7.2 The color of the sky

EFT's ① dof: photon field A_μ + atom dof ϕ

② symms: Lorentz, C, P, T, conservation of atom # $\phi_v \to e^{i\alpha}\phi_v$

③ cutoff: ΔE = gap to excited states of e^- in atoms.

$$E_\gamma \ll \Delta E \sim \frac{\alpha}{a_0} = m_e \alpha^2 \ll m_e \alpha = \frac{1}{a_0} \ll m_e \ll M_{atom}$$

Let ϕ_v create an atom w/ velocity v^μ

$v^\mu v_\mu = 1$ $\qquad v_\mu = (1, \vec{0})_\mu$

in the rest frame

$$L_{atom} = \phi_v^\dagger \, i v^\mu \partial_\mu \phi_v \overset{\text{rest frame}}{=} \phi^\dagger i \partial_t \phi$$

$$H = \pi \dot{\phi} - L = 0 \qquad \text{Rest energy} = 0.$$

(could have added $\gamma_v M_{atom} \phi_v^\dagger \phi_v$)

$$\gamma_v = \frac{1}{\sqrt{1-v^2}}.$$

Could add: $\phi_v^\dagger \frac{\nabla^2}{2M_{atom}} \phi_v$

$$L = L_{Maxwell}[A] + L_{atom}[\phi_v] + L_{int}[A, \phi_v]$$

local, real, gauge inv't Lorentz inv't

made from: $\cancel{\phi_v}$, $F_{\mu\nu}$, v_μ, ∂_μ

(not A)

atom # $\Rightarrow$ $\phi_v^\dagger \phi_v$.

$$L_{int} = c_1 \phi_v^\dagger \phi_v F_{\mu\nu} F^{\mu\nu} + c_2 \phi_v^\dagger \phi_v v^\sigma F_{\sigma\mu} v_\lambda F^{\lambda\mu}$$

$$+ c_3 \phi_v^\dagger \phi_v (v^\lambda \partial_\lambda) F_{\mu\nu} F^{\mu\nu} + \dots$$

$[\partial_\mu] = 1$, $[F] = 2$ $\Rightarrow$ $[\phi_v] = 3/2$ $[v] = 0$

$\Rightarrow$ $[c_1] = [c_2] = -3$ $[c_3] = -4$

all irrelevant.

expect: $c_{1,2} \sim \left(\frac{1}{\Delta E}\right)^3$ or a_0^3.

$$\sigma \propto c_i^2 \sim a_0^6 \qquad [\sigma] = -2$$

$$\Longrightarrow \sigma \propto a_0^6 \underline{\underline{E_\gamma^4}}$$

$$\left(A \propto E_\gamma^2 \quad \text{from } E \propto \partial_t A\right)$$

$$E_{blue} \sim 2 E_{red} \qquad \sigma_{blue} \sim 16\, \sigma_{red}.$$

$$c_1 \phi^2 (E^2 - B^2) + c_2 \phi^2 (E^2)$$

$$\sigma \propto E_\gamma^4 a_0^6 \left(1 + \underbrace{O\left(\frac{E_\gamma}{\Delta E}\right)}_{\text{important for UV light.}}\right)$$

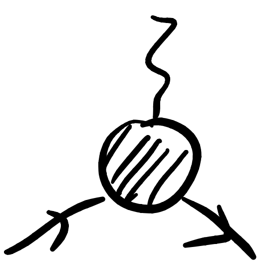
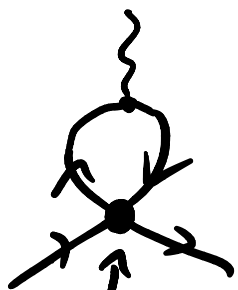
7.4 Loops in EFT

eg: $\mathcal{L}_F = \bar\psi(i\underset{\uparrow A}{\cancel{D}} - m)\psi + G_F\,\underline{\underline{\bar\psi\gamma\psi\,\bar\psi\gamma\psi}}$

$+ \mathcal{L}_{Max}(A) \qquad + \frac{1}{M_W^4}(\partial^2\bar\psi\gamma\psi)\psi\gamma\psi$

