# Lecture 3 Coriolis force

$$\vec{F}_{cor} = 2m\,\vec{v} \times \vec{\Omega}$$

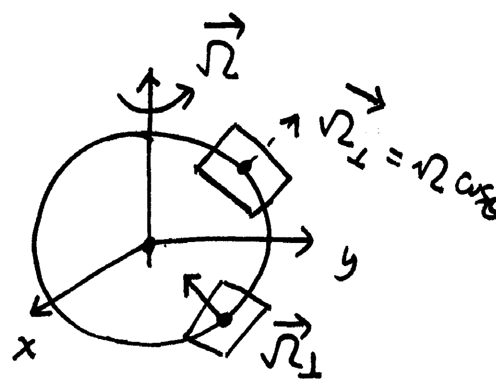
1. velocity-dependent force, which cannot be written as gradient of a scalar potential, but as a curl of vecter potential
2. dissipationless $\vec{F} \perp \vec{v}$ c.f. friction force $// \vec{v}$.

Analogy to Lorentz force $\vec{F}_L = q\,\frac{\vec{v}}{c} \times \vec{B}$. $\vec{B} \leftrightarrow \vec{\Omega}$

Example: vortices in type II superconducter $\leftrightarrow$ $\vec{B} = \nabla \times \vec{A}$

vortices in superfluid $^4$He $\leftrightarrow$ $\vec{\Omega} = \nabla \times \vec{A}_{\Omega}$

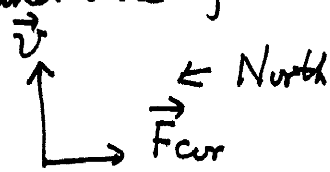
## § Direction of $\vec{F}_{cor}$ on the earth.




For object moving on the surface of the earth $\vec{v}$ is in the tangent plane, only the normal component of $\vec{\Omega}_{\perp}$ can result in an $\vec{F}_{cor}$ in-plane.

In the north-hemisphere $\Omega_{\perp}$ point out of the ground

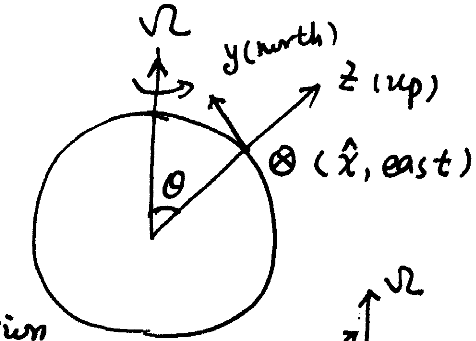
$$\vec{F}_{cor,\ inplane} = \cancel{2}\ \vec{v} \times \vec{\Omega}_{\perp} \rightarrow \text{right hand side of } \vec{v}$$



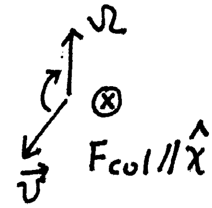

In the south-hemisphere, $\vec{F}_{cor} \rightarrow$ left hand sid of $\vec{v}$.

§2. Easterly deflection of a falling object

$$m\ddot{\vec{r}} = \underbrace{m\vec{g}_0 + \vec{F}_{cf}}_{m\vec{g}} + \vec{F}_{cor}$$




define up-direction ($\hat{z}$) as the oppositead direction of $\vec{g}$, north ($\hat{y}$) perpendicular to $\hat{z}$ and $\vec{\Omega}$ lies in th yz plane. $\hat{x}$ is the east.




$\vec{\Omega} = (0, \Omega\sin\theta, \Omega\cos\theta)$, $\hat{\theta}$ the angle betwee $\hat{\Omega}$ and $\hat{z}$-axis.

$$\dot{\vec{r}} \times \vec{\Omega} = (\dot{y}\Omega\cos\theta - \dot{z}\Omega\sin\theta, -\dot{x}\Omega\cos\theta, \dot{x}\Omega\sin\theta)$$

$$\Rightarrow \quad \ddot{\vec{r}} = -g\hat{z} + 2\dot{\vec{r}}\times\vec{\Omega}$$

$$\ddot{x} = 2\Omega(\dot{y}\cos\theta - \dot{z}\sin\theta), \quad \ddot{y} = -2\Omega\dot{x}\cos\theta, \quad \ddot{z} = -g + 2\Omega\dot{x}\sin\theta$$

- zero order $\Rightarrow$ $x = y = 0$ & $z = h - \frac{1}{2}gt^2$
- first order $\ddot{x} = +2\Omega gt\sin\theta \Rightarrow \boxed{x = \frac{1}{3}\Omega g t^3 \sin\theta}$

earth spins from west to east; the linear volecity ~~of~~ at higher place is larger than that at lower altitude.

