Lect 9 The wave nature of light

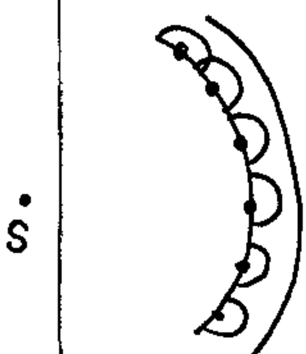
- particles (Newton) —

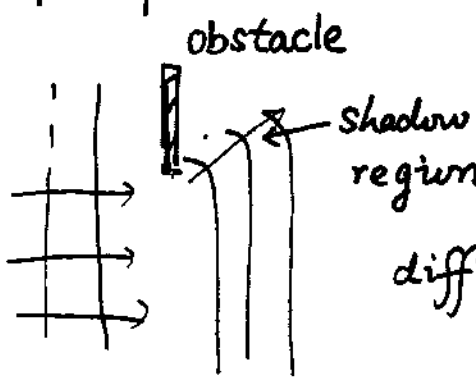
  waves (Huygens) —

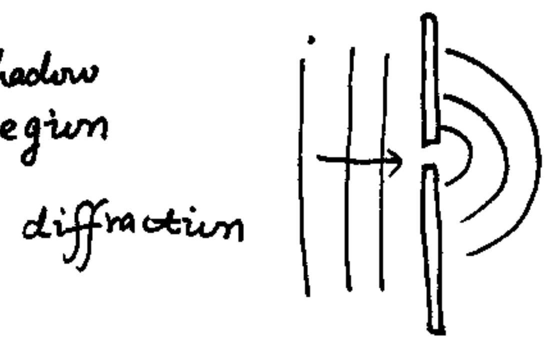
Since 1830, the interference of light has been observed → waves

what kind of waves? E & M waves (Maxwell).

- Huygen's principle: Every point on a wave front can be considered as a source of wavelets that spread out in the forward direction at the speed of the wave itself. The new wavefront is the envelope of the wavelets.



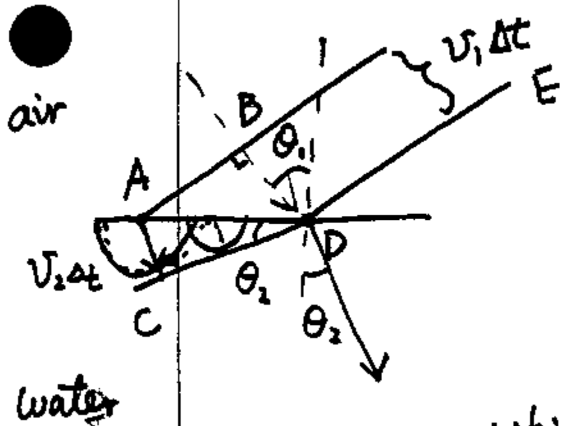




ray model of light does not explain diffraction. Normal openings and obstacles are much larger than light wave-length,

→ little refraction/ bending light

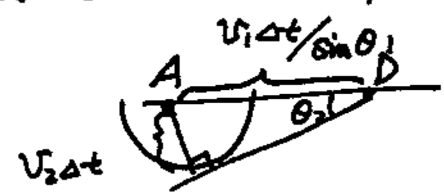
Huygen's principle and law of refraction



wave front in the air at time 0: AB

after Δt, the wave front in the air moves to DE, the normal direction perpendicular to the wave front is the wave vector, which has an incident angle θ_1.