$+ \mathcal{O}\left(\frac{\partial^2}{M_W^2}\right)^{\ell}$

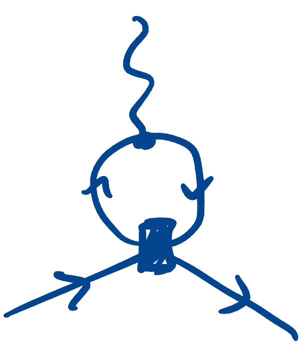
my: $|k_E| < \Lambda \sim M_W$

 $\ni$  $\sim I = \frac{1}{M_W^2}\underbrace{\int^{\Lambda} d^4k\,\frac{1}{k}\frac{1}{k}}_{\sim \int^{\Lambda}\frac{d^4k}{k^2} \sim \Lambda^2 \sim M_W^2}\,\mathrm{tr}(\ldots)$

$G_F \sim \frac{1}{M_W^2}$

$\sim \frac{\Lambda^2}{M_W^2} \sim \mathcal{O}(1).$

A

$\ni$  $\sim I_\ell = \frac{1}{M_W^2}\underbrace{\int d^4k\,\frac{1}{k^2}\left(\frac{k^2}{M_W^2}\right)^{\ell}}$

$= \int d^4k\,\frac{k^{2(\ell+1)}}{M_W^{2\ell}}$

$\sim \mathcal{O}(1).$

Fix: "mass-independent subtraction scheme".

eg: dim reg + $\overline{MS}$.

claim: $I \sim \frac{m^2}{M_W^2}\log\mu \ll I_\ell \sim \left(\frac{m^2}{M_W^2}\right)^{\ell+1}\log\mu$

$m \sim m_e$, or p, or Λ_{QCD}.

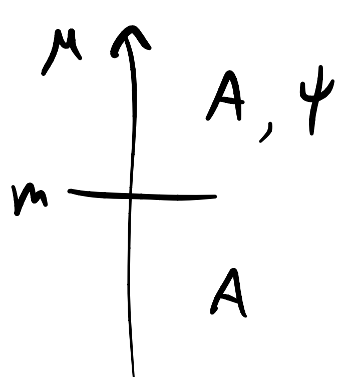
Some IR scale, **NOT** M_W.

"It respects the power-counting".

price:
- heavy particles of mass M don't decouple for $\mu < m$.

  eg β_g depends on heavy particles.
- $\Rightarrow$ pert. thy breaks down for $\mu \ll M$.

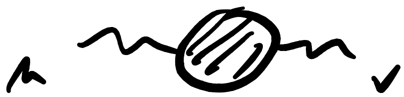
Soln:



integrate out heavy fields by hand & make a new EFT.

Comparison of EFT schemes in an example:

$$\mathcal{L}_{QED} = -\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} - \bar{\Psi}(i\not{D} - m)\Psi$$



$$\longrightarrow \Pi^{\mu\nu}(q^2) = \underline{\underline{\Pi(q^2)}}\, P^{\mu\nu}(q)$$

$$P^{\mu\nu}(q) = q^2 \eta^{\mu\nu} - q^\mu q^\nu .$$

Mass-dependent scheme:

subtract the value of Π at $p^2 = -M^2$

M: RG scale

$$\Pi(p^2 = -M^2) \overset{!}{=} 0$$

In dim reg: $\Pi_2^{\mu\nu} = P^{\mu\nu}\, \delta\Pi_2$

$$\delta\Pi_2(p^2) = -\frac{e^2}{2\pi^2}\int_0^1 dx\, x(1-x)\left(\frac{2}{\epsilon} - \log\frac{\Delta}{\mu^2}\right)$$

$$\Delta = m^2 - x(1-x)p^2 .$$

$$0 = \Pi_2^{(M)}(p^2 = -M^2) = \delta_{F^2}^{(M)} + \delta\Pi_2(p^2 = -M^2)$$

$\delta_{F^2}^{(M)}$: Counterterm for $F_{\mu\nu}F^{\mu\nu}$

$$\Pi_2^{(M)}(p^2) = \frac{e^2}{2\pi^2}\int dx\, x(1-x) \log\left(\frac{m^2 - x(1-x)p^2}{m^2 + x(1-x)M^2}\right)$$

(note: μ's go away)

