6/29/00

Apache Point Lunar Laser Ranging Overview

This overview summarizes some of the operational parameters associated with the APOLLR effort. It is intended as a baseline to aid in basic system design and proposal development.

The following topics are covered:

Science Goals - Nordtvedt Effect

The self-energy of a body is calculated as $M_{SE}c^2 = \iint \frac{G\rho(\vec{r})\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3\vec{r}\, d^3\vec{r}'$

For a rough idea, a uniform density sphere has $M_{SE}c^2 = G\rho^2 \iint \frac{d^3\vec{r}\, d^3\vec{r}'}{|\vec{r}-\vec{r}'|} \sim G\rho^2 \frac{V^2}{R}$

with V = volume, R = spherical radius $\rightarrow$ $M_{SE}c^2 \sim \frac{GM^2}{R}$

The fractional self energy is then $\frac{M_{SE}}{M} = \frac{GM}{Rc^2} \propto \frac{M}{R}$ earth $\rightarrow 7\times 10^{-10}$

Real density profile yields $\boxed{\frac{M_{SE}}{M} = 4.6\times 10^{-10}}$

The moon then has about $\frac{M_☾}{R_☾}\frac{R_⊕}{M_⊕} = \frac{R_⊕/R_☾}{M_⊕/M_☾} = \frac{3.67}{81.3} = \frac{1}{22}$ of earth's $\frac{M_{SE}}{M} \rightarrow 2\times 10^{-11}$

Assuming for a moment that the moon orbits the sun independent of the earth (nearly true), the earth's orbit is displaced from the moon's according to:

$\frac{GM_⊙ M_g}{r^2} = \frac{M_i v^2}{r}$ where M_g & M_i are gravitational & inertial masses.

Preserving earth's orbital period $\frac{v}{r} = \omega =$ const. $\rightarrow$ $\frac{GM_⊙ M_g}{r^2} = M_i \omega^2 r$

or $r^3 = \frac{GM_⊙}{\omega^2}\frac{M_g}{M_i}$, Kepler's law. The moon's orbit (and the earth's if $M_g = M_i$) has

$a = \left(\frac{GM_⊙}{\omega^2}\right)^{\frac{1}{3}}$, defining the astronomical unit. But if $M_i \neq M_g$, $r = a\left(\frac{M_g}{M_i}\right)^{\frac{1}{3}}$

If $M_g = M_i + \varepsilon$, $r \approx a(1+\frac{1}{3}\varepsilon)$ $\rightarrow$ $\boxed{\delta r = r - a = \frac{1}{3}\varepsilon a}$

Under full violation (self energy has no inertial mass), $\varepsilon = -\frac{M_{SE}}{M} \sim -4.6\times 10^{-10}$

$\rightarrow \delta_r = -23$ m (full violation $\rightarrow |\eta| = 1$)

"Real" calculations get $\boxed{\delta_r = 13.5\,\eta \text{ m}}$ (η measures magnitude of violation)

1 cm LLR data establish η to the 10^{-3} level; $\boxed{\text{1 mm data pushes to } 10^{-4}}$

or 4.6×10^{-15} in <u>total</u> $\frac{M_g}{M_i}$

Science Goals – $\dot{G}$ and the rest of it

Kepler's law, $GM = a^3\omega^2$ can be used to accurately determine GM as a function of semi-major axis (direct range measurement) and orbital period. Of course it's not so simple in a 3+ body system, but the idea is the same.

A precise measurement of GM allows us to study $\frac{d}{dt}GM \sim \dot{G}M \rightarrow \frac{1}{GM}\frac{d}{dt}GM \sim \frac{\dot{G}}{G}$

Current measurments (over decades) limits $\frac{\dot{G}}{G}$ to $< 10^{-12}$ per year – two orders of magnitude less than a secular effect scaling with the Hubble expansion.

Measurement of $\dot{G}$ requires two things: precise measurment of GM, and <u>time</u>. One year of mm precision LLR data will do little for $\dot{G}/G$ next to the 2+ decades of lower precision measurements thus far accumulated.

Assuming the past 5-10 years of few-cm level data is largely responsible for the current level of precision on $\dot{G}$, it will take us at least 5 years to beat this down by another factor of 10.

de Sitter Precession:

This small precession of the lunar orbit can be understood as a dragging of the earth-moon inertial frame as it revolves around the sun. This precession is totally dwarfed by Newtonian effects, amounting to 0.0192 arcsec/yr (compared to 20°/year classical precession).

The measurement of de Sitter precession, like $\dot{G}/G$ (and fine wines), improves with <u>time</u>. Presently the precession is verified to agree with GR to 0.35%.

Long Range Forces:

A Yukawa-like force-law with long-range attributes will effect the rate of perigee precession – a secular effect that, of course, is measured over <u>time</u>.

I'm not too familiar with this topic, and don't know what, if any, significant impact few-mm precision range data may offer in the short term.

Sources of Error – Atmospheric Path Delay

The path delay through the atmosphere depends on the integrated refractive index ecountered along the light path.

At sea level, $n_{air}(532nm) = 1.00027820$, $n_{air}(1064nm) = 1.00027397$ [$(1-n_{air})$ is 1.5% different for the two λ's]

$(n_{air}-1)\times 10^6 = \boxed{N \approx 78\frac{P}{T}}$ P in mB, T in K

$P_{SL} = 1013$ mB at sea level $\rightarrow P(h) = P_{SL}\exp(-\frac{h}{\alpha})$ α is scale height of atmosphere

A standard temperature profile can be approximated by two linear segments:

$$T = \begin{cases} 288 - 88\left(\frac{h}{15\,km}\right) & h < 15\,km \\ 170 + 2\left(\frac{h}{1\,km}\right) & h > 15\,km \end{cases} = \begin{cases} T_0 - T_L h & h<15 \\ T_0 - 15T_L - 30 + 2h & h>15 \end{cases} \quad (h \text{ in km})$$

Another inversion exists at mesopause, but by then P is so low $(n_{air}-1)$ is inconsequentially small

$\Delta l = 10^6\int_H^\infty N\,dh$ is the <u>one way</u> path delay through the atmosphere

$$\Delta l = 0.0790\left[\int_H^{15}\frac{e^{-h/\alpha}}{T_0 - T_L h}\,dh + \int_{15}^{\infty}\frac{e^{-h/\alpha}}{T_0 - 15T_L - 30 + 2h}\,dh\right]$$

T_0 is sea level temperature
T_L is lapse rate in troposphere

Under STP, and with scale-height = 7 km $\rightarrow$ from sea level $\Delta l = 2.2080$ m $H = 0$
from McDonald $\Delta l = 1.7052$ m $H = 2.075$ km
From APO $\boxed{\Delta l = 1.5611 \text{ m}}$ $H = 2.781$ km

<u>80%</u> of this delay occurs in tropopause.

