PHYS 110A

HW I

Taylor 1.34 6

$$\vec{L} = \sum_\alpha \vec{r}_\alpha \times \vec{p}_\alpha$$

$$\frac{d\vec{L}}{dt} = \sum_\alpha \frac{d}{dt}(\vec{r}_\alpha \times \vec{p}_\alpha) = \sum_\alpha \vec{r}_\alpha \times \frac{d\vec{p}_\alpha}{dt} + \sum_\alpha \frac{d\vec{r}_\alpha}{dt} \times \vec{p}_\alpha$$

(1 point for getting the product rule right)

By definition of velocity and momentum,

$\frac{d\vec{r}_\alpha}{dt} = \vec{v}_\alpha$, $\vec{p}_\alpha = m_\alpha \vec{v}_\alpha$, where m_α is the mass of the 'α'-th particle. (1 point for getting this right).

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_\alpha \vec{r}_\alpha \times \frac{d}{dt}\vec{p}_\alpha + \sum_\alpha \vec{v}_\alpha \times m_\alpha \vec{v}_\alpha$$

$$\vec{v}_\alpha \times m\vec{v}_\alpha = 0$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_\alpha \vec{r}_\alpha \times \frac{d}{dt}\vec{p}_\alpha$$

$$\frac{d\vec{p}_\alpha}{dt} = \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \vec{F}_{\alpha, ext} \quad \text{(by Newton's II law)}$$

$$= \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} \quad \text{(1 point for getting this right)},$$

$\vec{F}_{\alpha\beta}$ is the force by particle β on α. $\vec{F}_{\alpha, ext}$ is the external force on α.

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_\alpha \sum_{\beta \neq \alpha} \vec{r}_\alpha \times \vec{F}_{\alpha\beta}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_\alpha \sum_{\beta < \alpha} (\vec{r}_\alpha \times \vec{F}_{\alpha\beta} + \vec{r}_\beta \times \vec{F}_{\beta\alpha})$$

Now, $\vec{F}_{\alpha\beta} = -\vec{F}_{\beta\alpha}$ (by Newton's III law)

(1 point for this).

$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_\alpha \sum_{\beta < \alpha} (\vec{r}_\alpha - \vec{r}_\beta) \times \vec{F}_{\alpha\beta}$$

Given that the interparticle forces are central,

$$\Rightarrow \vec{F}_{\alpha\beta} \propto (\vec{r}_\alpha - \vec{r}_\beta)$$

$$\Rightarrow (\vec{r}_\alpha - \vec{r}_\beta) \times \vec{F}_{\alpha\beta} = 0$$

(1 point for this)

$$\Rightarrow \frac{d\vec{L}}{dt} = \vec{0}$$

Hence proved that $\vec{L}$ is conserved.

Taylor 1.40

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(a) Lets assume the ball has a mass 'm' and initial speed v_0.
Lets assume the ball is projected from the origin.
From Newton's second law:

$$\left.\begin{aligned} F_x &= m\ddot{x} = 0 &\Rightarrow \quad \ddot{x} &= 0 \\ F_y &= m\ddot{y} = -mg & \ddot{y} &= -g \end{aligned}\right\} \text{1 point}$$

Solving the equation, we get

$$\left.\begin{aligned} x(t) &= v_0 \cos\theta\, t \\ y(t) &= v_0 \sin\theta\, t - \frac{1}{2} g t^2 \end{aligned}\right\} \text{1 point}$$

Note that you can verify ~~that~~ these expressions as

$\dot{x}(t=0) = v_0 \cos\theta =$ initial x-velocity

$\dot{y}(t=0) = v_0 \sin\theta =$ initial y-velocity.

(b)
$$\left.\begin{aligned} r(t) &= \sqrt{x^2(t) + y^2(t)} \\ &= \sqrt{v_0^2 t^2 - g v_0 \sin\theta\, t^3 + \tfrac{1}{4} g^2 t^4} \\ \Rightarrow r^2 &= v_0^2 t^2 - g v_0 \sin\theta\, t^3 + \tfrac{1}{4} g^2 t^4 \end{aligned}\right\} \text{1 point}$$

If r increases continuously with time, so does r^2.

$$\Rightarrow \frac{dr^2}{dt} > 0 \quad \text{for all } t > 0$$

$$\Rightarrow t\left(g^2 t^2 - 3 g v_0 \sin\theta\, t + 2 v_0^2\right) > 0$$ (1 point for this eqn)

For the equation to be true for all $t>0$,

$$g^2t^2 - \frac{3}{\cancel{}}gv_0\sin\theta\, t + 2v_0^2 > 0$$

$$\Rightarrow \left(gt - \frac{3}{2}v_0\sin\theta\right)^2 > \frac{9}{4}v_0^2\sin^2\theta - 2v_0^2$$

For this to be true for all t,

$$\frac{9}{4}v_0^2\sin^2\theta - 2v_0^2 < 0$$

(Note that LHS of the previous equation is a perfect square, so it is always positive. If the RHS is negative, it ensures that the inequality holds for all t).

$$\Rightarrow \boxed{\sin^2\theta < \frac{8}{9}}$$ (2 points. for getting this)

$$\Rightarrow \sin\theta < \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \theta < 70.5^\circ$$

so, for $0<\theta<70.5^\circ$, r always increases.

1.49 Newton's second law, ~~for~~ for cylindrical coordinates gives

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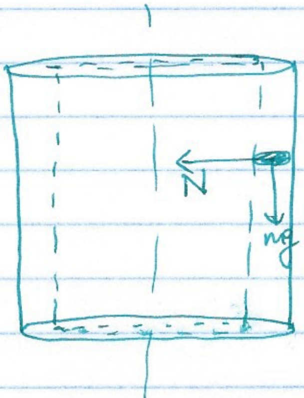
$$F_\rho = m(\ddot{\rho} - \rho\dot{\phi}^2) \quad \text{(1 point)}$$

$$F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) \quad \text{(1 point)}$$

$$F_z = m\ddot{z} \quad \text{(1 point)}$$



N is the net force by the cylinders on the puck.
mg is the force due to gravity.
There are no other forces since there is no friction.
Note that $\rho = R$ is fixed

1 point

$\Rightarrow \dot{\rho} = \ddot{\rho} = 0$ ~~(1 point)~~

$$\Rightarrow -N = -mR\dot{\phi}^2$$

$$R\ddot{\phi} = 0 \Rightarrow \ddot{\phi} = 0 \Rightarrow \dot{\phi} \text{ is constant. say } \dot{\phi} = \omega.$$

$$\ddot{z} = -g \Rightarrow \ddot{z} = -g$$

Solving the equation $\dot{\phi} = \omega$ & $\ddot{z} = -g$, we get

$$\left.\begin{aligned} \phi(t) &= \phi_0 + \omega t \\ z(t) &= z_0 + v_0 t - \frac{1}{2} g t^2 \end{aligned}\right\} \text{2 points}$$

where ϕ_0 is the initial azimuthal angle. ω is the angular velocity, z_0 is the initial z coordinate, v_0 is the initial velocity in z direction