$\boxed{\overline{MS}}$

$$\delta_{F^2}^{(\overline{MS})} = -\frac{e^2}{2\pi^2}\frac{2}{\epsilon}\int_0^1 dx\, x(1-x)$$

$$= -\frac{e^2}{6\pi^2}\frac{1}{\epsilon}.$$

$$\Rightarrow \Pi_2^{(\overline{MS})}(p^2) = \frac{e^2}{2\pi^2}\int_0^1 dx\, x(1-x) \log\left(\frac{m^2 - x(1-x)p^2}{\mu^2}\right)$$

$\boxed{M}$

$$\beta_e^{(M)} = \frac{e}{2} M \frac{\partial}{\partial M} \Pi_2^{(M)}(p^2)$$

why:

$$\mathcal{L}_{QED} = -\frac{1}{4 e_R^2 \mu^\epsilon} Z_{F^2} (F^0_{\mu\nu})^2$$

$\uparrow$ bare field

$$e_0 = e_R \mu^{\epsilon/2} Z_{F^2}^{-1/2} \Leftarrow \quad \equiv -\frac{1}{4 \underline{\underline{e_0}}^2} (F^0_{\mu\nu})^2$$

$$0 \stackrel{!}{=} M \frac{d}{dM}(e_0)$$

$$Z_{F^2} \equiv Z_3 = 1 + \delta_{F^2}.$$

$$0 = M\frac{d}{dM}(e_0) = M\frac{d}{dM}\left(e_R \cancel{\mu^{\epsilon/2}} Z_{F^2}^{-1/2}\right)$$

$$= e_0\left(\beta_e - \frac{1}{2}\frac{M}{Z_3}\frac{d}{dM}Z_3\right)$$

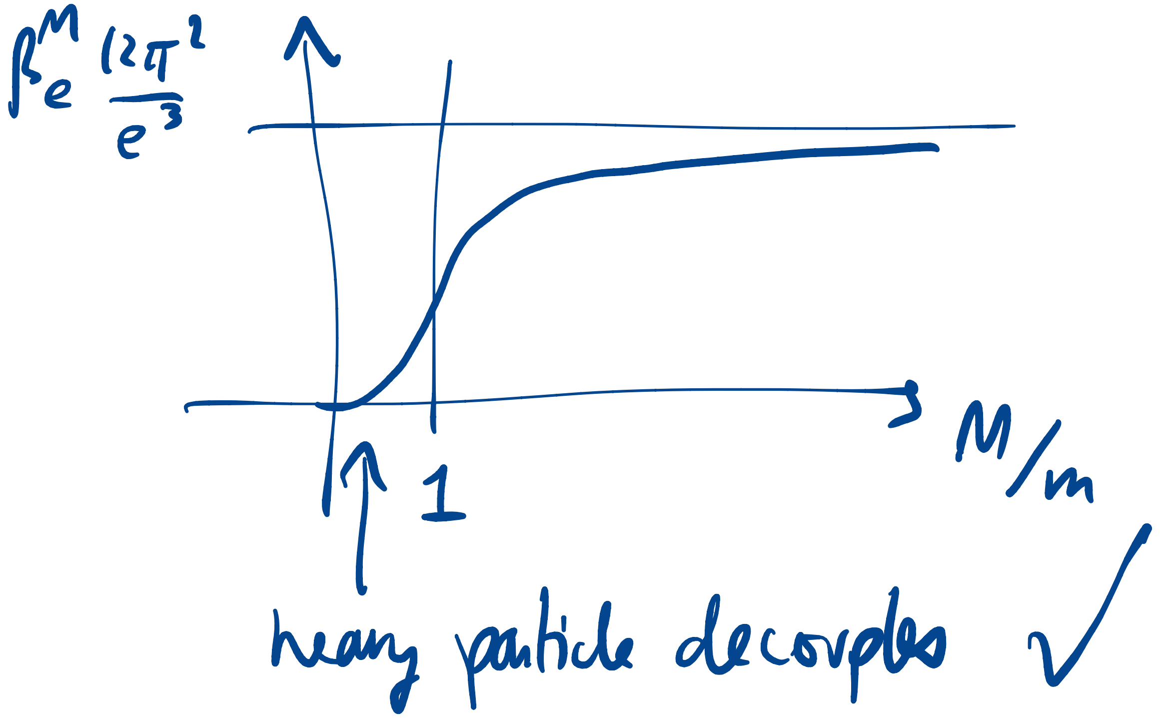
$$\Longrightarrow \beta_e = \frac{1}{2}e\frac{M}{Z_3}\frac{d}{dM}Z_3$$