Keep in mind the sec z term: twice the path delay at 30° elevation ($z = 60°$).

If the APO station is at 5°C, the nominal Δl is 1.5195 m. How does this change with changing conditions?

$\frac{\partial}{\partial P}\Delta l = \frac{1}{1013}\,1.5195\text{m} = \boxed{1.5 \text{ mm mB}^{-1}}$ for ΔP expressed as sea level change

$\frac{\partial}{\partial T}\Delta l = \boxed{-6.3 \text{ mm K}^{-1}}$

$\frac{\partial}{\partial T_L}\Delta l = \boxed{41 \text{ mm (K/km)}^{-1}}$ lapse rate, T_L assumed nominally 5.87 K km^{-1}

$\frac{\partial}{\partial \alpha}\Delta l = \boxed{316 \text{ mm km}^{-1}}$ scale height nominally 7 km

Typical pressure of ~ 0.1 inch over a few hundred miles (say 3 mB over 500 km) effectively changes scale height when looking at elevation θ in direction of gradient.

$\alpha' = \alpha\frac{\tan\theta}{\tan\theta - \alpha\frac{\Delta P}{\Delta x}\frac{1}{P}}$ $\rightarrow$ 7.0005 km for 30° elevation angle $\rightarrow$ <u>0.16 mm</u> delay difference
vs 7.000000

$\Rightarrow$ contrary to an earlier computation, we see that typical pressure gradients have little effect.

Sources of Error – Atmospheric Path Delay

It seems, therefore, that 1mm modelling of the atmosphere requires:

- measurement of T within 0.2°C
- measurement of P within 0.07 mB
- knowledge of lapse rate to 0.025 K/km (0.4%) → dep. on humidity
- accurate scale height to 3m!

Each of these seems rediculously impossible (even meaningless)

I can believe each of these can be had with 10× the error, delivering cm-level atmospheric delay. Seems like a modeling wall to me.

A source I just found indicates that the Marini-Murray model produces the following gradients at zenith:

$\frac{\partial \Delta l}{\partial P}$ = 1.5 mm mB^{-1} in agreement with my calculation

$\frac{\partial \Delta l}{\partial T}$ = 0.01 mm K^{-1} (as opposed to 6.3) !!

$\frac{\partial \Delta l}{\partial H}$ = 0.05 mm $\%^{-1}$ where H is relative humidity in %

Why the $\frac{\partial \Delta l}{\partial T}$ predictions vary by $\sim 10^4$ I cannot guess. Sure, my model is dirt-simple, but if the lapse rate is held fixed (ie., same humidity), and the troposphere is well mixed I'm not sure how my deal can go too wrong. – $\Delta l \sim \frac{P}{T}$ $\frac{\Delta T}{T} \sim \frac{1}{250}$ for 1° ΔT → expect 0.4%, or 6mm deviation.

For the moment accepting the gradient values reported in the paper, measurement of pressure is the critical uncertainty given typical measurement errors – which are reported at 0.5 - 1 mbar, 0.7 - 1.5 K, 5-10% RH.

I still consider the atmosphere a modeling "wall" below the 5-10mm level, but this certainly deserves more investigation.

Sources of Error – Crustal Deformations

The earth & moon both experience relatively predictable crustal motions – mainly due to tides. These bulk motions (30-50 cm on earth, I think) are fairly easy to model & fit.

Additional loading of earth's crust by sea and air are less easy to model, yet are said to exist at the several mm level.

My main questions: Is ocean loading relevant at Apache Point – ~~far~~ inland? Can we not throw in another term to the fit: $\alpha_{154}P$ to express the unknown atmospheric loading as a simple linear function of pressure (which we measure anyway)?

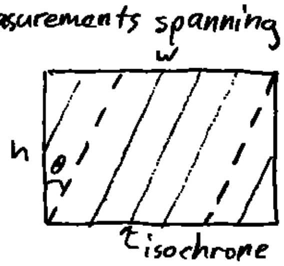
Barring this, an SLR tie-in may be the most effective means of eliminating these uncertainties.

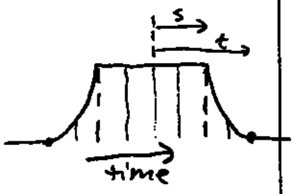
Sources of Error – Retro-reflector Array Orientation

This source of error ought to behave in a random rather than systematic sense, provided the reflector efficiency is uniform across the array.

The moon's libration – caused by variable revolution speed resulting from the elliptical orbit – effectively tilts the arrays away from normal incidence. The tilt is often around 5°-10° in magnitude, resulting in range measurements spanning ~ 10 cm.

In general, the rectangular array is spanned by a parallel family of constant-time (range) lines (isochrones?) at an angle determined by the libration angle relative to the array's north-south orientation





The return pulse profile will, in general, be composed of quadratic tails and a flat top, though either may be missing for special orientations.

If the laser pulse profile and timing jitter profiles are well understood (which is possible from calibration pulses) then we should <u>see</u> the retro-reflector array induced pulse broadening – not only verifying orientation angle, but also allowing us to characterize spatial throughput variations on the arrays. This will enable any systematic correction that may be necessary owing to non-uniform, skewed, pulse return shape.

Referring to above diagrams, the array is $h \times w$, with isochrone angle θ. The pulse is characterized by times s & t, measured from center → FW@ 0% Max = $2t$, FW@ 100% = $2s$

$$s = \tfrac{1}{2}(w - h\tan\theta)\cos\theta$$
$$t = s + h\sin\theta$$

$$\text{FWHM} = 2s + (2-\sqrt{2})h\sin\theta$$
$$\text{RMS} = \sqrt{\frac{s^2}{3} + \frac{3}{(t-s)^3}\left[\frac{t^5-s^5}{5} - \frac{t^5-ts^4}{2} + \frac{t^5-t^2s^3}{3}\right]}$$

for $\tan\theta < w/h$

if $\tan\theta > w/h$, exchange $h \leftrightarrow w$
$\sin\theta \leftrightarrow \cos\theta$
$\tan\theta \leftrightarrow \frac{1}{\tan\theta}$

Apollo 11 & 14 arrays have $h = w = 0.46$ m, Apollo 15 has $h = 0.6$ m, $w = 1.0$ m

If the librations in longitude & latitude fill a 7°×7° box, with equal probabilities at each point, we get the following return pulse characteristics:

		mean (ps)	median (ps)	σ (ps)	max (ps)	realistic mean* (ps)
Apollo 11, 14:	FWHM	198	202	77	$\boxed{374}$	$\boxed{212}$
	RMS	52	53	21	108	56
Apollo 15:	FWHM	374	351	176	$\boxed{812}$	$\boxed{424}$
	RMS	101	93	49	240	112

* Assumes sinusoidal variation of libration angles (independently) → spends more time @ high angles

Sources of Error – High Signal Bias

Unless we can figure out a meaningful way to centroid the time of return of a packet of photons (distributed over ~500 ps), we need to try to time-tag each detected photon, which almost certainly means each photon finds a different detector.

