Lect 10 — symmetry in crystals (I): Bravis lattices

\# 32 crystalline point groups 11 proper + 11 (improper w/ inversion + 10 (improper w/o inversion)

7 crystal systems:

$$
\begin{tikzcd}
\text{Cubic } (O_h) \arrow[r] \arrow[dr] & \text{tetragonal } (D_{4h}) \arrow[rr] & & \text{Orthorombic } (D_{2h}) \arrow[dd] \\
& \text{triagonal } (D_{3d}) & \text{hexagonal } (D_{6h}) \arrow[l] \arrow[ur] & \\
& & \text{triclinic } (C_i) \arrow[r, shift left] & \text{monoclinic } (C_{2h}) \arrow[l, shift left]
\end{tikzcd}
$$

14 Bravis lattices

230 space groups (73 symmorphic + 157 non-symmorphic)

We will explain the above numbers.

§1 Affine transformation — Euclidean group

$$\vec{r} \longrightarrow R\vec{r} + \vec{\alpha} = g(R, \vec{\alpha})\, \vec{r} = \vec{r}\,'$$

$$\begin{pmatrix} \vec{r}\,' \\ 1 \end{pmatrix} = \left(\begin{array}{c|c} R & \vec{\alpha} \\ \hline 0 & 1 \end{array}\right) \begin{pmatrix} \vec{r} \\ 1 \end{pmatrix}$$

For free space, R takes all the $O(3)$ operations,

$\vec{\alpha}$ any translation vector

Then $g(R\vec{\alpha})\, g(R'\vec{\beta}) \to \begin{pmatrix} R & \vec{\alpha} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R' & \vec{\beta} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} RR' & R\vec{\beta} + \vec{\alpha} \\ 0 & 1 \end{pmatrix}$

$= g(RR', \vec{\alpha} + g\vec{\beta})$

exercise:

$$[g(R,\vec{\alpha})]^{-1} = g[R^{-1}, -R^{-1}\vec{\alpha}]$$

The set of $\{g(R,\vec{\alpha})\}$ form the symmetry operations of 3D space, denoted as the Euclidean group E^3 (Euclidean geometry studies the invariant properties under translation, rotation, reflection, inversion etc),

Erlangen program: Characterization of geometries according to group theory.

⊛ When $\vec{\alpha} = 0$, $g(R, 0)$ — point operation

$R = E$ $\quad g(E, \vec{\alpha})$ — translation group

$$g(E\,\vec{\alpha})\,g(E\,\vec{\beta}) = g(E\,\vec{\beta})\,g(E\,\vec{\alpha})$$

translation group is an Abelean group.

Space group is Euclidean group's discrete subgroup. (but infinite).

§ 2: space group — crystal symmetry group

① Discrete translation: crystal (3D) is a periodical structure.

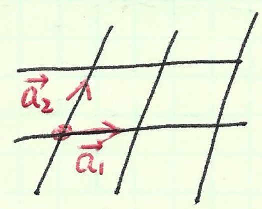
It's invariant under the discrete translation symmetry

$$\vec{r} \to \vec{r}\,' = T(l)\,\vec{r} = \vec{r} + \vec{l}, \text{ where } \vec{l} = \vec{a}_1 l_1 + \vec{a}_2 l_2 + \vec{a}_3 l_3 .$$

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ are primitive vectors, (l_1, l_2, l_3) are integer

$\{T(l)\}$ form the lattice translation group, and

the set of $\vec{l}$ form a lattie



② Space group: $\boxed{\text{Crystal}}$ may have other symmetries

$\vec{r}\,' = g(R, \vec{\alpha})\,\vec{r}$, where the crystal is invariant under

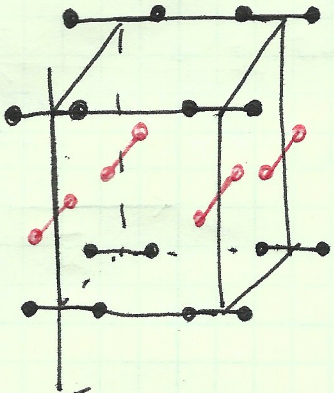
the affine transformation $g(R, \vec{\alpha})$.

① If $\vec{\alpha} = 0$, $g(R, 0) = R$, which is a proper or improper point operation

② If $R = E$, $\vec{\alpha}$ has to be integer coordinates on the basis of primitive vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

Actually, $\vec{\alpha}$ can be fractional. For example, in the case of screw rotation and glide reflection.

