Physics 239: Topology from Physics

OH: after lec. or by email appt
or email questions!

work: (A) problem sets < weekly

(B) Convey a nugget of truthy goodness
in 2 PRL pages.

(C) Find typos & errors in lecture notes
& problem sets & email me.

0.1 Introductory Remarks

Goals: Use physics to explain alg. topology.

chapter headings: (physical system) **and** (math. concept)

overview of topology in many-body physics:

Top. insulators: simple physics (free fermions) — fancy math

Intrinsic topological order: physics requiring interactions, yet to be conclusively discovered in Earth-rocks → §1 homology, cohomology → homotopy groups.

§1 — §4

Generally covariant physics:

topological $\overset{?}{\equiv}$ independent of a choice of metric.

eg 1: $Z_{GR} \overset{?}{=} \int \underline{\underline{\mathcal{D}g_{\mu\nu}}}\, e^{-I[g_{\mu\nu}]}$

$\uparrow$

ind. of $g_{\mu\nu}$

eg 2: $Z_{\text{Chern-Simons}}(M_3) = \int \mathcal{D}A\, e^{-I_{CS}[A]}$

$$I_{CS}[A] = \frac{k}{4\pi}\int_{M_3} \mathrm{tr}\left(\underline{\underline{A\wedge F}} + \frac{2}{3}\underline{\underline{A\wedge A\wedge A}}\right)$$

$F = dA + A\wedge A$

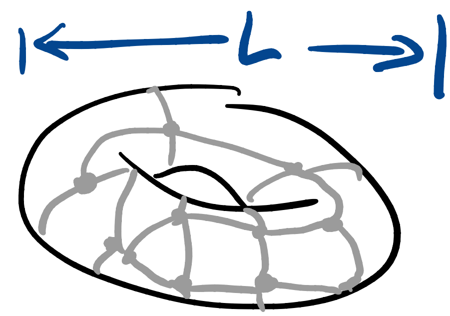
- self-dual Yang-Mills eqns
  $\longrightarrow$ Donaldson invariants of 4-mflds.

- take a supersymmetric field theory & twist
  eg 4d $N=2$ super-Yang-Mills $\to$ Donaldson thy.

§ 2.

eg 3: Gapped phases of quantum matter

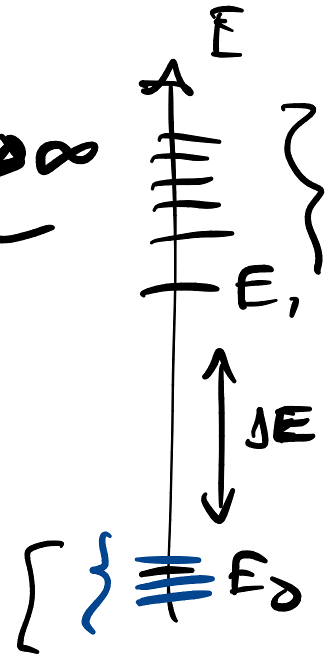
context: Take a manifold X
chop into simple pieces.



$\mathcal{H} = \bigotimes_x \mathcal{H}_x$ $\qquad H = \sum_x \underline{\underline{H_x}}$ acts only near x.

A <u>groundstate</u> is <u>gapped</u> if

$$\Delta E = E_1 - E_0 > 0 \quad \text{as } L \to \infty$$



(<u>vs:</u> massless field has

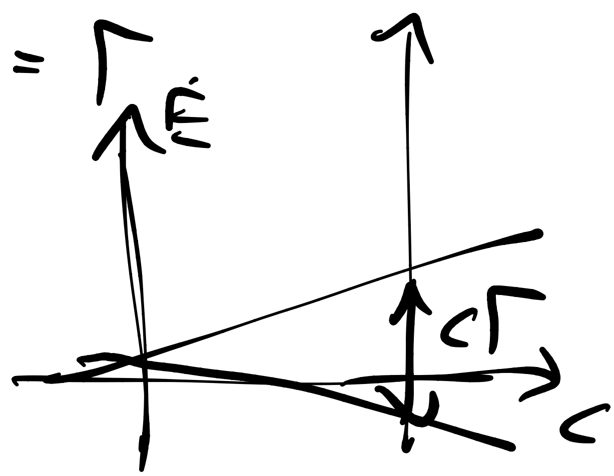
$$\Delta E \sim \frac{1}{L^2} \to 0 \quad \text{as } L \to \infty.$$
)