One way to do this is with a sequence of beam splitters, each with a 5-10% probability of reflection.

Another way is via an APD array oversampling the image of the return. This has many additional advantages, mainly owing to preservation of spatial information.

In either scheme, one wants each detector to have < 50% chance of catching a photon in each pulse. Otherwise, multiple events would be common, which distorts the inferred pulse return times (i.e., offsets the centroid). This is what I mean by "high signal bias."

The effect of this bias depends on how our system works. Namely, details of APD quenching combined with stop-pulse discrimination determine the effect of high-signal returns.

Two photons generating e-h pairs can have several outcomes. Assuming the first photo-electron triggers an avalanche breakdown, the second photo-electron can:

A) participate in the avalanche in progress, adding signal & duration to the pulse.
B) fail to initiate an avalanche, despite ripe conditions (still biased above breakdown).
C) find the APD quenched below breakdown, thereby generating a weak pulse at best

Case B is essentially indistinguishable from a normal single-photon event.

In Case A, the output pulse is shaped differently from a single-photon pulse. Either the pulse is simply broadened or there are two discernable peaks. In any case, the method of discrimination used by the interval counter in responding to a stop pulse will most likely ignore the second photon, responding only to the leading edge caused by the first photon.

In Case C, the second photon is sufficiently squelched that the stop pulse timing is based solely on the first photon.

Upshot: second photon is ignored, systematically biasing return pulse time to earlier values.

Treatments:

1) Throw out high-signal returns to avoid bias

2) Lower laser power if condition is persistent

3) If detector efficiencies are well understood, apply a correction factor, probably based on Monte Carlo simulations.

Hardware - Laser

Pulse width: no need to do much better than $\boxed{\sim 150\,ps}$ due to lunar libration

Rep rate: limited by detector dead time, atmospheric backscatter, need to multiplex with calibration pulses
Must keep system blocked off for ~ 2 × 200 km → 1.3 ms to avoid backscatter → dead time of detector insignificant by comparison

To have appreciable "windows", rep rate $\boxed{< 300\text{ Hz, preferably} < 100\text{ Hz}}$

How fast does round-trip time vary? $c\Delta t = 2(r_{\oplus\text{☾}} - R_{\oplus}(1-\cos\theta)) + \text{const.}$
where $r_{\oplus\text{☾}}$ is "static" distance, θ is hour angle of moon

time deriv $= -2R_{\oplus}\sin\theta \frac{d\theta}{dt}$ → if $\theta = 30°$ (2 hrs over), $\frac{dr}{dt} \sim 465$ m/s
typical 10 min. obs. → 280 km round-trip variation → $\boxed{1\text{ ms variation}}$

∴ windows ought to be large compared to 1 ms → < 100 Hz is comfy

Nd:YAG at 532 nm or 1064 nm deserves primary attention, for the following reasons:

- Ubiquitous, versatile, with excellent physical properties/durability
- Off-the-shelf Q-switched, mode locked, doubled (or not) items
- Plethora of optics to choose from (mirrors, lenses, filters) coated for 532 or 1064
- Turn-key operation, minimum cavity fussing — no need to become laser jocks

The following parameters assume a Nd:YAG laser

Pumping: Probably require flashlamps for high energy, but would be much happier with diode-pumped (Need to see if these can deliver bang for buck)

Pulse energy: Peak power limited to ~ 1 GW → pulse energy in mJ = pulse width in ns
→ 100 - 150 mJ/pulse

Average Power: Probably can't push beyond ~ 3 W, limiting rep rate to ~ 20 Hz

Hardware – Laser (continued)

<u>Nd:YAG wavelength:</u> to double or not to double

Doubling efficiency can be quite high, so no worries about power loss @ 532 nm

Reasons <u>to</u> use 532 nm:

- Detector efficiency high
- Thinner detector → less time jitter
- Easy to work with in lab / less of a hazard
- Can <u>see</u> beam backscatter, which may aid pointing

Reasons <u>not</u> to use 532:

- Startle factor for pilots still an issue even if eye safe
- Lunar surface at peak intensity @ 532
- half the photons per pulse compared to 1064
- backscatter from air 16 times worse than @ 1064
- sky transmission at absolute best at 1064 nm, but only 5-10% effect

Just found out that thick silicon APDs are only 3% efficient* @ 1064 and other materials are too far behind to be of much use.

⇒ Really must go <u>with 532</u>

* Thin APDs are only 0.1% efficient at 1064 nm.

Hardware - Detector

Importance of an array:

- with 5-10 photons/pulse (detected), need multiple detectors to catch individual photons
- array in image plane provides guider feedback, focus diagnostics, alignment diagnostics
- small area of each array element practically eliminates background (i.e., moon)

Given that we want a compact array, PMTs are out, <u>APDs in</u> [Microchannel Plates?]

APDs can have high detection efficiencies (30-50%) in single photon-counting Geiger mode. Cooling to just -20°C virtually eliminates dark noise (this statement deserves verification/exploration - it may still be as high as 10 counts/sec).*

APD arrays for Geiger mode operation are being developed. I'll bet a key difficulty is in achieving uniformity in thickness (and therefore breakdown voltage) across the array such that all elements operate in similar regime with same V.

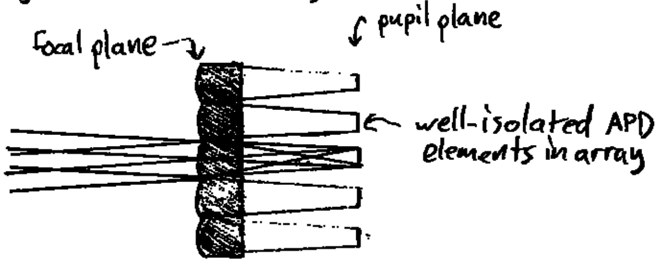
We'll need some understanding of multi-photon response of the APD in Geiger mode. In other words, aside from the dead-time after the avalanche, what is the time resolution? Will a second photon within 100 ps participate in the avalanche spurred by the first, or is the diode dead by then?

We may want to limit dark current by only applying breakdown voltage at times within a few 10's of ns of expected photon arrival time. This strategy would also be very effective at reducing false triggers from background photons.

If seeing, focus, etc. are good, I would vote for 0.2 arcsec pixels, dividing a 1" disk into ~ 20 elements — easily handling 5-10 photons detected per pulse. A 10×10 array would then be a convenient 2 arcsec across, allowing effective guider feedback, focus/alignment diagnostics, etc. A 5×5 array would not make these tasks impossible — just harder.

