Special Relativity

Einstein messes with space and time

How Fast Are You Moving Right Now?

- 0 m/s relative to your chair
- 400 m/s relative to earth center (rotation)
- 30,000 m/s relative to the sun (orbit)
- 220,000 m/s relative to the galaxy center (orbit)
- 370,000 m/s relative to the CMB cosmic wallpaper

Relative to What??

- This is part of the gist of special relativity
  - it's the exploration of the physics of relative motion
  - only relative velocities matter: **no absolute frame**
  - very relevant comparative velocity is c = 300,000,000 m/s

A world without ether

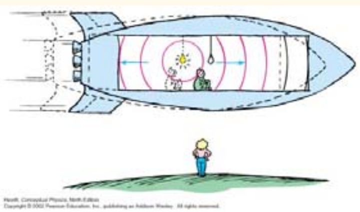
- For most of the 19th century, physicists thought that space was permeated by "luminiferous ether"
  - this was thought to be necessary for light to propagate
- Michelson and Morley performed an experiment to measure earth's velocity through this substance
  - first result in 1887
  - Michelson was first American to win Nobel Prize in physics
- Found that light waves don't bunch up in direction of earth motion
  - shocked the physics world: no ether!!
  - speed of light is not measured relative to fixed medium
  - unlike sound waves, water waves, etc.

Speed of light is constant: so what?

- Einstein pondered: what would be the consequences of a constant speed of light
  - independent of state of motion (if at const. velocity)
  - any observer traveling at constant velocity will see light behave "normally," and *always* at the same speed
- Mathematical consequences are very clear
  - forced to give up Newtonian view of space and time as completely separate concepts
  - provides rules to compute observable comparisons between observers with *relative* velocity
    - thus "relativity": means relative state of motion

Simultaneity is relative, not absolute

Observer riding in spaceship at constant velocity sees a flash of light situated in the center of the ship's chamber hit both ends at the same time



But to a stationary observer (or any observer in *relative* motion), the condition that light travels each way at the same speed *in their own frame* means that the events will not be simultaneous. In the case pictured, the stationary observer sees the flash hit the back of the ship before the front

One person's space is another's time

- If simultaneity is broken, no one can agree on a universal time that suits all
  - the *relative* state of motion is important
- Because the speed of light is constant (and finite) for all observers, space and time are ***unavoidably*** mixed
  - we've seen an aspect of this in that looking into the distance is the same as looking back in time
- Imagine a spaceship flying by with a strobe flashing once per second (as timed by the occupant)
  - the occupant sees the strobe as stationary
  - you see flashes in different positions, and disagree on the timing between flashes: space and time are mixed
    - see description of light clock in text
- Space and time mixing promotes unified view of *spacetime*
  - "events" are described by three spatial coordinates plus a time

The Lorentz Transformation

- There is a prescription for transforming between observers in relative motion

  $ct' = \gamma(ct - vx/c)$; $x' = \gamma(x - vt)$; $y' = y$; $z' = z$

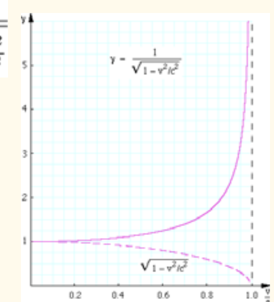
  - "primed" coordinates belong to observer moving at speed v along the x direction (relative to unprimed)
  - note mixing of x and t into x' and t'
    - time and space being nixed up
  - multiplying t by c to put on same footing as x
    - now it's a distance, with units of meters
  - the γ (gamma) factor is a function of velocity:

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The gamma factor

- Gamma (γ) is a measure of how whacked-out relativistic you are

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



- When $v = 0$, $\gamma = 1.0$
  - and things are normal
- At $v = 0.6c$, $\gamma = 1.25$
  - a little whacky
- At $v = 0.8c$, $\gamma = 1.67$
  - getting to be funky
- As $v \rightarrow c$, $\gamma \rightarrow \infty$

What does γ do?

- Time dilation: clocks on a moving platform appear to tick slower by the factor γ
  - at 0.6c, γ = 1.25, so moving clock seems to tick off 48 seconds per minute
  - standing on platform, you see the clocks on a fast-moving train tick slowly: people age more slowly, though to them, all is normal
- Length contraction: moving objects appear to be "compressed" along the direction of travel by the factor γ
  - at 0.6c, γ = 1.25, so fast meter stick will measure 0.8 m to stationary observer
  - standing on a platform, you see a shorter train slip past, though the occupants see their train as normal length

Why don't we see relativity every day?

