University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 239 Topology from Physics Winter 2021 Assignment 8

Due 5pm Friday March 5, 2021

Thanks in advance for following the guidelines on hw01. Please ask me by email if you have any trouble.

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1. **Cech cohomology brainwarmer.** Check that the Cech coboundary operator δ is nilpotent: $\delta^2 = 0$.

2. **Euler-Poincaré theorem for Cech cohomology.**

   Let X be a manifold with a finite good cover. Let β_p is the number of non-empty $(p+1)$-fold intersections $U_{\alpha_0 \cdots \alpha_p}$. Show that the Euler character is

$$\chi(X) = \sum_p (-1)^p \beta_p.$$

3. **Cech cohomology example.**

   Convince yourself that the computation of the Cech cohomology of the 2-sphere (with arbitrary coefficients) using the good cover described in lecture is the same as the computation of the homology of the tetrahedron cell complex.

4. **Homology of spheres.**

   (a) Use the Mayer-Vietoris sequence to compute $H^q(S^n)$ using the open cover $S^n = U_N \cup U_S$, where $U_N(U_S)$ is the complement of the north (south) pole. Start with S^2 and work your way up.

   (b) Consider the sphere $S^n(r) = \{\sum_{i=0}^n x_i^2 = r\} \subset \mathbb{R}^{n+1}$. Show that

$$\omega \equiv \frac{1}{r} \sum_{i=0}^{n} (-1)^i x_i dx_0 \wedge \cdots \widehat{dx_i} \wedge dx_n$$

   (the one with the hat is omitted) is not exact by integrating it over S^n. It is a generator of $H^n(S^n)$. [Hint: $dr \wedge \omega = dx_0 \wedge \cdots dx_n$ is the volume form on $\mathbb{R}^{n+1}$.]