One technological hurdle in APD array development is cross-talk between elements: an avalanche generates photons that can trigger the neighboring element to break down. We could potentially relax the areal fill-factor requirements, allowing the elements to be better isolated. A lenslet array (becoming commonplace & easy to find) can then make up for incomplete filling.

Additional benefit to lenslet array:
smaller active area → more precise timing.



* Dark current pales by comparison to lunar background, so it is of little concern.

Hardware - Clock

we need a clock.

Absolute precision necessary: Earth rotates 1 mm (at equator) in 2 μs

GPS - slaved Quartz clock provides time accurate to 110 ns accuracy 95% of time (without selective availability - the current condition - this is probably 20 - 40 ns).

Quartz has superior short-term stability, on par with the high-performance cesium standard over 1-10s timescales (lunar round-trip is 2.4 sec).

Quartz or cesium clocks have 5×10^{-12} stability over 1-10s → a few mm

At first I assumed we could not tolerate a 100 ns absolute time error, but I can't remember now what my specific objection was. The fastest relative motion between earth & moon stations is due to earth rotation. Even the 30 km/s earth orbital motion goes 1 mm in 30 ns, but absolute earth/moon position relative to the sun is irrelevant to this precision

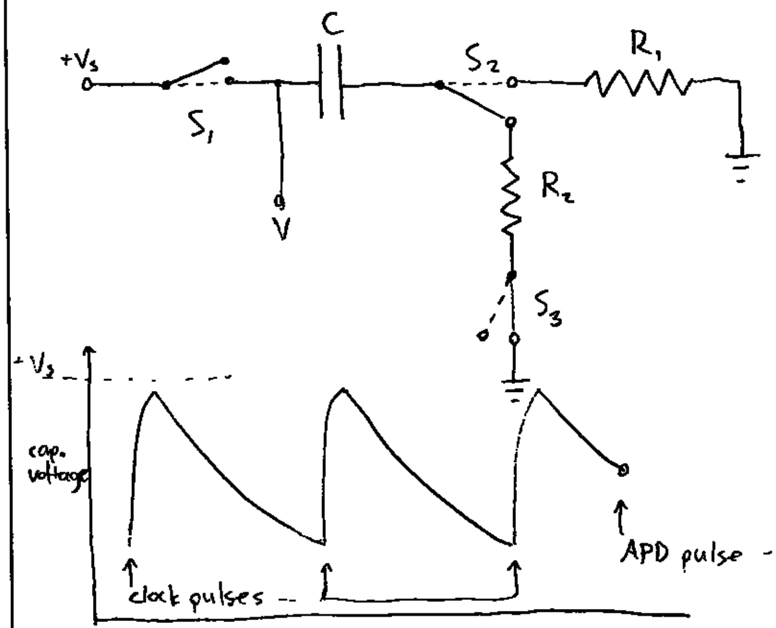
Price difference is extreme -- \$6 k vs \$70 k, both produce 10 MHz output reference.

Hardware – Small Interval Counter

We need to resolve the 10 MHz clock output to $\sim 5\times10^{-5}$ ($\rightarrow$ 5 ps) accuracy

The McDonald scheme uses a counter to count 10 MHz intervals, then a vernier to measure the time between the last clock pulse and the stop pulse from the detector.

A schematic vernier is as follows:



S_1 : normally open—clock pulse closes temporarily

S_2 : travels with S_1 - normally points to R_2

S_3 : normally closed - opened at APD pulse closed again @ clock pulse

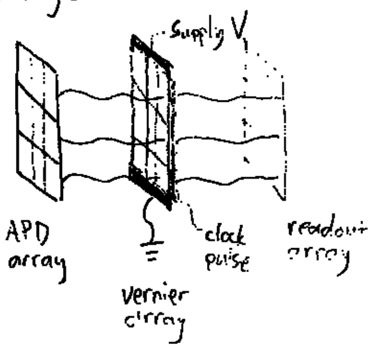
$R_2C \gtrsim 100$ ns

$R_1C \lesssim$ few ns

Of course the actual verniers are more complex/linear/reliable

As a black box, vernier has 5 inputs: supply voltage, clock pulse, APD pulse, ground, V_{meas}.

The perfect solution would be an array of micro-verniers fabricated on a sheet and placed between APD array and array of FET-based voltage measurement / A/D stage



$\Rightarrow$ output is array of voltages indicating time of signal pulse from APD

How realistic is this scheme? I wish I knew. Probably fabrication is too expensive both in time & money.

What do high energy guys do for timing arrays of detectors? My guess: racks & racks of electronics plus <u>lots</u> of wires.

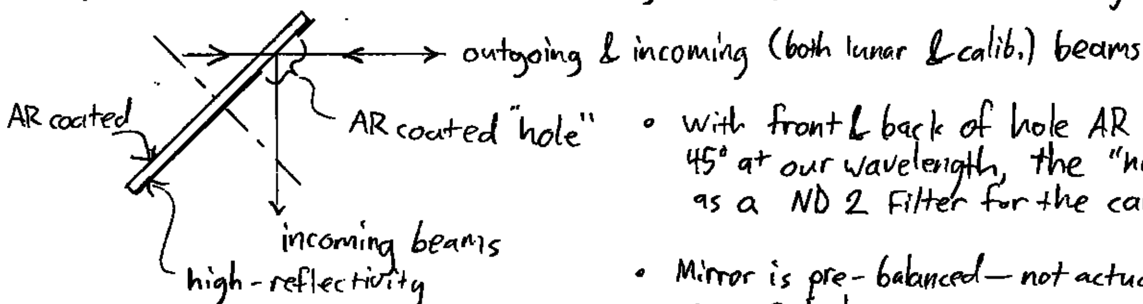
Hardware – Optics & Other Equipment

The simplest part of this system (by-and-large) is the optical system, as we only care about monochromatic light.

It would be most convenient to work with a collimated beam less than 25 mm in diameter, keeping the arrangement compact and the optics small. But with, say, a 20 mm beam, the outgoing pulse will deliver a peak power density of ~300 MW cm^{-2} — enough that normal AR coatings may be damaged, so that we'll want to have specially coated optics along the power train.

If we want a fully illuminated 3.5 m exit aperture (which leads to an eye-safe beam and an undamaged telescope), then there's no getting around a moving optic – e.g., a mirror with a hole through it, rotating.