① Screw rotation



$\leftarrow$ screw axis

$\boxed{\begin{array}{l}\text{crystal is different from the lattice} \\ \text{You can put decorations on the lattice}\end{array}}$

"$\pm$" represent inversion

4 represent 4-fold axis (4-fold = screw)

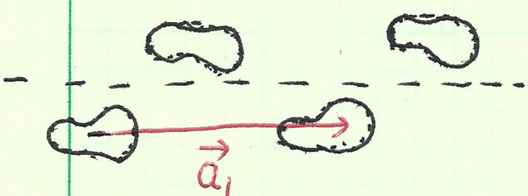
$00\tfrac{1}{2}$ $\leftarrow$ fractional translation

$P \pm 4\ 00\tfrac{1}{2}$

$\nearrow$ primitive lattice

$\boxed{\begin{array}{l} g(R,\vec{\alpha}): \ R = 4 \text{ fold rotation} \\ \qquad\qquad \vec{\alpha} = (00\tfrac{1}{2}) \\ \text{but } R \text{ itself is not a symmetry of the} \\ \text{crystal} \end{array}}$

② glide reflection



$\boxed{\begin{array}{l} g(\sigma, \tfrac{\vec{a}_1}{2}) \quad \sigma: \text{reflection with respect to} \\ \qquad\qquad\qquad \text{the dashed line} \\ \text{and } \sigma \text{ itself} \\ \quad \text{is not a symmetry of the crystal.} \end{array}}$

☆ But R is indeed the symmetry of the lattice formed by

$$\vec{l} = l_1\vec{a}_1 + l_2\vec{a}_2 + l_3\vec{a}_3 .$$

$$\boxed{\begin{array}{l} g(R\vec{\alpha})\, T(\vec{l})\, g^{-1}(R,\vec{\alpha}) \\ \quad = T(R\vec{l}) \end{array}} = g(R,\vec{\alpha})\, T(\vec{l})\, g(R^{-1}, -R^{-1}\vec{\alpha}) = T(R\vec{l})\, g(R\,\vec{\alpha})\, g(R^{-1}, -R^{-1}\vec{\alpha}$$

hence $R\vec{l}$ is also an element of the translation group.

i.e. the lattice is invariant under the $\{R\}$, operations.

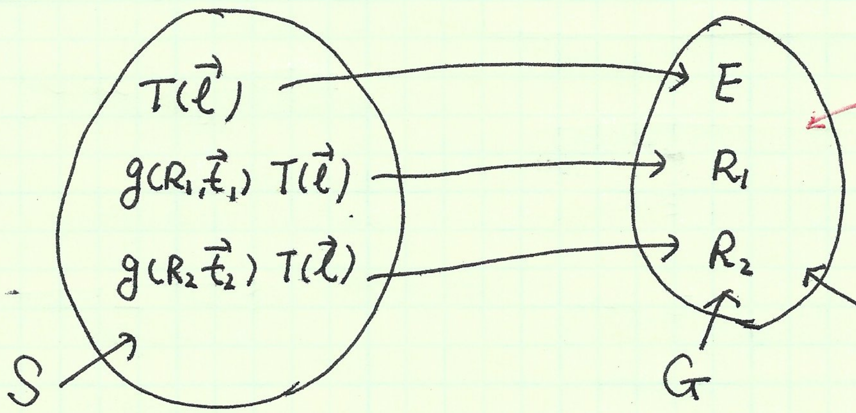
The above relation also shows that $T(\vec{l})$ form a normal subgroup of the space group $\{g(R,\vec{\alpha})\}$ denoted by S.

☆ quotient group — crystalline point group

Taking $T(\vec{l})$ as the kernel, we study the quotient group.

Assume $\vec{\alpha}_i = \vec{l}_i + \vec{t}_i$, $0 \le t_i < 1$, l_i integer coordinates

Then $\boxed{g(R,\vec{\alpha}) = T(\vec{l})\, g(R,\vec{t})}$ ← define the coset.



crystalline point group

$$\boxed{G = S/T}$$

G often is not the subgroup of space group, not the symmetry of the cryst

only when $\vec{t}=0$, R is a symmetry of the crystal.

when $\vec{t} \neq 0$, $g(\vec{R},\vec{t})$ is non-symmphoric symmetry.

If S can be represented as $g(R,0)\,T(\vec{l})$, then it's called symmorphic space group. Otherwise, if fractional $\vec{t}$ has to appear, it's non-symmorhic.