On the other hand, the wave front in the water,



$$AD = \frac{v_1\Delta t}{\sin\theta_1}$$

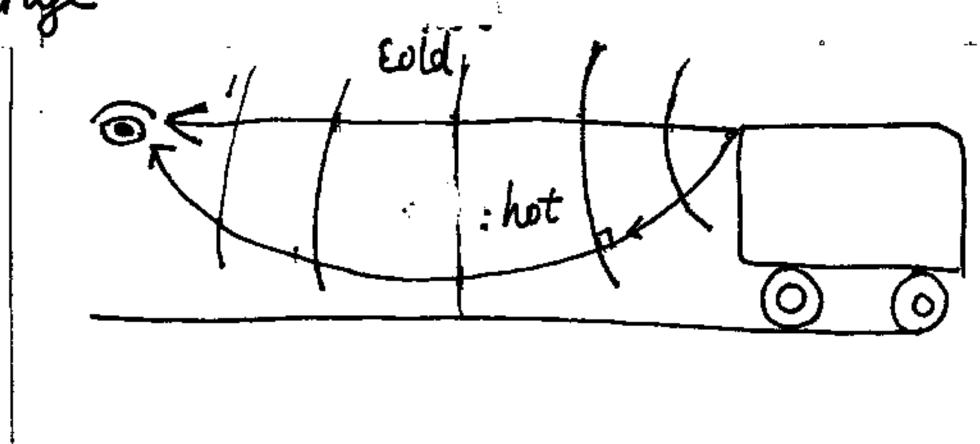
$$\frac{v_2\Delta t}{AD} = \sin\theta_2 \Rightarrow \frac{v_2}{v_1} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}$$

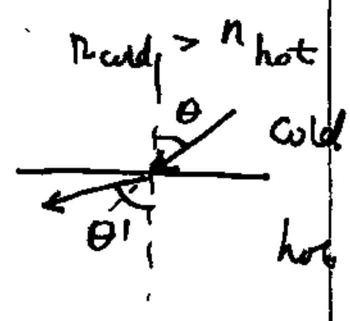
the wave length $\lambda = v/\omega \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$

$\Rightarrow \underline{\lambda_n = \lambda/n}$. The same frequency of light

$$\lambda_{water} = \frac{1}{n}\lambda_{air} = \frac{3}{4}\lambda_{air}.$$

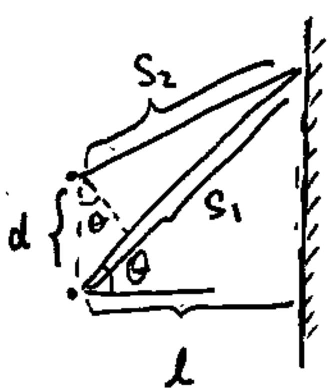
mirage

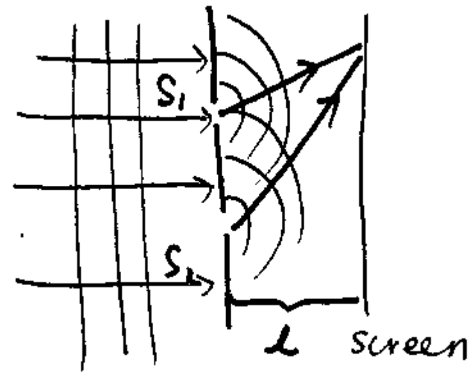




$\theta' > \theta$

★ Interference — Young's double-slit experiment





$S_1 - S_2 = d\sin\theta$. The phase differente between two paths is $\frac{d\sin\theta}{\lambda}\cdot 2\pi$. we will have anstructin inteference if $\frac{d\sin\theta}{\lambda}\cdot 2\pi = m\cdot 2\pi$ which is bright.

or $\frac{d\sin\theta}{\lambda}\cdot\cancel{2\pi} = m + \frac{1}{2} \longrightarrow$ destructive interference — dark

Vibration 1 $\psi_1 = \cos(\omega t + \varphi_1)$

vibration 2 $\psi_2 = \cos(\omega t + \varphi_2)$

$\left.\begin{matrix} \psi_1 = \cos(\omega t + \varphi_1) \\ \psi_2 = \cos(\omega t + \varphi_2) \end{matrix}\right\} \Rightarrow$ if $\Delta\varphi = \varphi_2 - \varphi_1 = 2n\pi \Rightarrow 2\cos(\omega t + \varphi_1) = \psi_1 + \psi_2$

if $\Delta\varphi = \pi \Rightarrow \psi_1 + \psi_2 = 0$.

Comment: $d\sin\theta = m\lambda \Rightarrow |m|\lambda < d \Rightarrow$ $\underbrace{\text{the value}}_{\text{number of}}$ of m is finite.

ex: a screen containing two slits 0.1 mm apart, and is 1.2 m from the viewing screen. Light of wavelength $\lambda = 500\,nm$. What is the distance between the bright fringes?

