Work, Energy, Power, Momentum

Measures of Effort & Motion;
Conservation Laws

Work, defined

- Work carries a specific meaning in physics
  - Simple form: work = force × distance

$$W = \boldsymbol{F} \cdot \boldsymbol{d}$$

- Work can be done *by* you, as well as *on* you
  - Are you the *pusher* or the *pushee*
- Work is a measure of expended energy
  - Work makes you tired
- Machines make work easy (ramps, levers, etc.)
  - Apply less force over larger distance for same work

Working at an advantage

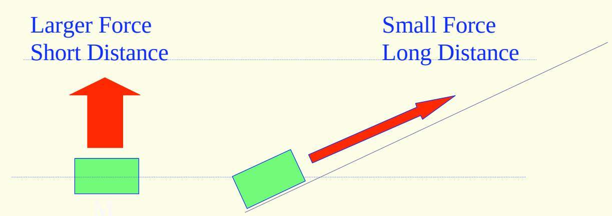
- Often we're limited by the amount of *force* we can apply.
  - Putting "full weight" into wrench is limited by your *mg*
- Ramps, levers, pulleys, etc. all allow you to do the same amount of work, but by applying a smaller force over a larger distance

Work = **Force** × Distance

= Force × **Distance**

Ramps

Exert a smaller force over a larger distance to achieve the same change in gravitational potential energy (height raised)

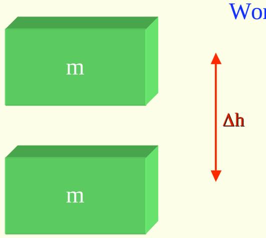


Gravitational Potential Energy

- Gravitational Potential Energy near the surface of the Earth:

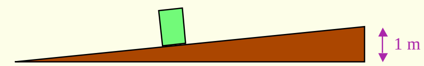
Work = Force × Distance

$\Delta W = mg \times \Delta h$



Ramp Example

- Ramp 10 m long and 1 m high
- Push 100 kg all the way up ramp
- Would require *mg* = 980 N (220 lb) of force to lift directly (brute strength)
- Work done is (980 N)×(1 m) = 980 N·m in direct lift



- Extend over 10 m, and only 98 N (22 lb) is needed
  - Something we can actually provide
  - Excludes frictional forces/losses

Work Examples "Worked" Out

- How much work does it take to lift a 30 kg suitcase onto the table, 1 meter high?
  - $W = (30\ \text{kg}) \times (9.8\ \text{m/s}^2) \times (1\ \text{m}) = 294\ \text{J}$
- Unit of work (energy) is the N·m, or Joule (J)
  - One Joule is 0.239 calories, or 0.000239 Calories (food)
- Pushing a crate 10 m across a floor with a force of 250 N requires 2,500 J (2.5 kJ) of work
- Gravity does 20 J of work on a 1 kg (10 N) book that it has pulled off a 2 meter shelf

Work is Exchange of Energy

- *Energy* is the *capacity* to do work
- Two main categories of energy
  - Kinetic Energy: Energy of motion
    - A moving baseball can do work
    - A falling anvil can do work
  - Potential Energy: Stored (latent) capacity to do work
    - Gravitational potential energy (perched on cliff)
    - Mechanical potential energy (like in compressed spring)
    - Chemical potential energy (stored in bonds)
    - Nuclear potential energy (in nuclear bonds)
- Energy can be converted between types

Conversion of Energy

- Falling object converts gravitational potential energy into kinetic energy
- Friction converts kinetic energy into vibrational (thermal) energy
  – makes things hot (rub your hands together)
  – irretrievable energy
- Doing work *on* something changes that object's energy by amount of work done, transferring energy *from* the agent *doing* the work

Energy is Conserved!

- The total energy (in all forms) in a "closed" system remains *constant*
- This is one of nature's "conservation laws"
  – Conservation applies to:
    - Energy (includes mass via $E = mc^2$)
    - Momentum
    - Angular Momentum
    - Electric Charge
- Conservation laws are fundamental in physics, and stem from symmetries in our space and time
  – Emmy Noether formulated this deep connection
  – cedar.evansville.edu/~ck6/bstud/noether.html

Energy Conservation Demonstrated

- Roller coaster car lifted to initial height (energy in)
- Converts gravitational potential energy to motion
- Fastest at bottom of track
- Re-converts kinetic energy back into potential as it climbs the next hill

Kinetic Energy

- The kinetic energy for a mass in motion is
  K.E. = $\frac{1}{2}mv^2$
- Example: 1 kg at 10 m/s has 50 J of kinetic energy
- Ball dropped from rest at a height h (P.E. = mgh) hits the ground with speed v. Expect $\frac{1}{2}mv^2 = mgh$
  – $h = \frac{1}{2}gt^2$
  – $v = gt \rightarrow v^2 = g^2t^2$
  – $mgh = mg\times(\frac{1}{2}gt^2) = \frac{1}{2}mg^2t^2 = \frac{1}{2}mv^2$ sure enough
  – Ball has converted its available gravitational potential energy into kinetic energy: the energy of motion

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Kinetic Energy, cont.