$$= \frac{e}{2}M\frac{d}{dM}\left(\frac{e^2}{2\pi^2}\int dx\, x(1-x)\left[\frac{2}{\epsilon} - \log\frac{m^2+x(1-x)M^2}{\mu^2}\right] + \dots\right)$$

$$= -\frac{1}{2}\frac{e^3}{2\pi^2}\int dx\, x(1-x)\left(\frac{-2M^2x(1-x)}{m^2+M^2x(1-x)}\right) + O(e^5)$$

$$\overset{m \ll M}{\simeq} \frac{e^3}{2\pi^2}\int dx\, x(1-x) = \frac{e^3}{12\pi^2}$$

$$\overset{m \gg M}{\simeq} \frac{e^3}{2\pi^2}\int dx\,(x(1-x))^2\frac{M^2}{m^2} = \frac{e^3}{60\pi^2}\frac{M^2}{m^2}.$$



$\overline{MS}$ $\quad \beta_e^{\overline{MS}} = \frac{e}{2}\mu\partial_\mu \Pi_2^{\overline{MS}}(p^2)$

$$0 = \mu\frac{d}{d\mu}(e^0) = e^0\left(\frac{\varepsilon}{2} + \frac{\beta_e}{e_R} - \frac{1}{2}\frac{\mu}{Z_3}\frac{d}{d\mu}Z_3\right)$$

$$\Rightarrow \beta_e = -\frac{\varepsilon}{2}e + \frac{1}{2}e\,\mu\frac{d}{d\mu}\log Z_3$$

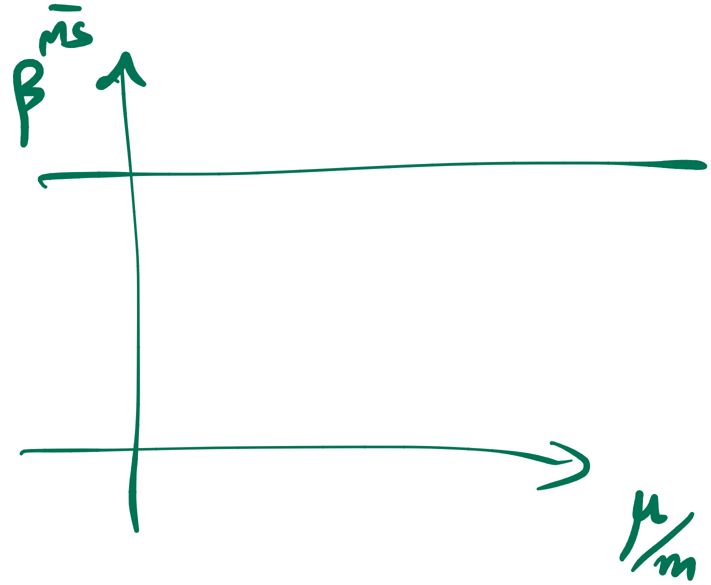
$$Z_3 = 1+\delta_3 = 1 - \frac{e^2}{6\pi^2}\frac{1}{\epsilon} + O(e^4)$$

$$\Rightarrow \beta_e = -\frac{\varepsilon}{2}e + \frac{1}{2}\beta_e\left(-\frac{e^2}{6\pi^2}\frac{1}{\epsilon}\right) + \dots$$