§ Crystalline point groups — constraints from translation symmetry

The crystalline point groups maintain the lattice invariant. Let us use $\vec{a}_1, \vec{a}_2, \vec{a}_3$ as a basis to form a representation. Since

$$R\vec{l} = \vec{l}' \Rightarrow \quad R\vec{a}_i = \vec{a}_j \, m_{ji} \quad \text{such that } m_{ji} \text{ are integers.}$$

then the character of $R \Rightarrow \mathrm{tr}\, R = \underset{\uparrow}{\pm}\left(1 + 2\cos\frac{2\pi}{N}\right) = \underset{\uparrow}{\sum m_{ii}}$

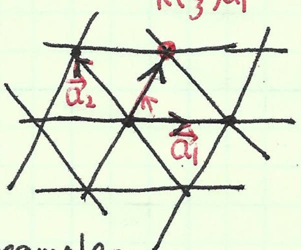
if improper — integer

hence $\boxed{2\cos\frac{2\pi}{N} = -2, -1, 0, 1, 2}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$\boxed{2, \; 3, \; 4, \; 6, \; 0\text{-fold axes.}}$

$R(\frac{\pi}{3})\vec{a}_1$



$$R(\tfrac{\pi}{3})\begin{pmatrix}\vec{a}_1\\ \vec{a}_2\end{pmatrix} = \begin{pmatrix}1 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\vec{a}_1\\ \vec{a}_2\end{pmatrix}$$

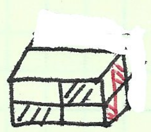
example:

crystalline point groups — 11 proper ones

$\boxed{\begin{matrix} C_1 \; C_2 \; C_3 \; C_4 \; C_6 \\ D_2 \; D_3 \; D_4 \; D_6 \\ T \; O \end{matrix}} \leftarrow 11$

☆ $C_{1,2,3,4,6}$ are denoted by $n = 1, 2, 3, 4, 6$ — n-fold axis.

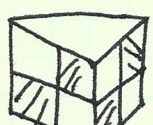
☆ D_2



2 2 2 or 2 2'

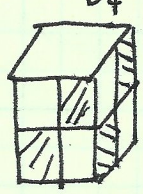
international

D_3



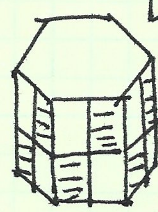
3 2 or 3 2'

D_4



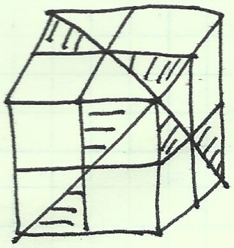
4 2 2 or 4 2'

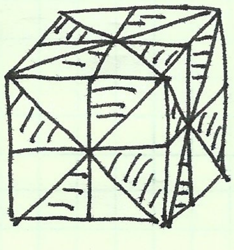
D_6



6 2 2 or 6 2'

we denote all other axes (2-fold)





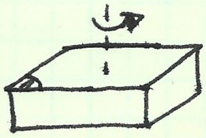
T, 3′2 O (3′42″)

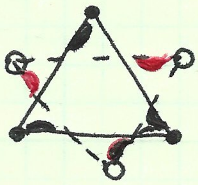
improper with inversion

(proper ⊗ {E, I})

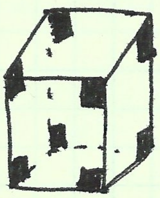
$$\boxed{\begin{array}{l} S_2 = C_i,\ C_{2h},\ S_6 = C_{3i},\ C_{4h},\ C_{6h} \\ D_{2h},\ D_{3d},\ D_{4h},\ D_{6h} \\ T_h,\ O_h \end{array}} \leftarrow 11$$

C_i, $\bar{1}$, C_{2h} 2/m, or ±2, $S_6 = C_{3i}$ $\bar{3}$



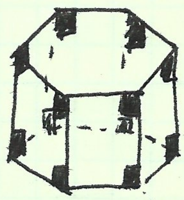


C_{4h}



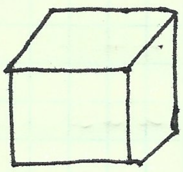
4/m, or ±4

C_{6h}



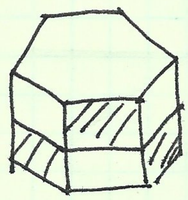
6/m, or ±6

D_{2h}

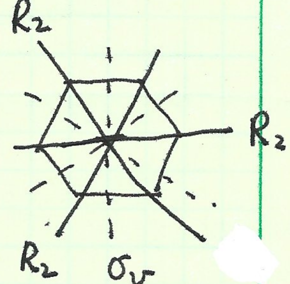


2/m m m, or

±2 2′



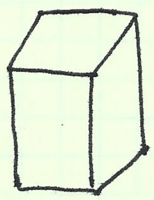
D_{3d}



$\bar{3}\,\frac{2}{m}$

$\bar{3}$ m

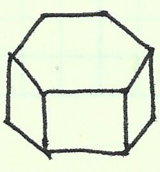
D_{4h}



4/m m m

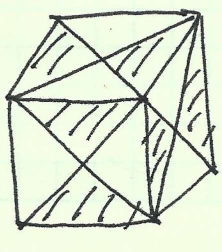
±4, 2′

D_{6h}



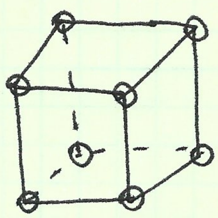
6/m m m

±6 2′



T_h ($\bar{3}$′22′)