- Kinetic energy is proportional to v^2…
- Watch out for fast things!
  - Damage to car in collision is proportional to v^2
  - Trauma to head from falling anvil is proportional to v^2, or to *mgh* (how high it started from)
  - Hurricane with 120 m.p.h. packs four times the punch of gale with 60 m.p.h. winds

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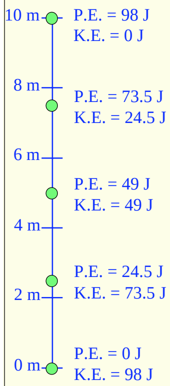
Energy Conversion/Conservation Example



- Drop 1 kg ball from 10 m
  - starts out with mgh = (1 kg)×(9.8 m/s^2)×(10 m) = 98 J of gravitational potential energy
  - halfway down (5 m from floor), has given up half its potential energy (49 J) to kinetic energy
    - $\frac{1}{2}mv^2 = 49$ J → $v^2 = 98$ m^2/s^2 → $v \approx 10$ m/s
  - at floor (0 m), all potential energy is given up to kinetic energy
    - $\frac{1}{2}mv^2 = 98$ J → $v^2 = 196$ m^2/s^2 → $v = 14$ m/s

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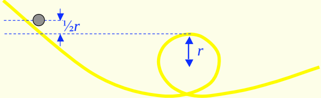
Loop-the-Loop

- In the loop-the-loop (like in a roller coaster), the velocity at the top of the loop must be enough to keep the train on the track:

$$v^2/r > g$$

- Works out that train must start ½*r* higher than top of loop to stay on track, ignoring frictional losses



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Heat: Energy Lost?

- Heat is a form of energy
  - really just randomized kinetic energy on micro scale
  - lattice vibrations in solids, faster motions in liquids/gases
- Heat is a viable (and common) path for energy flow
  - Product of friction, many chemical, electrical processes
- Hard to make heat energy *do* anything for you
  - Kinetic energy of hammer can drive nail
  - Potential energy in compressed spring can produce motion
  - Heat is too disordered to extract useful work, generally
    - notable exceptions: steam turbine found in most power plants
    - Solar core : heat is important in enabling *thermo*-nuclear fusion

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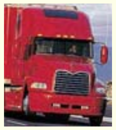
Power

- Power is simply energy exchanged per unit time, or how fast you get work done (Watts = Joules/sec)
- One horsepower = 745 W
- Perform 100 J of work in 1 s, and call it 100 W
- Run upstairs, raising your 70 kg (700 N) mass 3 m (2,100 J) in 3 seconds → 700 W output!
- Shuttle puts out a few GW (gigawatts, or 10^9 W) of power!

More Power Examples

- Hydroelectric plant
  - Drops water 20 m, with flow rate of 2,000 m^3/s
  - 1 m^3 of water is 1,000 kg, or 9,800 N of weight (force)
  - Every second, drop 19,600,000 N down 20 m, giving 392,000,000 J/s ≈ 400 MW of power
- Car on freeway: 30 m/s, A = 3 m^2 → F_{drag}≈1800 N
  - In each second, car goes 30 m → W = 1800×30 = 54 kJ
  - So power = work per second is 54 kW (72 horsepower)
- Bicycling up 10% (~6°) slope at 5 m/s (11 m.p.h.)
  - raise your 80 kg self+bike 0.5 m every second
  - mgh = 80×9.8×0.5 ≈ 400 J → 400 W expended

Momentum



- Often misused word, though most have the right idea
- Momentum, denoted p, is mass times velocity

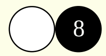
  $p = m \cdot v$
- Momentum is a conserved quantity (and a vector)
  - Often relevant in collisions (watch out for linebackers!)
- News headline: Wad of Clay Hits Unsuspecting Sled
  - 1 kg clay ball strikes 5 kg sled at 12 m/s and sticks
  - Momentum before collision: (1 kg)(12 m/s) + (5 kg)(0 m/s)
  - Momentum after = 12 kg·m/s → (6 kg)·(2 m/s)

Collisions

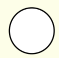
- Two types of collisions
  - Elastic: Energy not dissipated out of kinetic energy
    - Bouncy
  - Inelastic: Some energy dissipated to other forms
    - Sticky
- Perfect elasticity unattainable (perpetual motion)

Elastic Collision: Billiard Balls

- Whack stationary ball with identical ball moving at velocity v_{cue}



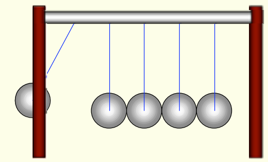
To conserve both energy *and* momentum, cue ball stops dead, and 8-ball takes off with v_{cue}



Momentum conservation: $mv_{cue} = mv_{cue,\ after} + mv_{8\text{-ball}}$
Energy conservation: $\frac{1}{2}mv^2_{cue} = \frac{1}{2}mv^2_{cue,\ after} + \frac{1}{2}mv^2_{8\text{-ball}}$

The only way $v_0 = v_1 + v_2$ and $v^2_0 = v^2_1 + v^2_2$ is if either v_1 or v_2 is 0.
Since cue ball can't move *through* 8-ball, cue ball gets stopped.

