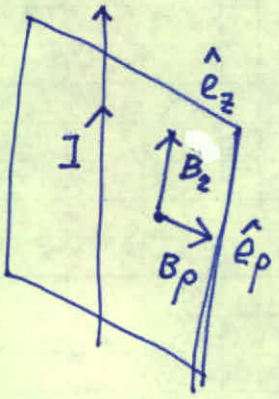
Solution Midterm 100B 2012

1 a) From symmetry analysis. B-field can only along the azimuthal ange direction $\vec{e}_\varphi$, because the system is symmetric with respect to any plane containing this wire.

For B-fields along $\hat{e}_\rho$ and $\hat{e}_z$, they change sign after the reflection with respect of the reflection to the z-ρ plane.



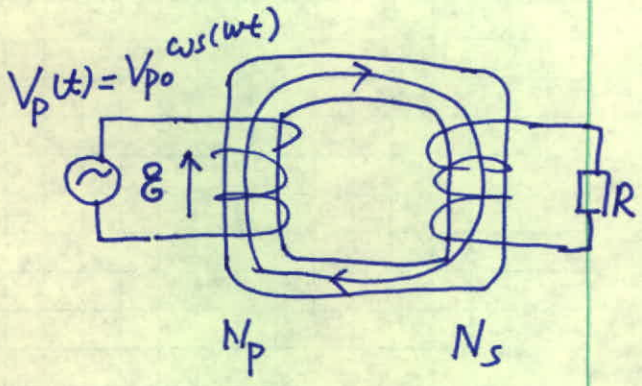
Due to the rotational sym around the z-axis B_φ is the same at the value of ρ.

⇒ b) $\oint \vec{B}\cdot d\vec{l} = 4\pi \iint \vec{j}\cdot d\vec{a}$

for $\rho > R$ ⇒ $2\pi\rho B_\varphi = 4\pi I$ ⇒ $\vec{B} = \frac{2I}{\rho}\hat{e}_\varphi$

$\rho < R$ $2\pi\rho B_\varphi = 4\pi \frac{I\cdot\rho^2}{R^2}$ ⇒ $\vec{B} = \frac{2I\rho}{R^2}\hat{e}_\varphi$

2)



a) The emf in the primary coil $\mathcal{E}_p(t)$ must a compansent the source voltage, and

$$\mathcal{E}_p(t) = -N_p \frac{d\Phi(t)}{dt} = V_p(t) = V_o \cos(\omega t)$$

the generated emf on the secnd coil

$$\mathcal{E}_s(t) = -N_s \frac{d\Phi(t)}{dt} \Rightarrow \frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p}$$

they have the same frequency ω

3)

3 a) The first step: $I_2 = 0$, for $0 < t < t_1$.

$$\mathcal{E}_1 = -L_1 \frac{dI_1}{dt} \Rightarrow W_1 = -\int_0^{t_1} \mathcal{E}_1 I_1 dt = \int_0^{t_1} L_1 I_1 \frac{dI_1}{dt} dt$$

$$= \int_0^{I_1} L_1 I_1 dI_1 = \frac{1}{2} L_1 I_1^2$$

5 b) for C_1: $\mathcal{E}_1 = -M_{12} \frac{dI_2}{dt} \Rightarrow W_1' = -\int_{t_1}^{t_2} \mathcal{E}_1 I_1 dt = M_{12} I_1 \int_0^{I_2} \frac{dI_2}{dt} dt$

$$= M_{12} I_1 I_2$$

for C_2 $\mathcal{E}_2 = -L_2 \frac{dI_2}{dt} \Rightarrow W_2' = +\int_{t_4}^{t_2} L_2 I_2 \frac{dI_2}{dt} dt = L_2 \int_0^{I_2} I_2 dI_2$

$$= \frac{L_2}{2} I_2^2$$

3 c) add together $\Rightarrow$ $W_{tot} = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2$

this amound of work is stored in the magnetic field energy generated by the currents.

2 d) repeat the above process, we only need to exchange

M_{12} to M_{21}

$\Rightarrow$

2 $W_{tot} = \frac{1}{2} L_2 I_2^2 + \frac{1}{2} L_1 I_1^2 + M_{21} I_2 I_1$ magnetic

e) These two ~~step~~ different method $\Rightarrow$ The same energy $\Rightarrow$ $M_{12} = M_{21}$