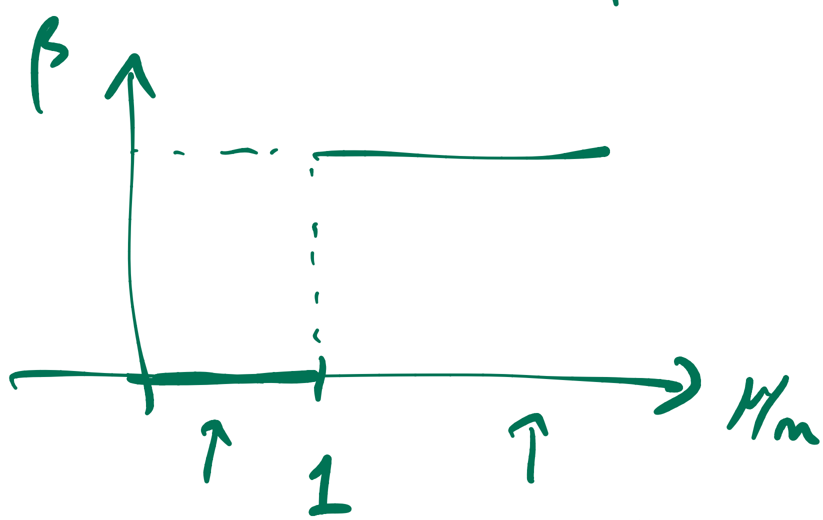
$$= -\frac{\varepsilon}{2}e + \frac{1}{2}\left(-\frac{\cancel{\varepsilon}}{2}e\right)\left(-\frac{e^2}{6\pi^2}\frac{1}{\cancel{\epsilon}}\right) + \dots$$

$$\Rightarrow \beta^{\overline{MS}} = -\frac{\epsilon}{2} e + \frac{e^3}{12\pi^2} + \mathcal{O}(e^5)$$

$$\overset{D \to 4}{=} \frac{e^3}{12\pi^2}$$



Resolution:



$S_{IR}[A]$ $\qquad$ $S_{UV}[A_\mu, \psi]$

match at scale $\mu = m$

Who?

Photon interactions at $E_\gamma < m_e$. (in vacuum)

① dofs : photon

② syms : Lorentz , C, P, T

no charges $\Rightarrow$ $\mathcal{L} = \mathcal{L}(F_{\mu\nu})$

③ cutoff : m_e.

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + c_1 F_{\mu\nu}\partial^\rho\partial_\rho F^{\mu\nu}$$

$$+ c_2 (F_{\mu\nu}F^{\mu\nu})^2$$

$$+ c_3 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \ldots$$

$[c_1] = -2$

$c_1 \propto \frac{1}{m_e^2}$.

$\Pi(q^2) = \Pi(0) + q^2\Pi'(0) + \ldots$

$\Pi'(0)$ determines c_1.

$[c_2], [c_3] = -4$

$c_{2,3} \sim \frac{1}{m_e^4}$

$F_\mu{}^\rho F_\rho{}^\nu F_\nu{}^\mu$ is forbidden by C : $A_\mu \to -A_\mu$.

In the UV theory (QED): IR theory:

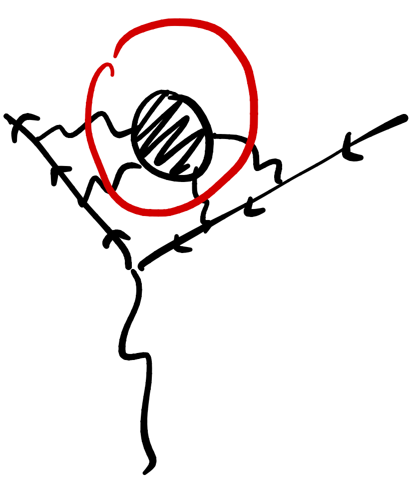
$c_{2,3}$

$$c_{2,3} \propto \frac{\alpha^2 \leftarrow \text{4 vertices}}{16\pi^2 \leftarrow \text{loop}}$$

"Naive Dimensional analysis".

$$\sigma(\gamma\gamma \to \gamma\gamma) \sim \frac{\alpha^4}{(16\pi^2)^2} \frac{E_\gamma^6}{m_e^8} \left(1 + G\left(\frac{E_\gamma}{m_e}\right)^2\right)$$

$$A \sim E_\gamma^4 \qquad \text{phase space} \sim \frac{1}{E_\gamma^2}.$$



7.5 The S.M. as an EFT

① dofs:

	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	ν_R	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	H
	Weyl fermions						scalar
SU(3)	·	·	·	□	□	□	·
SU(2)	□	·	·	□	·	·	□
hypercharge → $U(1)_Y$	$-\frac{1}{2}$	-1	·	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

$\boxed{\times 3}$

$Y_L + 3Y_Q = 0$ is required by anomaly cancellation.

$\Longrightarrow$ $p = \epsilon_{ijk} u_i u_j d_k$ has opposite electric charge from e^-.