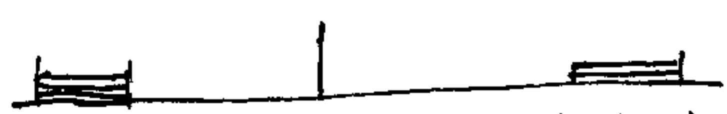
$d\sin\theta = m\lambda \Rightarrow d\cos\theta\,\Delta\theta \simeq \Delta m\,\lambda \approx d\,\Delta\theta = \Delta m\,\lambda$

the distance on the screen $x = l\tan\theta \Rightarrow \Delta x \approx l\,\Delta\theta$

$\Rightarrow \boxed{\Delta x \simeq \frac{l}{d}\lambda}$ for the adjacent bright fringes at $\theta \simeq 0$.

- increase λ but keep d, l the same $\Rightarrow$ patterns spreads out
- Keep λ unchanged, but enlarge d $\Rightarrow$ lines move closer.

- how about use white light:
  - The zeroth order fringe is white
  - The first order fringe develope dispersion

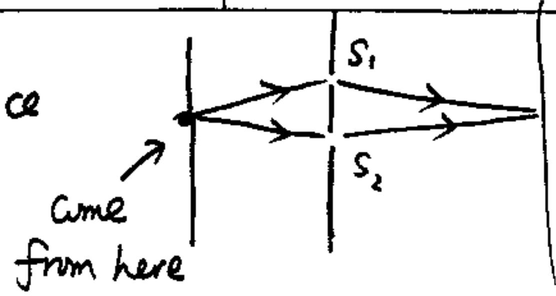


ex: $d = 0.5\,mm$, $l = 2.5\,m$ $\Rightarrow$ the first order friege red at $x = 3.5\,mm$
violet $x = 2.0\,mm$

$\Rightarrow \lambda = \frac{d\,x}{l} \Rightarrow \lambda_{red} = \frac{0.5}{2.5\times10^{3}} \cdot 3.5\times10^{-3} = 0.7\,\mu m$

$\lambda_{vio} = \frac{0.5}{2.5\times10^{3}} \times 2\times10^{-3} = 0.4\,\mu m$.

coherence v.s incoherent source



interference can only be seen from coherent source

- Interference intensity

$$E_1 = E_0 \sin\omega t \qquad \text{where } \delta = \frac{d\sin\theta}{\lambda}\cdot 2\pi$$
$$E_2 = E_0 \sin(\omega t + \delta)$$

$$\Rightarrow E_{tot} = E_1 + E_2 = 2E_0\left(\sin \omega t + \frac{\delta}{2}\right)\cos\frac{\delta}{2}$$

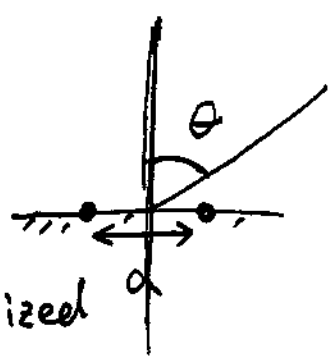
$$\Rightarrow I \propto E_{tot}^2 \propto \cos^2\frac{\delta}{2} \Rightarrow I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

$$\sin\theta \approx \theta \simeq \frac{x}{l} \Rightarrow \frac{I}{I_0} = \cos^2\left(\frac{\pi d}{\lambda l} y\right)$$

Antenna radiation:

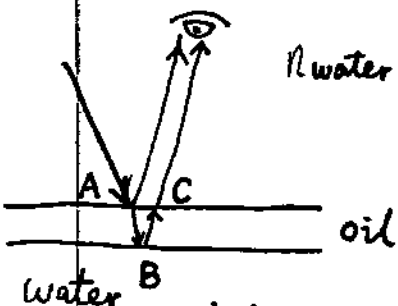
2-antena

If $d = \lambda \Rightarrow I/I_0 = \cos^2(\pi \sin\theta)$ $\theta = 0$, 90, 180, 270° maximized



$d = \frac{\lambda}{2} \Rightarrow I/I_0 = \cos^2\left(\frac{\pi}{2}\sin\theta\right)$ maximized $\theta = 0, 180°$

Lect 10. Interference in thin films



$n_{water} > n_{oil} > n_{air}$

one beam of light. splits into two $\begin{cases} A \rightarrow \text{eyes} \\ ABC \rightarrow \text{eyes} \end{cases}$

in the path difference ABC =

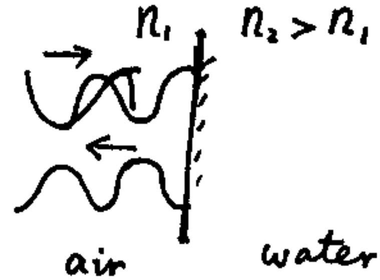
integer wavelength — cntructive inteference

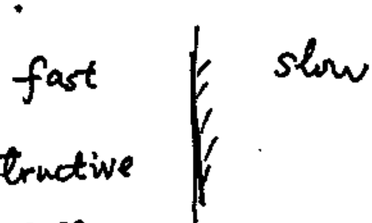
half integer wavelength — destructive inteference.

