 \frac{dE}{dt}$, evaluated at $r = r_0$. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other, so $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = \frac{1}{2}\varepsilon_0 r_0 E \frac{dE}{dt}$, inward. In calculating $\iint \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}}$ for energy flow into the capacitor volume, note that both $\vec{\mathbf{S}}$ and $d\vec{\mathbf{A}}$ point inward, and that S is constant over the curved surface of the cylindrical volume.

$$\iint \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = \iint S dA = S \iint dA = SA = S 2\pi r_0 d = \tfrac{1}{2}\varepsilon_0 r_0 E \frac{dE}{dt} 2\pi r_0 d = \varepsilon_0 d \pi r_0^2 E \frac{dE}{dt}$$

The amount of energy stored in the capacitor is the energy density times the volume of the capacitor. The energy density is given by Eq. 24-6, $u = \frac{1}{2}\varepsilon_0 E^2$, and the energy stored is the energy density times the volume of the capacitor. Take the derivative of the energy stored with respect to time.

$$U = u(\text{Volume}) = \tfrac{1}{2}\varepsilon_0 E^2 \pi r_0^2 d \quad \rightarrow \quad \frac{dU}{dt} = \varepsilon_0 E \pi r_0^2 d \frac{dE}{dt}$$

We see that $\iint \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = \frac{dU}{dt}$.

43. The electric field is found from the desired voltage and the length of the antenna. Then use that electric field to calculate the magnitude of the Poynting vector.

$$E_{\text{rms}} = \frac{V_{\text{rms}}}{d} = \frac{1.00 \times 10^{-3}\,\text{V}}{1.60\,\text{m}} = \boxed{6.25 \times 10^{-4}\,\text{V/m}}$$

$$S = c\varepsilon_0 E_{\text{rms}}^2 = c\varepsilon_0 \frac{V_{\text{rms}}^2}{d^2} = (3.00 \times 10^8\,\text{m/s})(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)\frac{(1.00 \times 10^{-3}\,\text{V})^2}{(1.60\,\text{m})^2}$$

$$= \boxed{1.04 \times 10^{-9}\,\text{W/m}^2}$$

48. (*a*) The rms value of the associated electric field is found from Eq. 24-6.

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E_{\text{rms}}^2 \rightarrow E_{\text{rms}} = \sqrt{\frac{u}{\varepsilon_0}} = \sqrt{\frac{4 \times 10^{-14}\,\text{J/m}^3}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}} = 0.0672\,\text{V/m} \approx \boxed{0.07\,\text{V/m}}$$

(*b*) A comparable value can be found using the magnitude of the Poynting vector.

$$\overline{S} = \varepsilon_0 c E_{\text{rms}}^2 = \frac{P}{4\pi r^2} \rightarrow$$

$$r = \frac{1}{E_{\text{rms}}}\sqrt{\frac{P}{4\pi\varepsilon_0 c}} = \frac{1}{0.0672\,\text{V/m}}\sqrt{\frac{7500\,\text{W}}{4\pi\left(8.85 \times 10^{-12}\,\text{C}^2/\text{n}\cdot\text{m}^2\right)\left(3.00 \times 10^8\,\text{m/s}\right)}}$$

$$= 7055\,\text{m} \approx \boxed{7\,\text{km}}$$

52. From the hint, we use Eq. 29-4, which says $\mathscr{E} = \mathscr{E}_0 \sin\omega t = NBA\omega\sin\omega t$. The intensity is given, and this can be used to find the magnitude of the magnetic field.

$$\overline{S} = \frac{E_{\text{rms}}B_{\text{rms}}}{\mu_0} = \frac{cB_{\text{rms}}^2}{\mu_0} \rightarrow B_{\text{rms}} = \sqrt{\frac{\mu_0\overline{S}}{c}} \;;\; \mathscr{E} = \mathscr{E}_0 \sin\omega t = NBA\omega\sin\omega t \rightarrow$$

$$\mathscr{E}_{\text{rms}} = NA\omega B_{\text{rms}} = NA\omega\sqrt{\frac{\mu_0\overline{S}}{c}}$$

$$= (320)\pi(0.011\,\text{m})^2\,2\pi\left(810\times10^3\,\text{Hz}\right)\sqrt{\frac{\left(4\pi\times10^{-7}\,\text{N}\cdot\text{s}^2/\text{C}^2\right)\left(1.0\times10^{-4}\,\text{W/m}^2\right)}{3.00\times10^8\,\text{m/s}}}$$

$$= \boxed{4.0\times10^{-4}\,\text{V}}$$

62. (*a*) Use the $\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$ from page A-4 in Appendix A.

$$E_y = E_0\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right]$$

$$= 2E_0 \sin\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right) = 2E_0 \sin(kx)\cos(-\omega t)$$

$$= \boxed{2E_0 \sin(kx)\cos(\omega t)}$$

$$B_z = B_0\left[\sin(kx - \omega t) - \sin(kx + \omega t)\right]$$

$$= 2B_0 \sin\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right) = 2B_0 \sin(-\omega t)\cos(kx)$$

$$= \boxed{-2B_0 \cos(kx)\sin(\omega t)}$$

(*b*) The Poynting vector is given by $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$.

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = \frac{1}{\mu_0}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2E_0 \sin(kx)\cos(\omega t) & 0 \\ 0 & 0 & -2B_0 \cos(kx)\sin(\omega t) \end{vmatrix}$$

$$= \frac{1}{\mu_0}\hat{\mathbf{i}}\left[-4E_0B_0 \sin(kx)\cos(kx)\sin(\omega t)\cos(\omega t)\right] = \boxed{-\frac{1}{\mu_0}\hat{\mathbf{i}}\, E_0B_0 \sin(2kx)\sin(2\omega t)}$$

This is 0 for all times at positions where $\sin(2kx) = 0$.

$$\sin(2kx) = 0 \;\rightarrow\; 2kx = n\pi \;\rightarrow\; \boxed{x = \frac{n\pi}{2k},\ n = 0, \pm 1, \pm 2, \ldots}$$