The McDonald folks had a calibration path differing from their lunar return path, which we may want to avoid if we require sub-mm metrology. Therefore it may be worth investing in the following arrangement for a rotating mirror:



- With front & back of hole AR coated for 45° at our wavelength, the "hole" also acts as a ND 2 filter for the calibration beam.
- Mirror is pre-balanced — not actually lacking glass @ hole

This type of mirror would need to be made of quality material, with good surfaces, as it must transmit outgoing pulse with immeasurable distortion. Also, the back surface ought to be slightly wedged at the hole location to separate front surface (keeper) from back surface (rejected) reflections. Balance issues can be compensated by symmetric treatment of rear surface. Manufacture of this wedge bit could get pricy, though. Perhaps a better solution is a thicker (self-shielding) mirror.

Narrow band interference filters with 1 nm FWHM, 30-50% throughput, and OD = 4 out-of-band rejection are easy to find. By tilting the filter for calibration pulse, this may act as a variable high-attenuation filter.

We probably also want a PLZT electronically activated filter – if these have high enough transmission. This is for the purpose of knocking out calibration photons

Another useful device to have on the optical bench is a streak camera. These are expensive, but potentially very important in understanding the systematics associated with the pulse width, shape, & stability. On the other hand, if our calibration scheme and APD array timing scheme work well, delivering ~10 photons per pulse, we'll have an in situ pulse profile measurement, with the ability to monitor stability over 10-30s (plus) time scales.

System Performance — Signal

The APO aperture, combined with good seeing/optics offers us the chance to routinely see 2-10 photons per pulse.

The number of detected photons (minus speckle-induced variability) is given by:

$$N_d = N_l \eta^2 f Q \left(\frac{n d^2}{r^2 \Phi^2}\right)\left(\frac{D^2}{r^2 \phi^2}\right)$$

N_l = # photons launched: 100 mJ/pulse → $\boxed{2.7\times10^{17}}$ photons @ 532, 5.4×10^{17} @ 1064

η = one way shared optical efficiency (steering/diverging optics, scope, atmosphere) → $\boxed{\sim 0.5}$

f = Narrow-band filter plus PLZT (?) : Worst case $\boxed{\sim 0.25}$

Q = quantum efficiency of detector in Geiger mode → $\boxed{0.3}$

d = corner-cube prism aperture : 0.038 m for all Apollo reflectors
n = # of retro-reflectors in array: 100 or 300
D = telescope aperture = 3.5 m
Φ = outgoing divergence ≈ 1 arcsec
ϕ = retroreflector divergence → as bad as 10 arcsec

These numbers range from 7 to 44 (532 nm on 100 element array vs. 1064 on 300) photons detected per pulse.

Speckle structure puts the median return at ~ ½ the average, with average or better about 30% of the time. ⅓ of average (~1 photon) ~ 70% of time.

Go ahead and knock the estimate down by another factor of two — still in 5 photon regime!

With the above numbers, overall "lab" efficiency is $\eta^2 f Q = 0.019$, which seems modest enough.

At 20 Hz, we'll get ~100 photons per second — the content of a typical "normal point".

One throughput variable not considered above is self-shadowing of the retroreflector array, which drops to $\boxed{60\%}$ throughput at a net libration angle of <u>10°</u>

System Performance – Background & Noise

The moon is brighter than the daytime sky, so the lunar surface brightness is the dominant source of background photons from natural sources.

The moon, at -13 mag ($\leftarrow -12.7$), has a surface brightness of 3 mag arcsec^{-2}

$V = 3$ mag $\rightarrow$ 2.17×10^{-9} W m^{-2} μm^{-1}

In 3.5 m aperture, this leads to 2.09×10^{-11} W nm^{-1} arcsec^{-2} $\rightarrow$ 5.6×10^{7} photons s^{-1} nm^{-1} arcsec^{-2}

or $\boxed{0.056 \text{ photons ns}^{-1}\text{ nm}^{-1}\text{ arcsec}^{-2}}$ arriving at telescope

Using $\eta f Q$ from throughput calculations takes this down to $\boxed{0.002 \frac{\text{photons}}{\text{ns}\cdot\text{nm}\cdot\text{arcsec}^2}}$ $\rightarrow$ detected $\leftarrow$

* Therefore our signal can not become confused by background photons.*

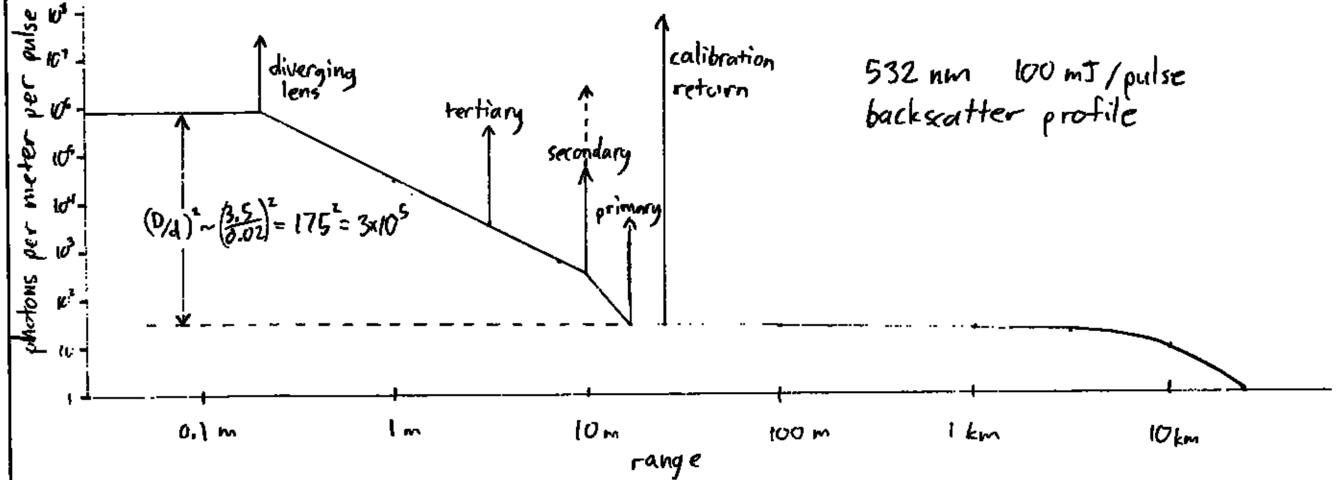
But we still have to worry about false triggers, shutting down the APD element for some time (dead times of 100–200 ns are typical)

Probability that background photon strikes somewhere in array (2×2 arcsec?) within one "dead time" of expected lunar photon arrival approaches 1!

Gating the detector (few 10's ns window when high voltage is applied) not only alleviates this problem, but also reduces probability of dark current hits. You just have to make sure the high voltage is on at the right time.