② symms: Poincaré ⊃ Lorentz, CPT.

③ cutoff: ?

$\mathcal{L}_{SM} =$ all gauge inv't, Lorentz inv't terms of dim ≤ 4.

$\mathcal{L}_{SM} \ni V(|H|) = m_H^2 |H|^2 + \lambda |H|^4$ $\qquad$ $m_H^2 < 0$.

$$SU(2)\times U(1)_Y \overset{\langle H\rangle}{\rightsquigarrow} U(1)_{EM}$$

$$\underbrace{SU(2)\times U(1)_Y}_{\text{4 generators}} \qquad \underbrace{U(1)_{EM}}_{\text{1 generator}}$$

$$\mathcal{L} \ni \left.|D_\mu H|^2\right|_{H=\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}}$$

SU(2) generators $a=1,2,3$

$$D_\mu H = \left(\partial_\mu - i g W_\mu^a \tau^a - \frac{1}{2} i g' Y_\mu\right) H$$

$$= M_W^2 W_\mu^+ W^{-\mu} + M_Z^2 Z_\mu Z^\mu$$

$$M_W = \frac{vg}{2}.$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ Y_\mu \end{pmatrix} = O\begin{pmatrix} W \\ Y \end{pmatrix}$$

choose O to diagonalize M^2.

$$\Rightarrow \tan\theta_W = \frac{g'}{g} \qquad M_\gamma = 0,\ M_Z = \frac{M_W}{\cos\theta_W} < M_W.$$

$$\mathcal{L}_{\text{Yukawa}} = -Y^{l}_{ij}\, \underbrace{\bar{L}_i H}_{SU(2)\text{ singlet}}\, e^j_R - Y^{u}_{ij}\, \underbrace{\bar{Q}^i H}\, d^j_R$$

$$- Y^{d}_{ij}\, \underbrace{\bar{Q}^i (i\tau^2 H^*)}\, u^j_R$$

$$+ h.c.$$

i j are flavor indices

$$\mathcal{L}_{\text{Yukawa}}\Big|_{\langle H\rangle} = - m_e\, \bar{e}_L e_R + h.c.$$

$$m_e = y_e\, v/\sqrt{2}$$

<u>Mnemonic</u>: SU(3) $\rightarrow$ $\left(\begin{array}{c|c} T^A_{SU(3)} & 0 \\ \hline 0 & 0 \end{array}\right)$, $\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & T^a \end{array}\right)$ $\leftarrow$ SU(2),

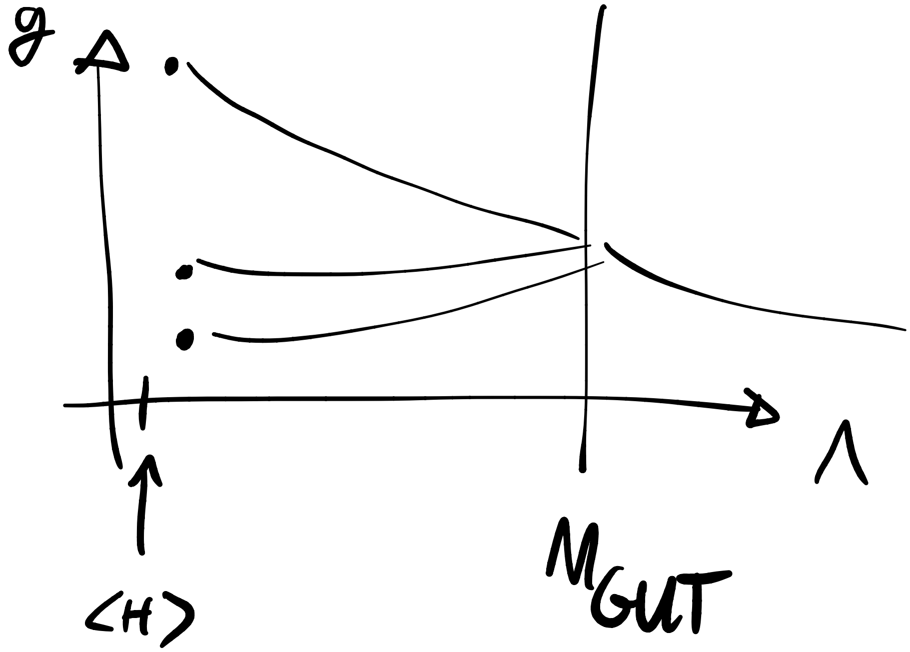
$$\left(\begin{array}{ccc|cc} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ \hline & & & 1/2 & \\ & & & & 1/2 \end{array}\right) \leftarrow U(1)_Y \qquad \in \left\{ \begin{array}{c} \text{generators} \\ \text{of } SU(5) \end{array} \right\}$$