$\frac{2}{m}$3



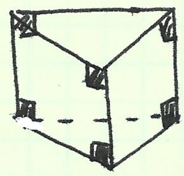
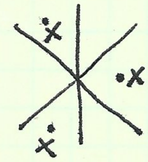
O_h ($\bar{3}$′42″)

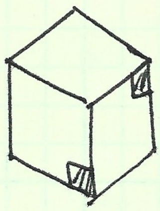
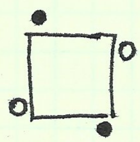
$\frac{4}{}$ 3 $\frac{2}{}$

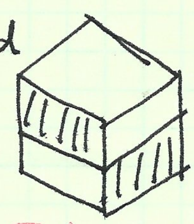
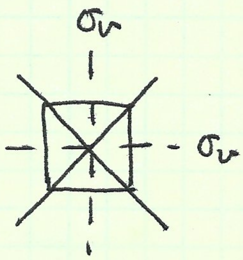
improper operations without inversion

C_h, C_{3h}, S_4
C_{2v} C_{3v} C_{4v} C_{6v} ← 10
D_{3h}, D_{2d}, T_d

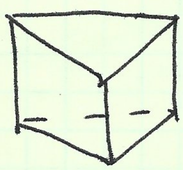
C_h : m, or $\bar{2}$

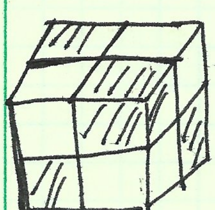
C_{3h}  $\bar{6}$  (Wierd! strange convention)

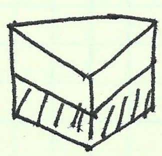
S_4   $\bar{4}$

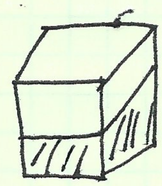
D_{2d}  

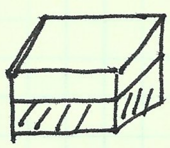
$\bar{4}2'$ or $\bar{4}2m$

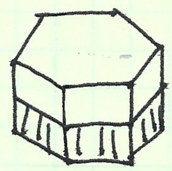
D_{3h}  $\bar{6}\,2'$ or $\bar{6}2m$

 T_d or $3'\bar{4}2''$ or $\bar{4}3m$

 C_{3v} or $3\,\bar{2}'$, $3m$

 C_{4v} or $4\bar{2}'$, $4mm$

 C_{2v} $2mm$

 C_{6v} $6mm$

§ Crystal systems

Bravis lattice: An array of points (feature less) that are expressed as $\vec{l} = l_1\vec{a_1} + l_2\vec{a_2} + l_3\vec{a_3}$. A crystal is said to belong to a given Bravis lattice if the crystal is invariant under $g(E, \vec{l})$, or $T(\vec{l})$.

holohedry: The point group of the Bravis lattice.

Bravis lattice has inversion symmetry, hence, we need to pick up among

$C_i, C_{2h}, C_{3i}, C_{4h}, C_{6h}, D_{2h}, D_{3d}, D_{4h}, D_{6h}, T_h, O_h$.

Another property of holohedry is that: If it contains a n-fold axis C_n with $(n \geq 3)$, then it also possesses a vertical mirror plane passing the axis. (The proof is omitted, actually it's not so easy).

We need to exclude those without vertical reflection plane, i.e.

$S_6 (C_{3i})$, C_{4h}, C_{6h}, and T_h (reflection plane does not pass the 3-fold axis).

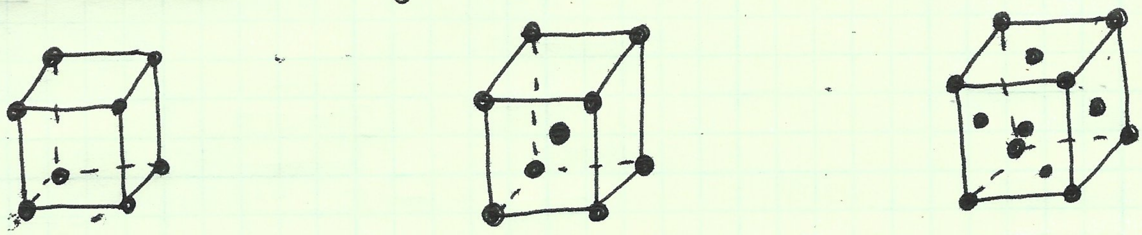
Then we only have

$$\boxed{C_i, \quad C_{2h}, \quad D_{2h}, \quad D_{4h}, \quad D_{3d}, \quad D_{6h}, \quad O_h}$$

Hence, we have 7 crystal systems. Bravis lattices possessing the same holohedry belong to the same crystal systems.