Backscatter @ 532 nm is 1.2×10^{-6} m^{-1} sr^{-1} at APO altitude

For a collimated "laboratory" beam diameter of 20 mm ($175\times$ smaller than exit beam) this corresponds to 10^{6} photons per meter sent back to a 2" square array from the collimated beam within the lab. Once beyond the scope, it's 30 photons per meter sent back toward the detector, falling off as atmospheric density decreases



* For McDonald, detected background in 5" aperture $\rightarrow$ 2×10^{-3} photons ns^{-1} nm^{-1}
$\rightarrow$ one in five difficult to distinguish from signal (which is 10^{-2} per pulse).

System Performance – Background & Noise (continued)

Integrated air backscatter toward detector (2"×2" area)

- collimated section: $0.2\,\text{m} \times 10^{6}\,\text{m}^{-1}$ = 2×10^{5} photons $\propto (D/d)^2$
- enroute to secondary: ⇒ 2×10^{5} photons $\propto (D/d)^2$
- secondary to primary: 3×10^{3} photons $\propto (D/d)^2$
- atmosphere 3×10^{5} photons

$D = 3.5$ m
d = collimated beam diameter (assumed 20mm)

Keeping the section of outgoing collimated beam that is visible to detector <u>short</u> is important

A bigger collimated beam helps the backscatter, at the substantial cost of scaling all optics

To put backscatter into perspective, retro-reflector for collimation sends

$$N_{cal} \approx N_\ell \times \left(\frac{d_{cc}}{D}\right)^2 \left(\frac{\phi_{det}}{\lambda/d_{cc}}\right)^2 = N_\ell \left(\frac{\phi_{det}\, d_{cc}^2}{\lambda D}\right)^2$$

where N_ℓ = # launched, ϕ_{det} = angular scale of detector (2")
d_{cc} is corner cube diameter, λ = wavelength

$\rightarrow \sim 7\times10^{10}\left(\frac{d_{cc}}{1\text{cm}}\right)^4$ photons, overwhelming integrated backscatter

Therefore, the filtering used for calibration will completely block any backscatter.

Lens surfaces, and especially dusty ones, will also send many photons back.
Worst case: 1% of surface scatters into 4π →

$$N_{scat} = 0.01 N_\ell \frac{(D/d\ \phi_{det})^2}{4\pi} \sim 6\times10^{8} \text{ photons} \rightarrow \sim 1\% \text{ of calibration strength}$$

One last source of background to consider: our laser <u>returns from lunar surface.</u> Using range equation with 0.5 albedo, 2π scatter → $\boxed{\text{0.026 photons per pulse}}$ could be <u>detected</u>. The range will be many ns off, but can still trigger the APDs! This noise source is more significant than the sunlit lunar surface for us, owing to the fact that these photons have the right passwords for our spatial/spectral/temporal filters.

Oh – one question of relevance: does the APO secondary have a central hole? If not, approximately 1.3×10^{9} photons head back toward the detector real estate directly off the secondary. This could potentially serve as a calibration source (in lieu of a retro-reflector) if its position is known absolutely as a function of telescope focus. Otherwise, it is a relevant noise source as the calibration is concerned.

System Performance — Eye Safety

The number quoted by Eric Silverberg regarding eye safety was 1 μJ. There was some question as to whether this is in 1 cm^2 or into an eyeball.

Taking the conservative 1 $\mu J\ cm^{-2}$ limit, the APO area of $\sim 9 \times 10^4\ cm^2$ allows a dosage of ~ 110 mJ. I assume the figures relate to total intake. If only one pulse is taken in, we're probably fine with a "laboratory" laser energy of ~150 mJ per pulse. Various inefficiencies will eat this number down to acceptable levels by the time it hits United Flight 264.

But are we dealing with one pulse or more? At 20 Hz rep rate, and a beam diameter of 3.5 m, and a 60° ~~zenith~~ angle (worst case), a plane going slower than 140 m/s (~ 280 knots) could be exposed to two pulses. Thus commercial airliners are in the one-pulse category, small commercial planes two-pulse, and small general aviation planes (at ~ 100 knots) are subject to three. Fortunately, situated at 9,500 feet, the cone up to 12-14,000 feet (where small planes fly) has very little volume. Plus, flying over 9000+ foot mountains at night isn't a popular activity — smart pilots fly over the valleys.

Before we deem our system eye-safe, we should get the official word on what safe is.

System Performance - Calibration

Good, real time calibration is of fundamental importance to our proposed measurement. We want every shot to be accompanied by a calibration return, comparable in photon density to the lunar return (~0.3 photons detected per array element).

The advantage to an array detector is that even if each element receives less than one calibration photon on average, as long as element-to-element timing offsets are static, the entire array is calibrated by a handful of detections.

The array-based calibration also provides real-time (or nearly so) pulse width/shape analysis, quantifies timing jitter, etc.

A typical scheme involves placing a retro-reflector at the telescope aperture and carefully surveying its position relative to the fixed intersection of telescope axes. At the mm level we'll have to worry about thermal expansion of the telescope structure, and surveying may be something of an art.

With a diameter d_{cc}, the cornercube smears light out over at least λ / d_{cc} radians, such that the total return to the detector spanning ϕ_{det} radians on the sky is:

$N_{cc} = N_\ell \left(\frac{d_{cc}}{D}\right)^2 \left(\frac{\phi_{det}}{\lambda / d_{cc}}\right)^2 = N_\ell \frac{d_{cc}^4 \phi_{det}^2}{\lambda^2 D^2}$: N_ℓ = # photons launched, D is telescope aperture

for a 2" detector & 532 nm, we get $7 \times 10^{10} \left(\frac{d_{cc}}{1\,\text{cm}}\right)^4$ photons sent back.

Using $\eta f Q \sim 4\%$ (as was used in signal calculations) $\rightarrow$ $\boxed{2.8 \times 10^9 \left(\frac{d_{cc}}{1\,\text{cm}}\right)^4}$ detected $h\nu$, which is down to 2×10^8 photons for a 0.5 cm corner cube.

Therefore, the corner cube approach necessitates $\sim 10^8$ attenuation

Another potential callibration scheme can work if the telescope secondary mirror has no central hole. In this case, a fraction of the outgoing beam is returned as a diverging beam toward the receiver optics.

Assuming an f/10 telescope with an f/2 primary, the focal plane will be approx. 10 m back from the secondary $\rightarrow$ ϕ_{det} on sky looks like $\frac{35\,\text{m}}{10\,\text{m}} \phi_{det} = 3.5 \phi_{det}$ from secondary. The secondary spreads light into an f/2 beam ($\frac{1}{2}$ radian), so the fractional solid angle spanned by the detector array is $\sim (7\phi_{det})^2$. If $\phi_{det} = 2'' = 10^{-5}$ radian $\rightarrow$ 5×10^{-9} of photons return to detector. With 2.7×10^{17} photons launched $\rightarrow$ 1.3×10^9 photons head back, and after $\eta f Q = 4\%$, $\boxed{5.4 \times 10^7}$ are detected.