allow multiple ground states not related by local <u>operators</u>

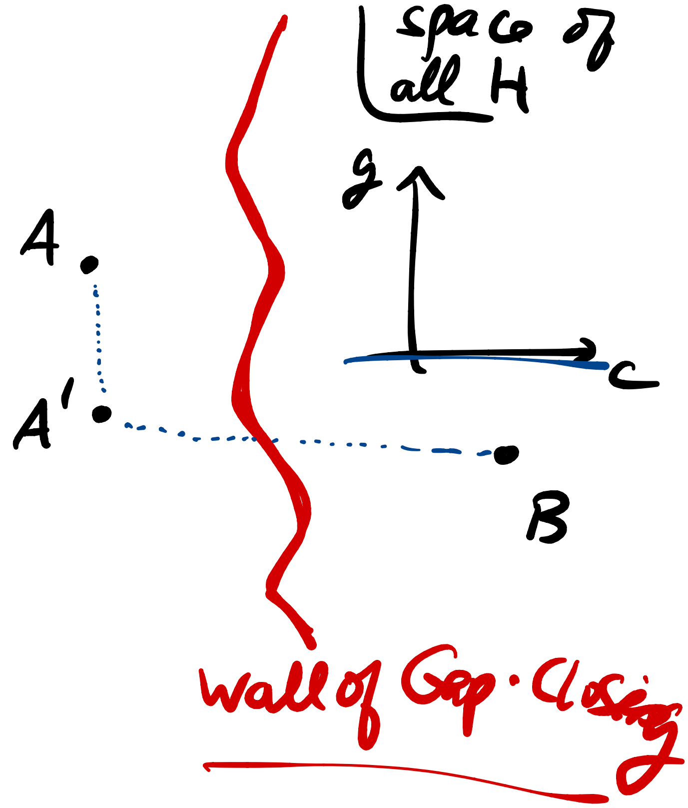
$$\langle E_i | \mathcal{O}_x | gs \rangle \neq 0 \quad \text{only for excited states } E_i.$$

<u>suppose:</u> $\langle gs_1 | \mathcal{O}_x | gs_2 \rangle = \Gamma$

$$\Delta H = \sum_x c\, \mathcal{O}_x$$



$$[A] = [A'] \neq [B]$$



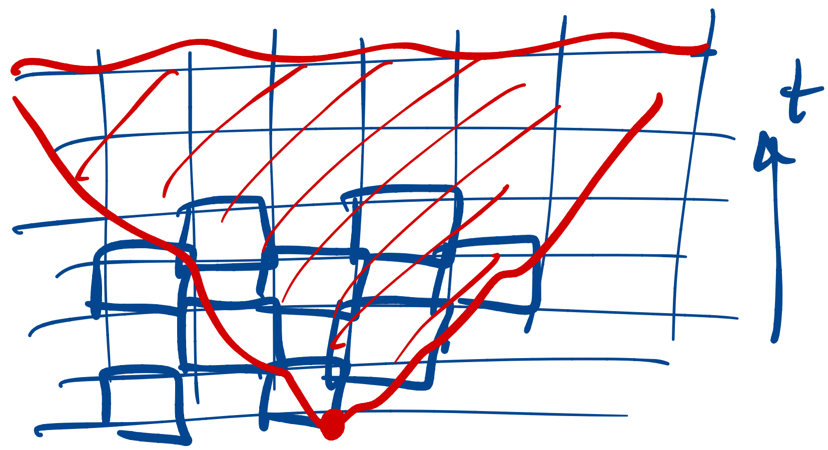
$$\underbrace{T e^{i\int_0^T dt\, H(t)}} |\text{gs of } A\rangle$$
$$= |\text{gs of } A'\rangle + \dots$$