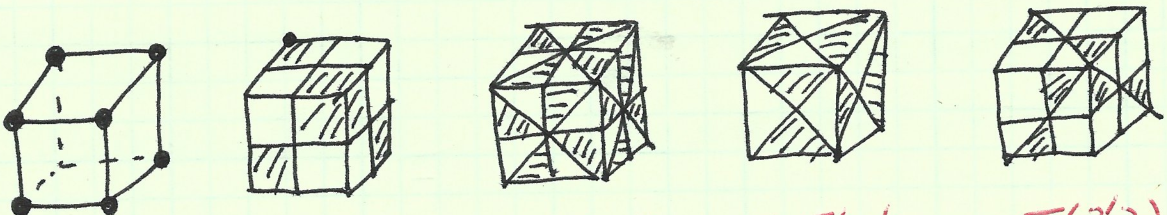
C_i: triclinic	C_{2h}: monoclinic	D_{2h}: Orthorombic	
D_{4h}: tetragonal	O_h: octahedral	D_{3d}: triagonal	D_{6h}: hexagonal

① First consider the cubic systems. There are three Bravis lattices with the cubic (O_h) symmetry.



P- (primitive) or simple cubic (SC) | I- body centered (bcc) | F- (face-centered fcc)

Although the lattice has O_h symmetry, the crystal may not have. We can choose the following decorations.



$O_h(\bar{3}'42'')$ | $T_d(3'\bar{4}2'')$ | $O(3'42'')$ | $T_h(\bar{3}'22')$ | $T(3'2)$

Hence we can define symmorphic cubic space groups

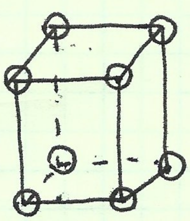
$P\bar{3}'42''$	$I\bar{3}'42''$	$F\bar{3}'42''$
$P3'\bar{4}2''$	$I3'\bar{4}2''$	$F3'\bar{4}2''$
$P3'42''$	$I3'42''$	$F3'42''$
$P\bar{3}'22'$	$I\bar{3}'22'$	$F\bar{3}'22'$
$P3'2$	$I3'2$	$F3'2$
		$T(\vec{l})$ $g(R,0)$

$g(R,0)\,T(\vec{l})$

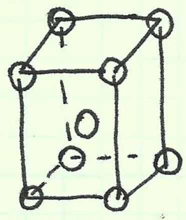
← 15

② From the cubic symmetry, we can deform it to the tetragonal symmetry. There are only two Bravis lattices

$\alpha=\beta=\gamma=90^\circ$
$a=b\neq c$

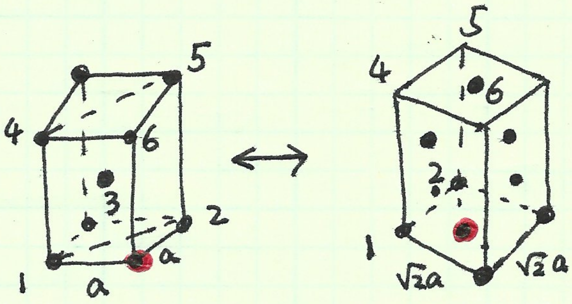


P



I

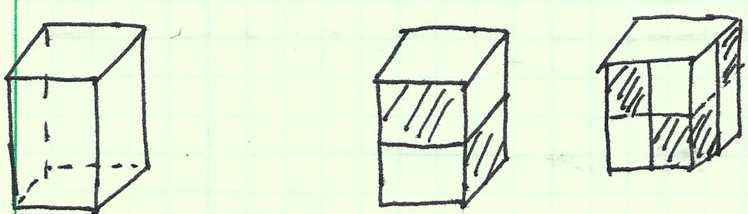
FCC in tetragonal lattice is the same as I



bcc ≡ fcc

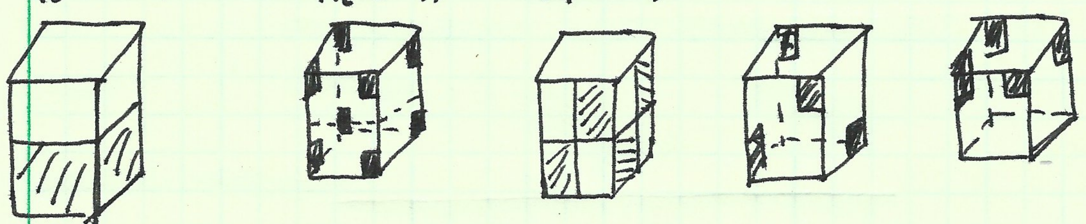
The point group symmetries

D_{4h} $(\pm 42')$, D_{2d} $(\bar{4}2')$ $(\bar{4}2'')$



(two different choices of proper axes / reflection planes)

