University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 239 Topology from Physics Winter 2021 Assignment 1

Due 12:30pm Monday, January 11, 2020

- Homework will be handed in electronically. Please do not hand in photographs of hand-written work. The preferred option is to typeset your homework. It is easy to do and you need to do it anyway as a practicing scientist. A LaTeX template file with some relevant examples is provided here. If you need help getting set up or have any other questions please email me. I am happy to give TeX advice.
- To hand in your homework, please submit a pdf file through the course's Canvas website, under the assignment labelled hw01.

Thanks in advance for following these guidelines. Please ask me by email if you have any trouble.

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1. **Groundstate degeneracy and 1-form symmetry algebra.**

   (a) Suppose we have a system with Hamiltonian H with string operators W_C and $V_{\check{C}}$ supported on closed curves, and commuting with H, and satisfying $W^N = V^N = 1$. Suppose $[W_C, W_{C'}] = 0, [V_{\check{C}}, \check{C}']$ for all curves but

   $$W_C V_{\check{C}} = \omega^{\#\left(C \cap \check{C}\right)} V_{\check{C}} W_C$$

   where $\omega \equiv e^{\frac{2\pi \mathbf{i}}{N}}$ and $\#\left(C \cap \check{C}\right)$ is the number of intersection points of the curves. How many groundstates does such a system have on the two-torus (that is, with periodic boundary conditions on both spatial directions)?

   This is what happens in the $\mathbb{Z}_N$ toric code.

   (b) Now suppose in a different system we have just one set of string operators W_C satisfying

   $$W_C W_{C'} = \omega^{\#(C \cap C')} W_{C'} W_C,$$

   with the same definitions as above. How many groundstates does this system have on the two-torus?

   This is what happens in the Laughlin fractional quantum Hall state with filling fraction $\frac{1}{N}$.

   (c) [Bonus problem] Redo the previous problems for a genus g Riemann surface, *i.e.* the surface of a donut with g handles.

In all parts of this problem you should make the assumption that the string operators are *deformable*: W_C acts in the same way as $W_{C+\partial p}$ on groundstates.

2. **Anyons in the toric code.**

   (a) Show that when acting on a toric code groundstate the operator

   $$W_C = \prod_{\ell \in C} X_\ell$$

   creates a state which violates only the star operators at the sites in the boundary of C, ∂C, a pair of e-particles.

   (b) Show that when acting on a toric code groundstate the operator

   $$V_{\check{C}} = \prod_{\ell \perp \check{C}} Z_\ell$$

   creates a state which violates only the plaquette operators in the boundary of $\check{C}$, $\partial \check{C}$.

   (c) Show that a boundstate of an e particle and an m particle in the 2d toric code must be a fermion.