$$SU(3) \times SU(2) \times U(1)_Y \subset SU(5) \subset SO(10)$$

$$\text{one generation} = \underline{10} \oplus \underline{\bar{5}} \oplus \underline{1} = \underline{16}$$

$\underline{1} \leftarrow \nu_R$

$$\underset{d_R}{(3,1)_{1/3}} \oplus \underset{L^c}{(1,2)_{+1/2}} = \bar{5}$$



$$\langle H_{GUT} \rangle$$

$$SU(5) \xrightarrow{\langle H_{GUT} \rangle} SU(3) \times SU(2) \times U(1)_Y$$

$$d_3 = dt\,\frac{\partial}{\partial t} + d_2$$

$$a = d_2\phi \qquad a_0 = 0.$$

$$a \wedge d_3 a \qquad \boxed{d_2^2 = 0.}$$

$$dt\,\frac{\partial}{\partial t}(d_2\phi) \neq 0.$$

$$S[\phi] = \int_{\partial\Sigma} \mathcal{L}(\phi)$$

$$=$$

<u>Stokes</u>: $\displaystyle\int_\Sigma d(\omega) = \int_{\partial\Sigma} \omega$

$$\int dt \int_{UHP} d_2\omega = \int dt \int_{y=0} \omega$$

$$u(3) = \underbrace{SU(3)}_{\text{8 traceless}} \times \underbrace{U(1)}_{\text{1 trace part}}$$

baryon #.

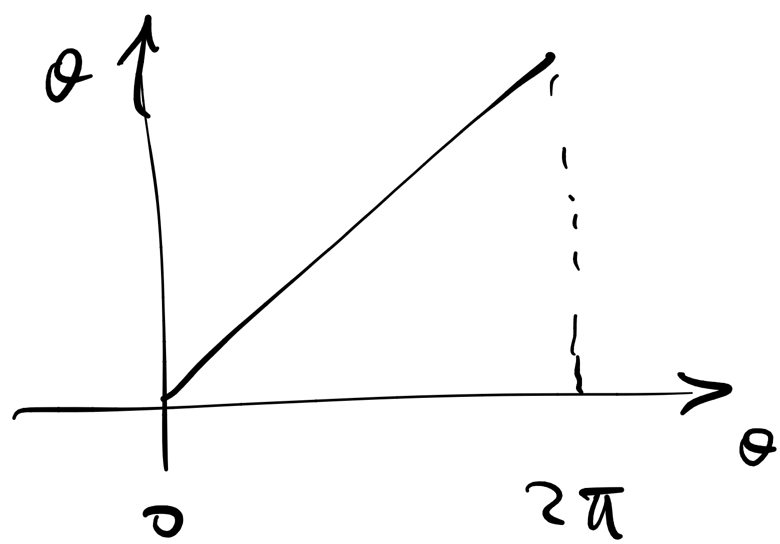
$e^{i\alpha^A T^A}$

3x3 hermitian

$q^i \to e^{i\alpha} q^i$

$$n \oint d\theta = n$$

$$= \int_{S^1} -i g^{-1} dg$$



winding #. $\in \mathbb{Z}$

$$e^{\underbrace{S_{\text{eff}}[A]}} = \int Da \, e^{\int \frac{k}{4\pi} a\, da + J A}$$

$$J^{\mu}(x) = \frac{\delta S}{\delta A_{\mu}(x)} = \underline{\frac{\delta}{\delta A_{\mu}} \log Z}$$

$$= \langle \underline{J^{\mu}(x)} \rangle$$

$$S_{\text{eff}}(A) = \int A\, D A \qquad \sigma^{xy}$$

$$J_x = \frac{\delta S}{\delta A_x} = (DA)_x \quad \propto \sigma E_y$$

$$\partial_0 A_y$$