$H(0) = H_A$, $H(T) = H_{A'}$

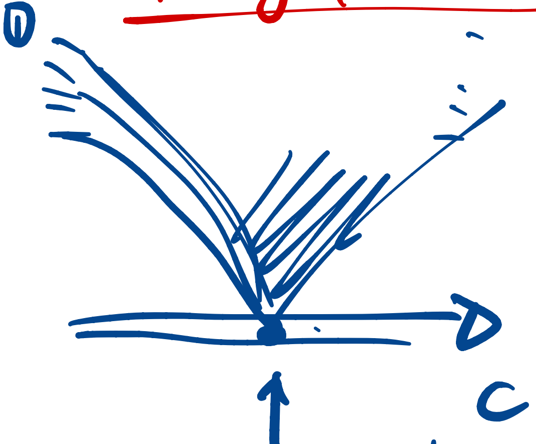
with $T \sim O(L^0)$

$U \equiv$ finite-depth unitary.

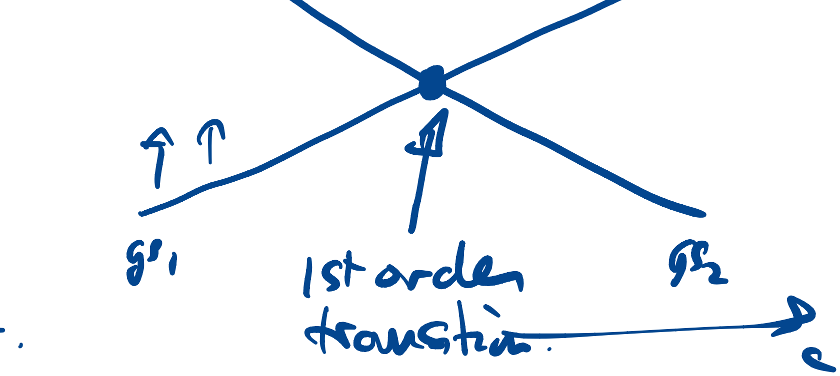
within each sector of gdstate subspace evolution by U is independent.

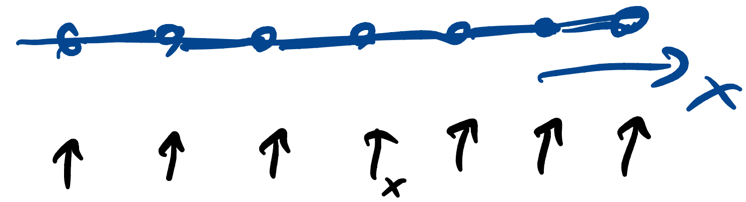


ways gap can close:



gapless! quantum critical pt.





$$H_1 = \sum_x H_x$$

$$\Delta H = -\sum_x Z_x$$

acts as σ^z on τ_x

$$|\text{gs of } \Delta H\rangle = \bigotimes_x |\uparrow\rangle_x$$

$$|\text{gs of } H_1 + \Delta H\rangle = |\text{gs of } H_1\rangle \bigotimes_x |\uparrow\rangle_x$$

Now we can further adiabatically deform $H_1 + \Delta H$

gapped phase of $|\psi\rangle$ $\equiv$ all gapped states reachable from $|\psi\rangle$ by adiabatic evolution & adding decoupled bits.

$\sim$

eg: $H_0 = -\sum_x Z_x$ gs is $\bigotimes_x |\uparrow\rangle_x$.

orbit under $\sim$ $\equiv$ trivial phase

Crucial desideratum: Topological Label

$\equiv$ a qtty we can compute from $|gs\rangle$ which can't change smoothly.