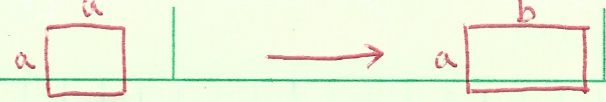
C_{4v} $(4\bar{2}')$ C_{4h} (± 4) D_4 $(42')$ S_4 $(\bar{4})$ C_4 (4)



We can combine the P and I (translation symmetry) and the point group symmetries. We have

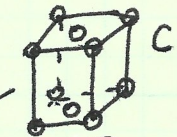
$P\pm42'$	$P\bar{4}2'$,	$P\bar{4}2''$,	$I\pm42'$,	$I\bar{4}2'$	$I\bar{4}2''$
$P4\bar{2}'$	$P\pm4$	$P42'$	$I4\bar{2}'$,	$I\pm4$	$I42'$
$P\bar{4}$	$P4$		$I\bar{4}$	$I4$	

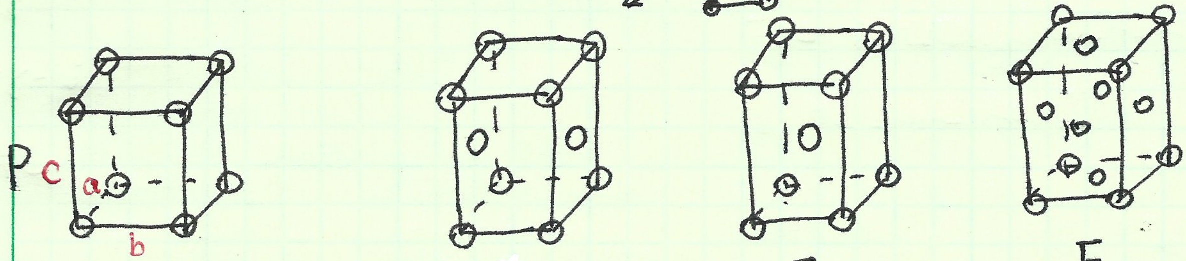
16 tetrahedral (symmorphic) space groups
crystal system



③ From tetragonal symmetry ⟶ orthorombic symmetries

There are four Bravis lattices



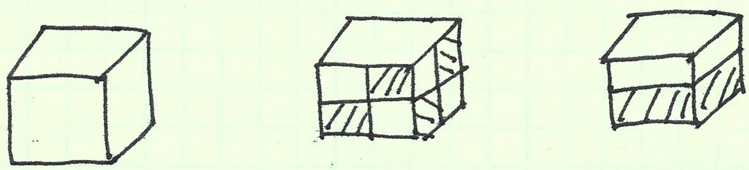


$a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$ A, B, I F

(exercise: convince yourself that bcc and fcc are not equivalent any more, and the base centered is also independent).

the point group symmetries

D_{2h} $(\pm 2 2')$ D_2 $(2 2')$ C_{2v} $(2 \bar{2}')$



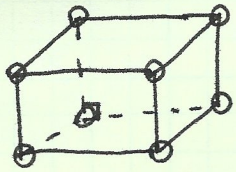
For D_{2h} and D_2, the A B C - base centered lattices have no differences.
But for C_{2v}, A and B have no difference, but C is different.

we have

$P \pm 2 2'$	$C \pm 2 2'$	$I \pm 2 2'$	$F \pm 2 2'$
$P\, 2 2'$	$C\, 2 2'$	$I\, 2 2'$	$F\, 2 2'$
$P\, 2 \bar{2}'$	$C\, 2 \bar{2}'$	$I\, 2 \bar{2}'$	$F\, 2 \bar{2}'$
	$A\, 2 \bar{2}'$		

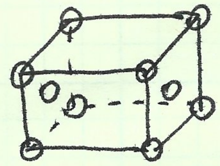
13 orthorombic symmorphic space groups

4 monoclinic — two different Bravais lattices



$a \neq b \neq c$

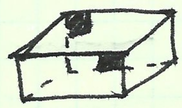
$\alpha = \beta = 90°$



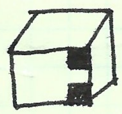
A-type base

(exercise: Show that F and I-type lattices are not indepedent, They can be reduced to the A-type by a suitable choice of basis vectors).

