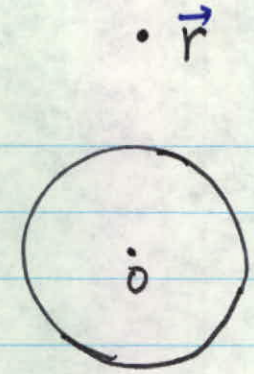
Prob one



1) for any point $\vec{r}$, the system has an rotational symmetry with respect to the axis from the center O to $\vec{r}$.

If $\vec{E}(\vec{r})$ is not paralleel to $\vec{r}$, then we apply the rotation, $\vec{E}(\vec{r})$ will be changed. Since we know that $\vec{E}(\vec{r})$ ~~only~~ is fixed, it has to be invariant under this rotation

$\Rightarrow$ thus $\vec{E}$ has to be paralleel to $\vec{r}$.

2) we choose a sphere with radial r, from Gauss's law and the fact proved in 1) $\Rightarrow$

$$\oint \vec{E} \cdot d\vec{a} = 4\pi r^2 E(r) = \begin{cases} 0 & \text{if } r < R_1 \\ 4\pi Q & \text{if } r > R_1 \end{cases}$$

$$\Rightarrow E(r) = \begin{cases} 0 & r < R_1 \\ \frac{Q}{r^2} & r > R_1 \end{cases}$$

$$V(r) = -\int_{\infty}^{r} E(r)\,dr = \int_{r}^{\infty} E(r)\,dr = \begin{cases} \frac{Q}{r} & \text{for } r > R_1 \\ \frac{Q}{R_1} & \text{for } r < R_1 \end{cases}$$

3) because $\vec{E}(\vec{r})$ has discontinuiety at $r = R_1^-$ and R_1^+

$$d\vec{f} = \frac{\vec{E}(R_1^-) + \vec{E}(R_1^+)}{2}(\sigma\, da) = \frac{Q}{2R_1^2}\frac{Q}{4\pi R_1^2}\,\hat{e}_r\, da$$

$$= \frac{Q^2}{8\pi R_1^4}\, da\, \hat{e}_r,$$ the direction is outward along $\hat{e}_r$.

d)

$$W = \int_{R_1}^{R_2} d\vec{r} \int d\vec{f}$$

$$= \int_{R_1}^{R_2} dr \frac{Q^2}{8\pi r^4} \cdot 4\pi r^2 = \int_{R_1}^{R_2} dr \frac{Q^2}{2 r^2}$$

$$= \frac{Q^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

e): perspective of charge

$$\Delta E = E_{ini} - E_{final}. \quad E_{ini} = \frac{1}{2}\int \rho\, V_{ini}\, d\tau = \frac{Q}{2} V_{in}$$

$$= \frac{Q}{2}\frac{Q}{R_1}$$

$$E_{final} = \frac{1}{2}\int \rho\, V_{final}\, d\tau = \frac{Q}{2} V_{final} = \frac{Q^2}{2R_2}$$

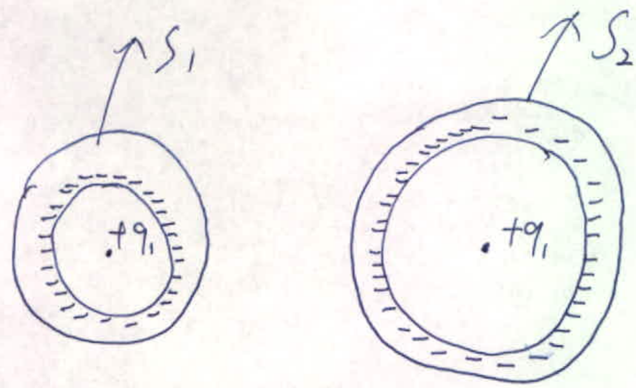
$$\Rightarrow \Delta E = \frac{Q^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

perspective of field

$$\Delta E = \int d\Omega \int_{R_1}^{R_2} r^2\, dr\, \frac{E^2(r)}{8\pi} = \frac{4\pi}{8\pi}\int_{R_1}^{R_2} r^2\, dr \left(\frac{Q}{r^2}\right)^2 = \int_{R_1}^{R_2} dr \frac{Q^2}{r^2} = \frac{Q^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Problem 2

(a)



Choose the surfaces as the figure.

$$\int_{S_1} \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} = \frac{q_1 + q_{ind}}{\epsilon_0}$$

$\vec{E} = 0$ inside metal $\Rightarrow \quad 0 = \dfrac{q_1 + q_{ind}}{\epsilon_0} \Rightarrow \boxed{q_{ind} = -q_1}$ (on the surface of cavity one)

Similarly $\displaystyle\int_{S_2} \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} = \frac{q_2 + q_{ind}}{\epsilon_0}$

$\vec{E} = 0 \Rightarrow \quad 0 = \dfrac{q_2 + q_{ind}}{\epsilon_0} \Rightarrow \boxed{q_{ind} = -q_2}$ (on the surface of cavity two)

Since $+q_1$ and $+q_2$ are at the centers, the induced charges must distribute uniformly to make the field outside the cavities vanish.

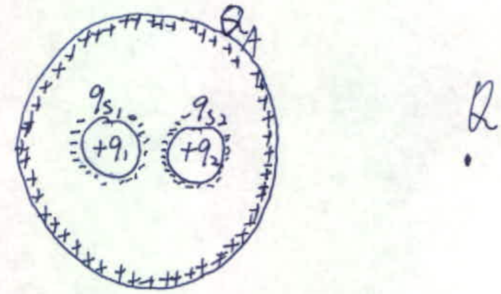
(b) The net charge in metal is zero, so the charge on the outer surface must compensate the charge on the inner surfaces

$\Rightarrow \boxed{Q_A = q_1 + q_2}$ (on the outer surface)

(c) For q_1, Q and q_2 are screened by metal

$\Rightarrow \begin{cases} \text{the field from } Q \text{ and } Q_A \text{ is zero.} \\ \text{the field from } q_2 \text{ and } q_{s_2} \text{ is zero.} \end{cases}$

And, the field from q_{s_1} is zero, since q_{s_1} is uniform on S_1.



$\Rightarrow$ The total force on q_1 is zero

For the same reason,

The total force on q_2 is zero

For Q, q_1 and q_2 are screened.

The only force is from $Q_A = q_1 + q_2$.

If Q is far away from A, Q_A is approximately a point charge

$$\boxed{\vec{F_Q} = Q\vec{E_A} = Q \times \frac{(q_1+q_2)}{4\pi\epsilon_0 r^2}\hat{r}}$$

The force on the ball A is $-\vec{F_Q}$ because of Newton's third law.

$$\vec{F_A} = -\vec{F_Q} = \frac{-Q(q_1+q_2)}{4\pi\epsilon_0 r^2}\hat{r}$$

(d) F_{q_1} and F_{q_2} are exact.

F_A and F_Q are correct only for large r.