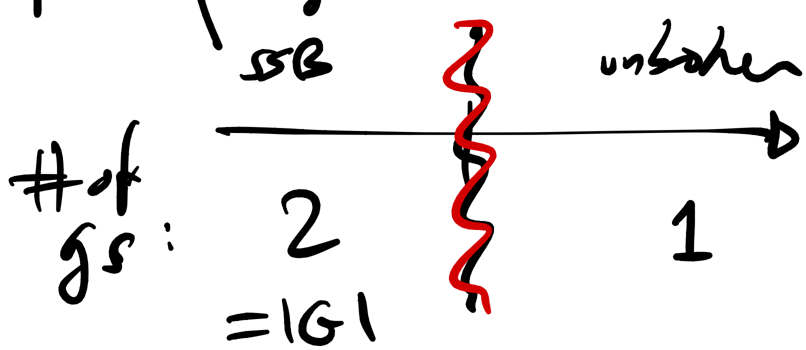
eg an integer.

$\exists$ 3 classes of labels: (1) symmetry breaking
(2) topological order
(3) edge modes.

(1) eg $\mathbb{Z}_2$. Symmetry is represented on $\mathcal{H}$ by U.

$[H,U]=0$. is spontaneously broken

$$U|gs\rangle \not\propto |gs\rangle$$

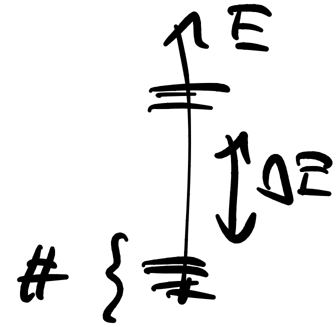


$\otimes|\uparrow\cdots\uparrow\rangle$

$\otimes|\downarrow\cdots\downarrow\rangle$.

defects in ordered phases §4.

(2) intrinsic T.O. $\equiv$ # of gs depends on topology of X.