If you view from a perticular angle, the path difference ABC will match a particular wevelength. If you change the viewing angle, another color will/light be matched.

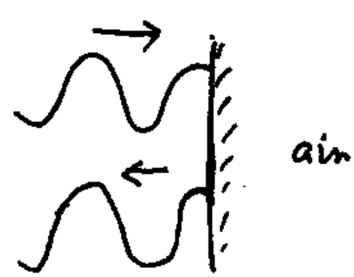
★ half-wave loss:

a beam of light reflected by a material with larger n than that of the material in which it is traveling, changes the phase by 180°.





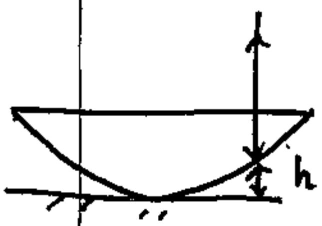
destructive inteference between incoming & reflected waves,



slow fast

constructive interference between incoming & reflected waves.

Ex Newton ring

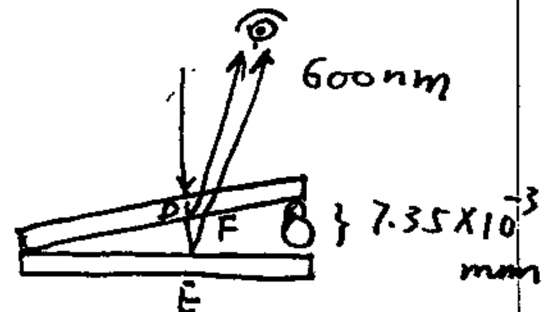


air gap

$$\Delta\varphi = \frac{2h}{\lambda}\cdot 2\pi + \pi \quad \begin{cases} \text{the first reflection no phase flip} \\ \text{the second one: with } \pi\text{-flip} \end{cases}$$

$\Rightarrow$ the center / the touching point is black.

Ex the film of air — wedge shaped



the phase difference = DEF + π-phase

$2t = m\lambda \leftarrow t$ is the thickness of the air gap

dark bands

$t < 7.35\times10^{-3}$ mm

$$\Rightarrow \quad m < \frac{2t}{\lambda} = \frac{14.7\times10^{-3}\times10^{-3}}{6\times10^{2}\times10^{-9}} = \frac{147}{6} = 24.5$$

24
6) 147
12
27
24
30

at $t=0$, it starts with a dark $m=0$

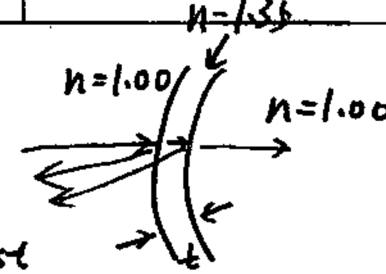
$\vdots$

dark $m=24$

bright $m=24.5$

☆ If the glass plates are not exactly flat at the order of light wavelength, the pattern will not be straight lines. Similarly, if the lens is not spherical, the Newton rings will not be circular —— precision measurement

- Soap bubble



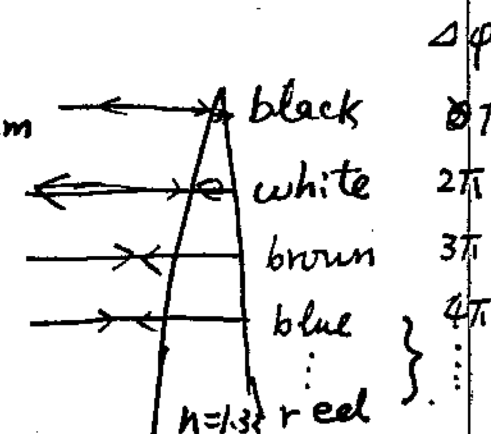
a soap bubble appears green ($\lambda = 540\,nm$) at the point on its front surface nearest to the viewer. what's the smallest thickness?