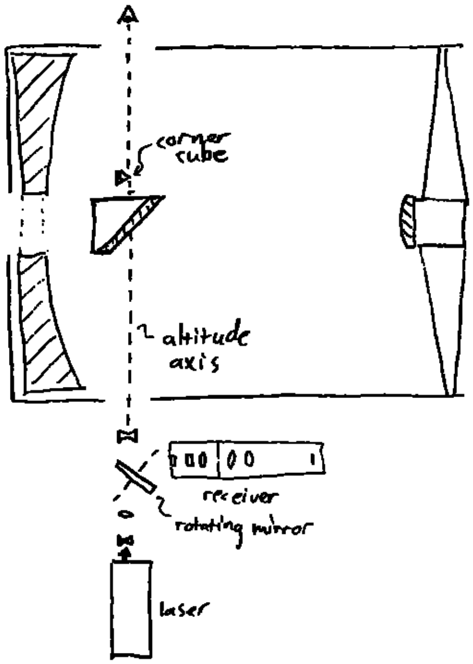
The secondary return certainly competes with the corner cube return so that if the corner cube scheme is used, the corner cube return must be made larger than the secondary return by at least a factor of 10.

The actual spot size on the secondary is ~50 μm, assuming a ~0.7 m secondary diam. Hope there's no dust on that particular spot!

If the secondary provides the calibration signal, special care must be taken to know where the secondary is in an absolute sense. Complicating matters, the secondary moves for focus control. However, this motion generally compensates thermal expansion/contraction of the telescope structure. If the optics are low-expansion glass material (surely they are) then the physical spacing between primary & secondary doesn't change appreciably if focus is maintained.

Note that when focus is maintained, the secondary position is more stable than that of a corner cube at the exit aperture (near top of telescope).

An alternate placement of the corner cube near the intersection of axes may eliminate much confusion about thermal expansion and flexure. If I remember correctly, the APO scope has a Nasmyth focus, which means there's a tertiary mirror very near the axis intersection. Attaching the retro-reflector behind this mirror, if carefully done, will put the corner cube precisely on the altitude axis. This minimizes the need for surveying — just watch how it moves when the telescope moves in altitude.



Besides ease of locating w.r.t. the telescope's fixed coordinates and simplicity of surveying, this placement also puts the retro-reflector in the good central part of the beam where illumination is likely more uniform than out near the edge.

I think we want the back tip of the corner cube on the axis, as this represents the optical midpoint between entrance surface & exit surface (same physical surface). We'll need to remember to compute the refractive delay within the prism.

In this position, care must be taken that the prism doesn't shadow itself. The mounting arm must allow light to pass through from the secondary to the primary between the corner cube and the tertiary. The mount may also need to be retractable so-as not to interfere with other observations.

Now with photons coming back from the corner cube and/or secondary (would it really be so bad if we got both?), we need to address the pesky issue of:

Attenuation

With no attenuation, we expect 3×10^9, 2×10^8, or 5×10^7 ~~detected~~ photons across the array (2" array assumed) for 10 mm diameter, 5 mm diameter corner cubes, or secondary, respectively. The rotating mirror employing AR coatings instead of an "air hole" may knock 10^2 off this, if the 45° incidence AR coat can transmit 99% of the photons. This leaves about 10^6 of necessary attenuation for the corner cubes, or 10^5 for the secondary. The goal is to record 10–30 photons per pulse on the array (assuming 10×10 array).

System Performance - Calibration (continued)

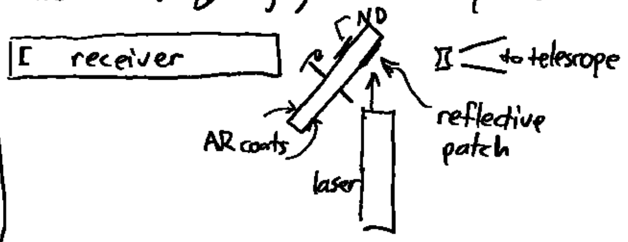
A single device capable of swinging from > 50% transmission to 10^{-6} is probably non-existent.

Two PLZT's may work. I haven't been able to find any information on PLZT properties, but my <u>guess</u> is that they run from 70-80% to 10^{-3}-10^{-4} transmission.

Alternatively, if the narrow-band interference filter is tilted 10°-20°, it becomes an O.D. 3-4 filter (10^{-3}-10^{-4}) at the laser wavelength. But rapidly jerking this optic this-way-and-that lacks appeal.

There is always the "dog bone" option of a rotating ND filter in sync. with the rotating mirror, but one moving mechanism seems like enough.

One nice idea that had not occurred to me when I schemed up a layout was to swap the positions of the laser & receiver relative to the rotating mirror, and invert AR & highly reflective parts:



Now the laser does not go through the rotating mirror, eliminating one high-power surface with scaterers. Furthermore, the receiver is better isolated from the beam due to the highly reflective surface.

Now the back of the rotating mirror can have a high O.D. (or reflective) patch coated behind the mirror patch - further eliminating scattered light and knocking down the calibration signal. Only one moving part, easily balanced, and accomplishing several functions at once.

I'll bet between the two coatings we can get the 10^8 we need. A supplemental PLZT may still be desireable for control of the calibration signal level.

Note that the highly reflective surface on the rotating mirror is not aluminized but a dielectric interference coating, which means that the light not reflected is transmitted rather than absorbed as would happen with a metal coating.

I believe the above configuration (denoted by brace) is a winner - solving the attenuation and backscatter protection problems.

System Performance - SLR tie in

In my opinion, the cm-level uncertainties in atmospheric delay and crustal motion are unlikely to become mm-level uncertainties by our bold modeling efforts.

If we let the SLR community tackle these problems, they are sure to result in knowledge of the two LAGEOS orbits to <5mm precision. Potentially, their modeling advances will be transferable to our modeling effort. Failing this easy transference, we still have an accurate orbit as a reference.