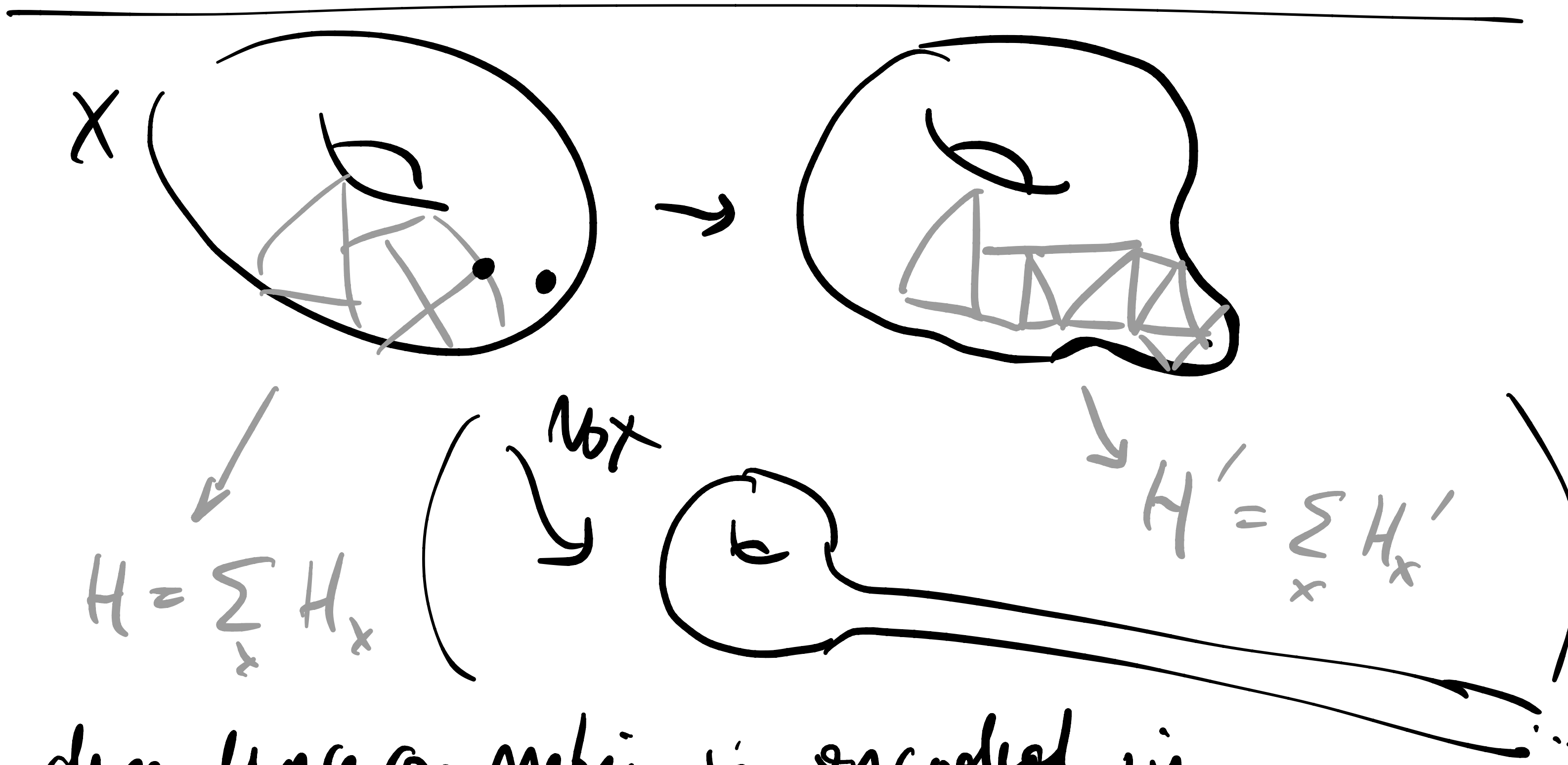


[Wen] ~

$\subset$ SSB. (of 1-form symmetries.)

(3) edge modes are a signature of "invertible topological phase".

LATER.



dependence on metric is encoded in change of couplings & adding decoupled bits.

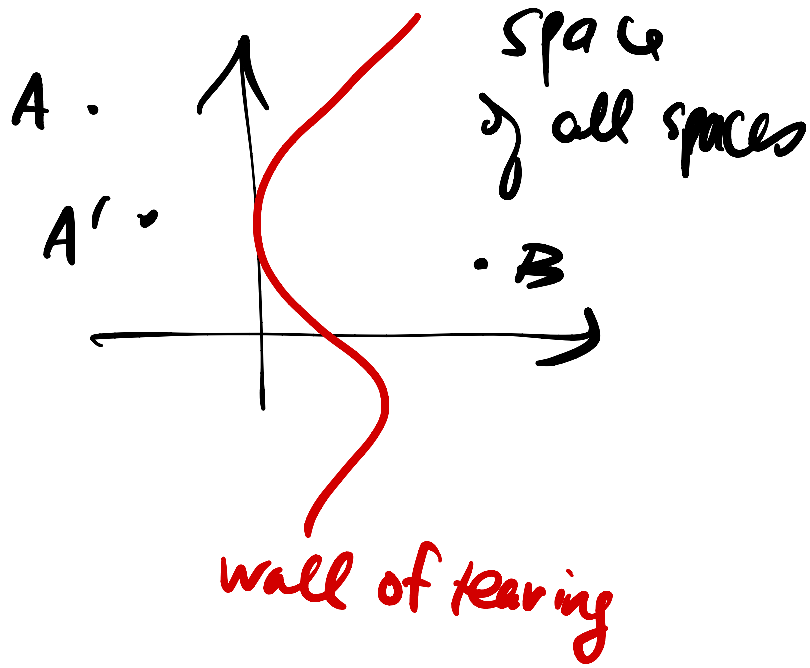
$\Rightarrow$ universal properties of a phase don't change!

$\Rightarrow$ topological labels on phases are top. invts of X.

IF top. label of phase on $X \neq$ top. label of phase on Y

then X & Y have different topology.

topology situation:



1. The toric code & homology.

... to be continued.