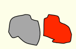
Desk Toy Physics



- The same principle applies to the suspended-ball desk toy, which eerily “knows” how many balls you let go…
- Only way to simultaneously satisfy energy *and* momentum conservation
- Relies on balls to all have same mass

Inelastic Collision

- Energy not conserved (absorbed into other paths)
- Non-bouncy: hacky sack, velcro ball, ball of clay



Momentum before = $m_1 v_{initial}$
Momentum after = $(m_1 + m_2)v_{final} = m_1 v_{initial}$ (because conserved)

Energy before = $\frac{1}{2}m_1 v^2_{initial}$
Energy after = $\frac{1}{2}(m_1 + m_2)v^2_{final}$ + heat energy

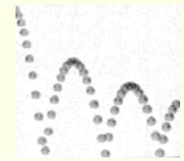
Questions

- Twin trouble-makers rig a pair of swings to hang from the same hooks, facing each other. They get friends to pull them back (the same distance from the bottom of the swing) and let them go. When they collide in the center, which way do they swing (as a heap), if any? What if Fred was pulled higher than George before release?
- A 100 kg ogre clobbers a dainty 50 kg figure skater while trying to learn to ice-skate. If the ogre is moving at 6 m/s before the collision, at what speed will the tangled pile be sliding afterwards?

Real-World Collisions

- Is a superball elastic or inelastic?
  - It bounces, so it's not completely inelastic
  - It doesn't return to original height after bounce, so *some* energy must be lost
- Superball often bounces 80% original height
  - Golf ball → 65%
  - Tennis ball → 55%
  - Baseball → 30%
- Depends also on *surface*, which can absorb some of the ball's energy
  - down comforter/mattress or thick mud would absorb

Superball Physics



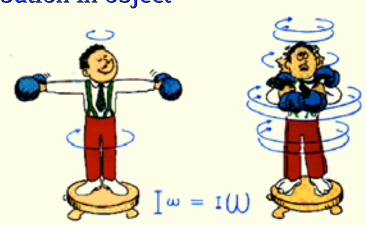
- During bounce, if force on/from floor is purely vertical, expect constant horizontal velocity
  - constant velocity in absence of forces
  - like in picture to upper right
- BUT, superballs often behave contrary to intuition
  - back-and-forth motion
  - boomerang effect

Angular Momentum

- Another conserved quantity is *angular momentum*, relating to *rotational inertia:*
- Spinning wheel wants to keep on spinning, stationary wheel wants to keep still (unless acted upon by an external rotational force, or torque)
- Newton's laws for linear (straight-line) motion have direct analogs in rotational motion

Angular Momentum

- Angular momentum is proportional to rotation speed (ω) times rotational inertia (I)
- Rotational inertia characterized by (mass)×(radius)^2 distribution in object



Angular Momentum Conservation

- Speed up rotation by tucking in
- Slow down rotation by stretching out
- Seen in diving all the time
- Figure skaters demonstrate impressively
- Effect amplified by moving large masses to vastly different radii

Do cats violate physical law?

- Cats can quickly flip themselves to land on their feet
- If not rotating before, where do they get their angular momentum?
- There are ways to accomplish this, by a combination of contortion and varying rotational inertia

For more on falling cats:

- Websites:
  - www.pbs.org/wnet/nature/cats/html/body_falling.html
    - play quicktime movie
  - www.exploratorium.edu/skateboarding/trick_midair_activity.html

Announcements/Assignments

- Midterm review next week (Thu. evening?)
- Exam Study Guide online by this weekend
- Should have read Hewitt 2, 3, 4, 5, 6, 7 assignments by now
- Read Hewitt chap. 8: pp. 125–128, 138–140, 143–146
- HW #3 due 4/25:
  - Hewitt 2.E.22, 2.E.29, 2.E.33, 3.E.27, 3.P.3, 3.P.4, 3.P.10, 4.E.1, 4.E.6, 4.E.10, 4.E.30, 4.E.44, 4.P.1, 5.E.17, 5.P.2, 7.R.(4&7) (count as one), 7.R.16, 7.E.40, 7.P.2, 7.P.4
- Next Question/Observation (#2) due Friday 4/25