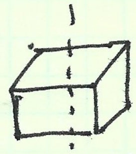
Point group symmetries:



$C_2\ (2)$



$C_h\ (\bar{2})$

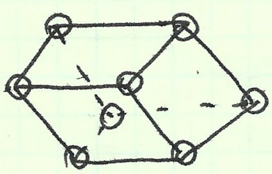


$C_{2h}\ (\pm 2)$

Hence

P2	A2
P$\bar{2}$	A$\bar{2}$
P$\pm$2	A$\pm$2

5 · triclinic : point group $C_1\ (1)$, $C_i\ (\bar{1})$

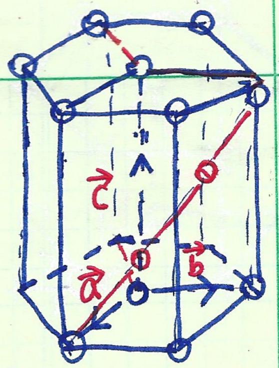


$a \neq b \neq c$

$\alpha \neq \beta \neq \gamma$

$\boxed{P1 \text{ and } P\bar{1}}$

- triagonal crystal system



3R

$a = b \neq c$

$\alpha = \beta = 90°, \gamma = 120°$

rhombohedral Bravis lattice

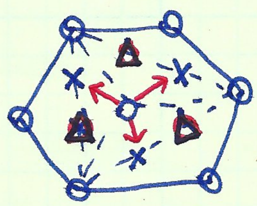
– centered hexagonal

extra points $\frac{1}{3}\vec{a} + \frac{2}{3}(\vec{b}+\vec{c})$

$\frac{2}{3}\vec{a} + \frac{1}{3}(\vec{b}+\vec{c})$

and their symmetric partners

top view



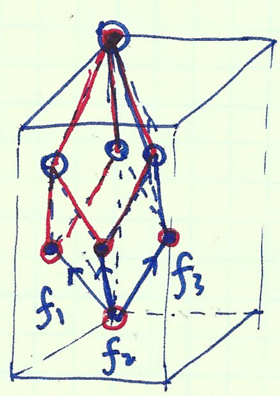
o – 1st layer

x – 2nd layer

Δ – 3rd layer

If we use $\boxed{\begin{aligned} \vec{f}_1 &= \frac{2}{3}\vec{a} + \frac{\vec{b}}{3} + \frac{\vec{c}}{3} \\ \vec{f}_2 &= -\frac{\vec{a}}{3} + \frac{\vec{b}}{3} + \frac{\vec{c}}{3} \\ \vec{f}_3 &= -\frac{1}{3}\vec{a} - \frac{2}{3}\vec{b} + \frac{\vec{c}}{3} \end{aligned}}$

then rhombohedral (R)

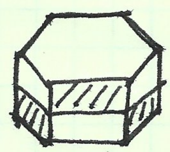


$|f_1| = |f_2| = |f_3|$

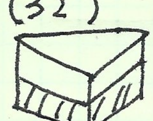
$\alpha = \beta = \gamma$

point group symmetries

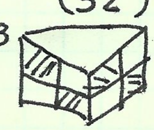
D_{3d} ($\bar{3}2'$)



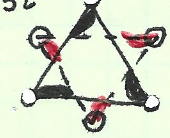
C_{3v} ($3\bar{2}'$)



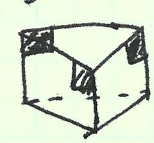
D_3 ($32'$)



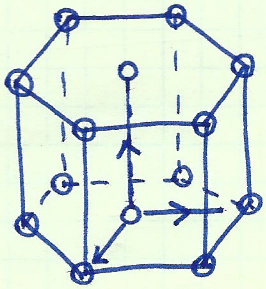
C_{3i} ($\bar{3}$)



C_3 (3)



$\boxed{R\bar{3}2',\ R3\bar{2}',\ R32',\ R\bar{3},\ R3}$



P

$P3 \quad P\bar{3} \quad P32', \; P32'', \; P3\bar{2}' \; P3\bar{2}''$

$P\bar{3}2', \; P\bar{3}2''$

primitive hexagonal lattice.

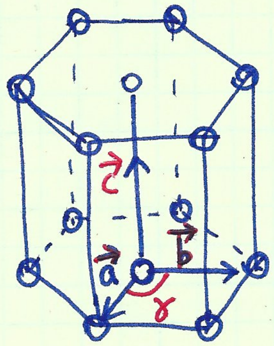
but the crystal only has 3-fold axis

- hexagonal

$D_{6h}(\pm 62'), D_{3h}(\bar{6}2') \; C_{6v}(6\bar{2}')$

$D_6(62') \quad C_{6h}(\pm 6) \quad C_{3h}(\bar{6}) \quad C_6(6)$

6-fold axis for primitive hexagonal lattice



P

$a = b \neq c$

$\alpha = \beta = 90^\circ \quad \gamma = 120^\circ$

$P6, \; P\bar{6}, \; P\pm 6, \; P62', \; P6\bar{2}'$

$P\bar{6}2', \; P\bar{6}2'', \; P\pm 62'$

symmorphic space group:

triclinic	2	tetragonal	16
monoclinic	6	cubic	15
orthorhombic	13		
triagonal	13	subtotal	73
hexagonal	8		