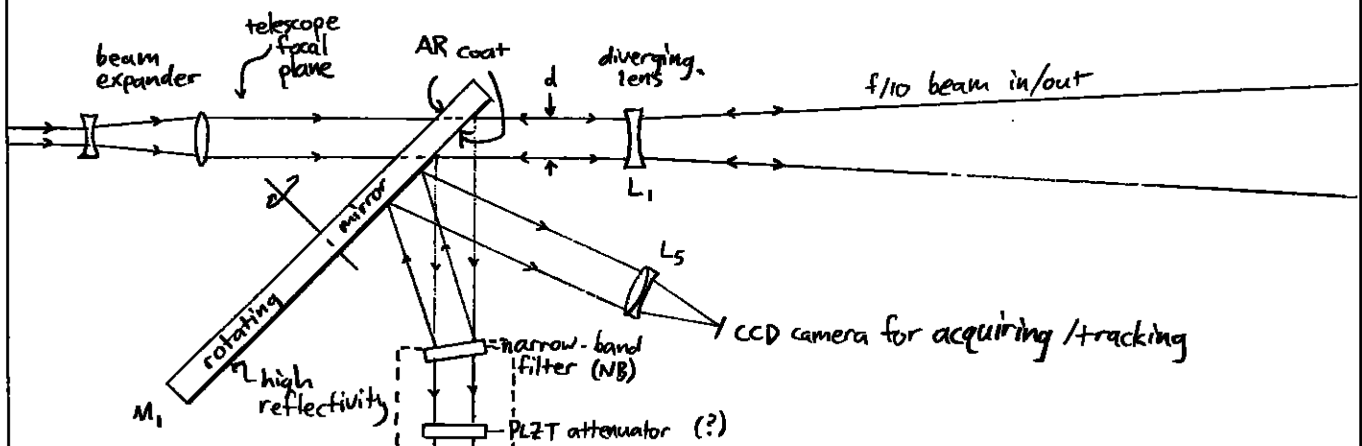
Ranging to the LAGEOS satellites from APO near the epoch of lunar ranging will enable us to measure the atmospheric delay & crustal displacement relative to the well established orbit (which is measured daily around the world).

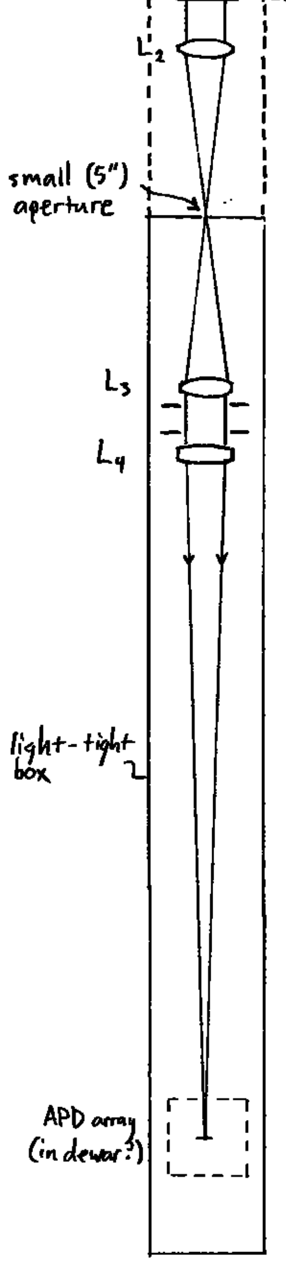
It would be nice to have SLR capability with the same instrumental setup, though we may not want to burden the 3.5m with a task easily performed by a separate, compact unit. We should get info on this SLR 2000 system. Can we afford one? Can we convince someone interested in SLR that they should set up at Apache Point? Can astronomers cope with a nearby, independent laser operation?

Our sensitive lunar setup would detect ~ 10^8 photons per pulse from LAGEOS! We could degrade this almost arbitrarily with telescope defocus (also eases pointing/tracking demands). We could also bump the laser power down significantly either via laser tuning or by reflecting most of the beam to a dump.

Even if we have a separate, independent SLR setup, establishing the offset between the two systems would be greatly facilitated by SLR capability from the 3.5m.

A secondary, but very important benefit of doing some SLR from the 3.5 m is that we can use SLR to iron out the system before tackling the moon. Going full power on LAGEOS, we'll see the return pulse in our CCD tracking camera! That's a heck of a good way to verify system operation – when you know you are or are not illuminating the target.





Telescope is aperture D, focal length F

For collimated beam diameter d, L_1 has $f_1 = \frac{d}{D}F$

For 5″ stop to be 1 mm, L_2 has $f_2 = \frac{d}{D}\frac{1}{5''}$ mm

L_3 is same as L_2

For 1″ on array to be 1 mm, L_4 has $f_4 = \frac{d}{D} 206265$ mm

$f_4 = 5 f_3$

ϕ_d is span of detector on sky

detector spans $\frac{D}{d}\phi_d$ from within collimated beam

Guideline numbers:

$d = 20$ mm $\rightarrow \frac{D}{d} = 175 \rightarrow f_1 = 200$ mm

$f_2 = f_3 \simeq 200$ mm $\rightarrow$ 1 mm $\approx 6''$ on sky at stop

$f_4 \approx 1$ m $\rightarrow$ 1 mm $\approx 1.2''$ on sky at detector

It would pay to increase distance between L_3 & L_4 allowing meaningful baffling to be placed here.

Light-tight box should really include the narrow-band filter – perhaps most effectively as its window

Bottom Line Summary

see page

- ~1 mm precision data tests self-energy equivalence principle to $\cancel{10^{-5}}$ 10^{-4} level — 2
  Further improvements by watching over many lunar orbits.
- To achieve factor of 10 better limits on $\dot{G}$, de Sitter precession, long-range forces (?) — 3
  requires TIME. Perhaps 5 years for good baseline. with NO EP viol (precession)
- Atmospheric path delay ~ 1.6 m one-way, 80% in troposphere. — 4
- Large-scale pressure gradients produce negligible error. — 4
- Big confusion over path delay dependence on temperature. — 5
- Is ocean-loading relevant in NM? Can pressure term model atmospheric loading? — 6
- Apollo 11 & 14 arrays have FWHM ~ 210 ps, sometimes ≳350 ps — 7
  Apollo 15 has FWHM ~ 420 ps, as big as 800 ps
- Too many return photons and/or too few detectors → bias to shorter range measurements. — 8
- Laser parameters: ~ 150 ps FWHM, < 100 Hz rep. rate, highly recommend Nd:YAG — 9
- Gotta go with 532 nm rather than 1064 nm — 10
- At all costs, develop APD array — APDs probably best for us — 11
- Maybe a cheap GPS-slaved clock is adequate for us — 12
- I don't know what the hell I'm talking about when it comes to small interval timing — 13
- Have to have at least one moving optic; need special high-power coatings — 14
- Photons-a-plenty — 15
- Moon background no problem w.r.t. confusion; may still want to gate detector — 16
- Cool backscatter plot — 16
- Backscatter completely overshadowed by calibration pulse, secondary may be significant — 17
- Probably eye-safe with 3.5 m beam — 18
- Corner cube vs. secondary photon returns for calibration — 19
- Corner cube placement & relative reliability of secondary vs. corner cube calibration — 20
- Clever rotating mirror → calibration/backscatter attenuation taken care of — 21
- Should do some SLR from 3.5 m, but also good to supplement with SLR 2000 — 22