- We're soooo slow (relative to c), that length contraction and time dilation don't amount to much
  - 30 m/s freeway speed has $v/c = 10^{-7}$
    - γ = 1.000000000000005
  - 30,000 m/s earth around sun has $v/c = 10^{-4}$
    - γ = 1.000000005
    - but precise measurements see this clearly

Velocity Addition

- Also falling out of the requirement that the speed of light is constant for all observers is a new rule for adding velocities
- Galilean addition had that someone traveling at v_1 throwing a ball forward at v_2 would make the ball go at v_1+v_2
- In relativity, $$v_{\rm rel} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$
  - reduces to Galilean addition for small velocities
  - can never get more than c if v_1 and v_2 are both $\leq c$
  - if either v_1 OR v_2 is c, then $v_{rel} = c$: light always goes at c

Classic Paradoxes

- The twin paradox:
  - one twin (age 30) sets off in rocket at high speed, returns to earth after long trip
  - if v = 0.6c, 30 years will pass on earth while only 24 will pass in high speed rocket
  - twin returns at age 54 to find sibling at 60 years old
  - why not the other way around?
- Pole-vaulter into barn
  - high-speed runner with 12 meter pole runs into 10 meter barn; barn door closes, and encompasses length-contracted 9.6 m pole (at 0.6c)
  - but runner sees barn shrunken to 8 m, and is holding 12 m pole!
  - can the barn door close before the pole crashes through the back?
  - resolution in lack of simultaneity: "before" is *nuanced*

If I'm in a car, traveling at the speed of light…

- If I turn on my headlights, do they work?
- Answer: of course—to you, all is normal
  - you are in an un-accelerated (inertial) frame of reference
  - all things operate normally in your frame
- To the "stationary" outsider, your lights look weird
  - but then again, so do you (because you're going so fast)
  - in fact, *at* the speed of light, all forward signals you send arrive at the same time *you* do
- And the outside, "stationary" world looks weird to you
- But I must inquire: how did you manage to get all the way up to the speed of light?!

What would I experience at light speed?

- It is *impossible* to get a massive thing to travel truly at the speed of light
  - energy required is γmc^2, where $\gamma \rightarrow \infty$ as $v \rightarrow c$
  - so requires infinite energy to get all the way to c
- But if you are a massless photon…
  - to the outside, your clock is *stopped*
  - so you arrive at your destination in the same instant you leave your source (by *your* clock)
    - across the universe in a perceived instant
  - makes sense, if to you the outside world's clock has stopped: you see no "ticks" happen before you hit

E = mc² as a consequence of relativity

- Express 4-vector as (ct, x, y, z)
  - describes an "event": time and place
  - time coordinate plus three spatial coordinates
  - factor of c in time dimension puts time on same footing as space (same units)
- We're always traveling through time
  - our 4-velocity is (c, 0, 0, 0), when sitting still
    - moving at speed of light *through time dimension*
  - stationary 4-momentum is $p = mv \rightarrow (mc, 0, 0, 0)$
  - for a moving particle, $\boldsymbol{p} = (\gamma mc, \gamma p_x, \gamma p_y, \gamma p_z)$
    - where p_x, etc. are the standard momenta in the x, y, and z directions
    - the time-component times another factor of c is interpreted as energy
  - conservation of 4-momentum gets energy and momentum conservation in one shot

E = mc², continued

- γ can be approximated as

  $\gamma = 1 + \frac{1}{2}v^2/c^2 + \ldots$(small stuff at low velocities)
- so that the time component of the 4-momentum × c is:

  $m\gamma c^2 = mc^2 + \frac{1}{2}mv^2 + \ldots$
  - the second part of which is the familiar kinetic energy
- Interpretation is that total energy, $E = m\gamma c^2$
  - mc^2 part is ever-present, and is called "rest mass energy"
  - kinetic part adds to total energy if in motion
  - since γ sticks to m in 4-momentum, can interpret this to mean mass is effectively increased by motion: $m \rightarrow m\gamma$
  - gets harder and harder to accelerate as speed approaches c

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Experimental Confirmation

- We see time dilation in particle lifetimes
  - in accelerators, particles live longer at high speed
    - their clocks are running slowly as seen by us
    - seen daily in particle accelerators worldwide
  - cosmic rays make muons in the upper atmosphere
    - these muons only live for about 2 microseconds
    - if not experiencing time dilation, they would decay before reaching the ground, but they *do* reach the ground in abundance
- We see length contraction of the lunar orbit
  - squished a bit in the direction of the earth's travel around the sun
- $E = mc^2$ extensively confirmed
  - nuclear power/bombs
  - sun's energy conversion mechanism
  - bread-and-butter of particle accelerators

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References

- Relativity Visualized
  - by Lewis Carroll Epstein
- http://www.anu.edu.au/physics/Searle/ movie

- Assignments
  - Q/O #3 due today by midnight
  - Partial read of Chapters 9 & 10 (pages on assignment page)
  - Read Chapters 35 & 36 on relativity
  - HW5: 9.R.13, 9.E.9, 9.E.14, 9.E.43, 9.P.7, 10.E.16, 35.R.27, 35.E.6, 35.E.19, 35.E.20, 35.E.37, 35.P.3, 35.P.10, 36.R.7, 36.E.2, 36.E.6

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