Non-primitive basis vector

In many situations, the primitive unit vectors do not represent the point group symmetry of the Bravis lattice. In order to show the symmetry explicitly, we use non-primitive ones, $\vec{a}_1 \vec{a}_2 \vec{a}_3$ such that some fractional combinations is also a Bravis lattice vector, $\vec{f} = f_1 \vec{a}_1 + f_2 \vec{a}_2 + f_3 \vec{a}_3$ with $0 \le f_i < 1$. The corresponding translations are denoted as $T(\vec{f}) \equiv T(f_1, f_2, f_3)$. The integer values $\vec{l} = l_1 \vec{a}_1 + l_2 \vec{a}_2 + l_3 \vec{a}_3$ form an invariant subgroup T_l. The coset of the translation group $T / T_l = T(\vec{f})$. According to the structure of $T(\vec{f})$ we have

① Primitive translation group $T = T_l$ P

② Body centered $T = T_l \otimes \{E, T(\tfrac{1}{2} \tfrac{1}{2} \tfrac{1}{2})\}$ I

③ Base centered $T = T_l \otimes \{E, T\{0, \tfrac{1}{2}, \tfrac{1}{2}\}\}$; A

$\otimes \{E, T\{\tfrac{1}{2}, 0 \tfrac{1}{2}\}\}$ B

$\otimes \{E, T\{\tfrac{1}{2}, \tfrac{1}{2}, 0\}\}$ C

If both A, B are symmetry operations of a Bravis lattice, the C is also a symmetry operation, In this case, it is called face-centered translation group F.

④ face-centered $T = T_l \otimes \{E, T(0 \tfrac{1}{2} \tfrac{1}{2}), T(\tfrac{1}{2} 0 \tfrac{1}{2}), T(\tfrac{1}{2} \tfrac{1}{2}, 0)\}$. F

These $\boxed{P, I, A, B, C, F}$ types translational symmetries, when combined with the holohedry groups: $\boxed{C_i, C_{2h}, D_{2h}, D_{4h}, D_{3d}, D_{6h}, O_h}$

- 7 crystal systems $\longrightarrow$ 14 Bravais lattices.

(*) Symbols for Mauguin-Hermann, international point/space group

"n" — proper axis. "$\bar{n}$" — improper cyclic point group

"±" — inversion C_i, "#" without a prime: axis along the a_3-axis

"#'" — high order axis along another direction, or 2-fold axis along $\vec{a}_1$

"#''": 2-fold axis along another direc

"m": mirror plane

"n/m" or "$\frac{n}{m}$": n-fold axis, and a mirror plane perpendicular to that axis

Sch	Int-point	MH	Int-space	Sch	Int-point	MH	Int-space
C_1	1	1	1	C_{3v}	3m	3m1	$3\bar{2}'$
C_i	$\bar{1}$	$\bar{1}$	$\bar{1}$	C_{3v}	3m	31m	$3\bar{2}''$
C_2	2	2	2	D_{3d}	$\bar{3}$m	$\bar{3}\frac{2}{m}1$	$\bar{3}2'$
C_s	m	m	$\bar{2}$	D_{3d}	$\bar{3}$m	$\bar{3}1\,2/m$	$\bar{3}2''$
C_{2h}	2/m	2/m	±2	D_4	422	422	$42'$
C_3	3	3	3	C_{4v}	4mm	4mm	$4\bar{2}'$
C_{3i}	$\bar{3}$	$\bar{3}$	$\bar{3}$	D_{2d}	$\bar{4}$2m	$\bar{4}$2m	$\bar{4}2'$
C_4	4	4	4	D_{2d}	$\bar{4}$2m	$\bar{4}$m2	$\bar{4}2''$
S_4	$\bar{4}$	$\bar{4}$	$\bar{4}$	D_{4h}	4/mmm	4/m 2/m 2/m	$\pm 42'$
C_{4h}	4/m	4/m	±4	D_6	622	622	$62'$
C_6	6	6	6	C_{6v}	6mm	6mm	$6\bar{2}'$
C_{3h}	$\bar{6}$	$\bar{6}$	$\bar{6}$	D_{3h}	$\bar{6}$m2	$\bar{6}$2m	$\bar{6}2'$
C_{6h}	6/m	6/m	±6	D_{6h}	6/mmm	$\frac{6}{m}\frac{2}{m}\frac{2}{m}$	$\pm 62'$
D_2	222	222	$22'$	T	23	23	$3'22'$
C_{2v}	2mm	2mm	$2\bar{2}'$	T_h	m3	$\frac{2}{m}3$	$\bar{3}'22'$
D_3	32	321	$32'$	O	432	432	$3'42''$
D_3	32	312	$32''$	T_d	$\bar{4}$3m	$\bar{4}$3m	$3'\bar{4}\bar{2}''$
				O_h	m3m	$\frac{4}{m}3\frac{2}{m}$	$\bar{3}'42''$