Review chapter 31

☆ Maxwell equation

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \longrightarrow \nabla\cdot\vec{E} = \rho/\varepsilon_0$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \longrightarrow \nabla\cdot\vec{B} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \longrightarrow \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

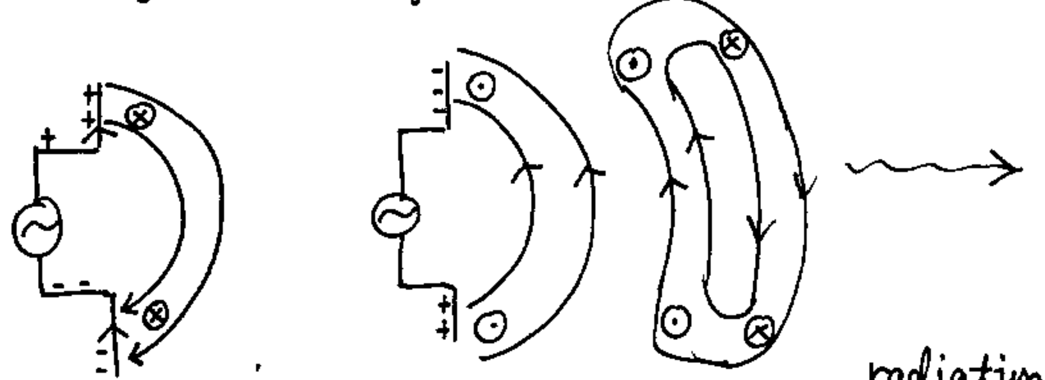
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \frac{1}{c^2}\frac{d\Phi_E}{dt} \longrightarrow \nabla\times\vec{B} = \mu_0\vec{j} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$$

introduction of displacement current : charge conservation

$$\nabla\cdot(\nabla\times\vec{B}) = \mu_0\left[\nabla\cdot\left(\vec{j} + \frac{\partial}{\partial t}\varepsilon_0\vec{E}\right)\right] = \mu_0\underbrace{\left[\nabla\cdot\vec{j} + \frac{\partial}{\partial t}\rho\right]} = 0$$

$$\Leftrightarrow \quad \nabla\cdot\vec{j} + \frac{\partial}{\partial t}\rho = 0 \quad \text{or} \quad \frac{\partial}{\partial t}Q + \oiint d\vec{A}\cdot\vec{j} = 0$$

☆ generation of E-M wave through charge oscilation



propagation direction

radiation field $E \propto \frac{1}{r}$

$B \propto \frac{1}{r}$

* propagation of E & M field

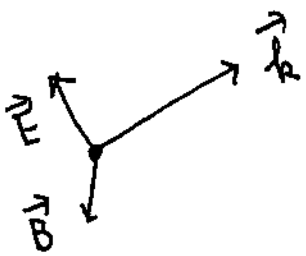
$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \vec{E} = 0$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\vec{E} \qquad \nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\vec{E} = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$
$$\vec{B} = \vec{B}_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{E}_0 \perp \vec{k}$$
$$\vec{B}_0 \perp \vec{k}$$
$\Rightarrow$ transverse wave

$$\vec{B}_0 = \frac{1}{c}\hat{k} \times \vec{E}_0$$



$$\vec{E}_0 = -\frac{(\mu_0\varepsilon_0)^{-1}}{c}\hat{k} \times \vec{B}_0 = -c\,\hat{k} \times \vec{B}_0$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

* Poynting vector

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} = \varepsilon_0 E^2$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$\overline{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{E_0 B_0}{2\mu_0} = \frac{E_{rms} B_{rms}}{\mu_0}$$

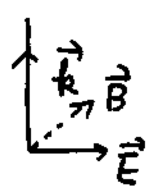
* radiation pressure

$$\Delta p = \frac{\Delta W}{c} \text{ (absorb)} \quad \text{or} \quad \frac{2\Delta W}{c} \text{ (reflect)}$$

$$\overline{\mathbb{P}} = \frac{\overline{S}}{c} \qquad \overline{\mathbb{P}} = \frac{2\overline{S}}{c}$$

§ Polarization

① linear polarization



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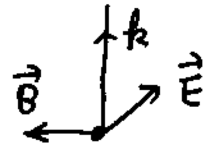
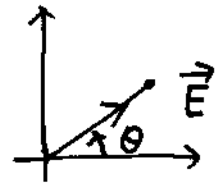


② circular polarization

$$\vec{E}_0 = \frac{E_0}{\sqrt{2}}\left[\hat{x} + i\hat{y}\right] \Rightarrow \vec{E}(\vec{r},t) = \frac{E_0}{\sqrt{2}}\left[\hat{x} + i\hat{y}\right] e^{i(kz-\omega t)}$$

or
$$E_x(r,t) = \frac{E_0}{\sqrt{2}}\cos(kz-\omega t)$$
$$E_y(r,t) = -\frac{E_0}{\sqrt{2}}\sin(kz-\omega t)$$



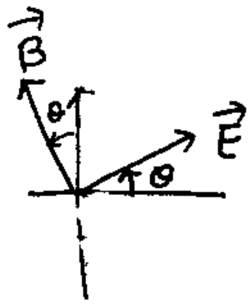
$$\underline{E = \sqrt{E_x^2+E_y^2} = E_0 \ , \ \theta = \omega t - kz \ .}$$ Right-hand

~~B~~

$$\vec{B}_0 = \frac{1}{c}\hat{k}\times\vec{E}_0 = B_0\left[\hat{y} - i\hat{x}\right], \quad B_0 = \frac{E_0}{c}$$

$$\Rightarrow \vec{B}(\vec{r},t) = \frac{B_0}{\sqrt{2}}\left(\hat{y} - i\hat{x}\right) e^{ikz-\omega t}$$

or
$$B_x(r,t) = \frac{B_0}{\sqrt{2}}\sin(kz-\omega t)$$
$$B_y(r,t) = \frac{B_0}{\sqrt{2}}\cos(kz-\omega t)$$



$\Rightarrow$ $\vec{E} \perp \vec{B}$. but rotating

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{E_0 B_0}{2\mu_0}\hat{k} \quad \text{const with time.}$$

left handed $\quad \vec{E}_0 = \frac{E_0}{\sqrt{2}}\left(\hat{x} - i\hat{y}\right), \quad \vec{B}_0 = B_0\left[\hat{y} + i\hat{x}\right]$