$$\frac{2t}{\lambda n} \cdot 2\pi + \pi = m \cdot 2\pi, \quad \text{set } m=1 \Rightarrow \frac{2t}{\lambda n} = \frac{1}{2}$$

$$\Rightarrow t = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = \frac{540\,nm}{4 \times 1.35} \approx 100\,nm.$$

$$m=2 \Rightarrow t = \left(\frac{1}{2} + \frac{1}{4}\right)\frac{\lambda}{n} = 300\,nm.$$

- Colors in a thin soap film $t < 50\,nm$



① at the top $2t \simeq 0$, the phase difference $\Delta\varphi = \frac{2t}{\lambda n} \cdot 2\pi + \pi \sim \pi$ for all the light. we see black.

② below the black area, where

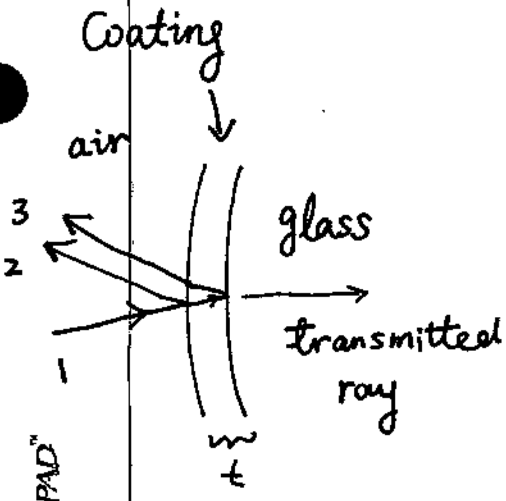
$2t \sim \frac{\lambda}{2n}$ where λ from 400nm to 600nm

$\Rightarrow$ when $t \sim 100\,nm$, which nearly match the constructive inteference for all the wavelength

we see bright.

③ Then the $\underbrace{\text{second darkness}}_{\text{browness}}$, — a little dispersion

④ $2t \sim \left(1 + \frac{1}{2}\right)\left(\frac{\lambda}{n}\right)$ then the dispersion of the constructive inteference becomes noticble — rainbow



both reflections have half-wave loss.

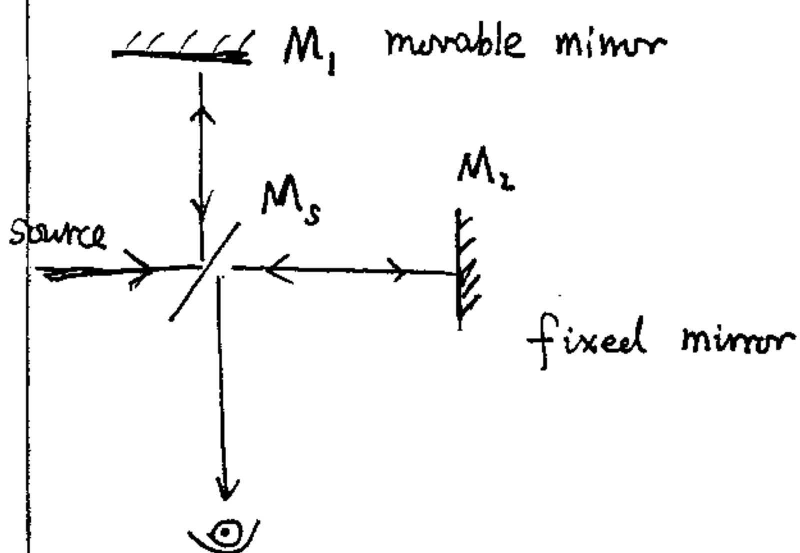
the light distance difference is

$$\frac{2t}{\lambda_n} = \left(\frac{1}{2} + n\right)$$

$$\Rightarrow t_{min} = \frac{\lambda_n}{4} = \frac{\lambda}{4n} .$$ for 550nm $n = 1.5$

$$\rightarrow t \approx 99.6 \text{ nm}$$

Michelson interferometer



a movement of M_1 at $\frac{\lambda}{4}$, produces a light distance difference of $\frac{\lambda}{2}$, charge constructive $\leftrightarrow$ destructive interference