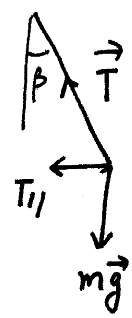
Estimat: for $\theta = 90^\circ$, $h = 100m$

$$\Delta x = \frac{1}{3}\Omega g\left(\frac{2h}{g}\right)^{\frac{3}{2}} \approx 2\,cm.$$

## §4: Foucault pendulum: ( See the spin of the earth )

The rotation of the oscillation plane.

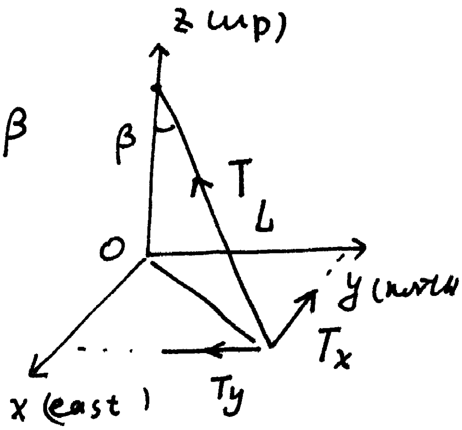
$$m\ddot{\vec{r}} = \vec{T} + \underbrace{m\vec{g}_0 + m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}}_{m\vec{g}} + 2m\,\dot{\vec{r}}\times\vec{\Omega}$$




$$T_{\parallel} \simeq T\cdot\beta \simeq mg\beta$$

$$T_x \simeq -mg\,\frac{x}{L}$$

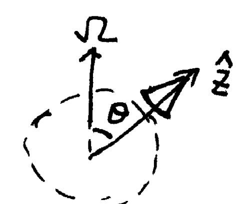
$$T_y \simeq -mg\,\frac{y}{L}$$




$$\Rightarrow \left.\begin{aligned} \ddot{x} &= -\frac{gx}{L} + 2\dot{y}\Omega\cos\theta \\ \ddot{y} &= -\frac{gy}{L} - 2\dot{x}\Omega\cos\theta \end{aligned}\right\}$$

$\Omega_z = \Omega\cos\theta$ is the component of $\vec{\Omega}$ perpendicular the $xy$ plane

$\theta$ the angle between $\hat{\Omega}$ & $\hat{z}$




define $\omega_0^2 = g/L \Rightarrow$

$$\left.\begin{aligned} \ddot{x} - 2\Omega_z\dot{y} + \omega_0^2 x &= 0 \\ \ddot{y} - 2\Omega_z\dot{x} + \omega_0^2 y &= 0 \end{aligned}\right\}$$

define $\eta = x + iy \Rightarrow \ddot{\eta} + 2i\Omega_z\dot{\eta} + \omega_0^2\eta = 0$

eigen-frequency $\quad e^{-i\alpha t} \Rightarrow -\alpha^2 + 2\Omega_z\alpha - \omega_0^2 = 0$

$$\alpha = \Omega_z \pm \sqrt{\Omega_z^2 + \omega_0^2} \approx \Omega_z \pm \omega_0$$

$$\Rightarrow \eta(t) = e^{-i\Omega_z t}\left[C_1 e^{i\omega_0 t} + C_2 e^{-i\omega_0 t}\right]$$

if we choose linear polarization at $t=0$, i.e. $C_1 = C_2$

$$\Rightarrow x(t) + iy(t) = \boxed{e^{-i\Omega_z t}}\cos\omega_0 t$$

rotation of oscillation plane at $\Omega_z = \Omega\cos\theta$.