University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215B QFT Winter 2017
Assignment 3

Due 11am Tuesday, January 31, 2017

1. **Brain warmer.** Prove the Gordon identity by evaluating $\bar{u}(p') \left(\not{p}'\gamma^\mu + \gamma^\mu \not{p}\right) u(p)$ in two different ways.

2. **Numerator algebra.** Check that you understand the steps leading to the expression for the numerator of the integrand for the QED vertex correction (equation (6.28) of the lecture notes). It uses $x + y + z = 1$, the Dirac equation $\not{p}u(p) = m_e u(p), \bar{u}(p')\not{p}' = \bar{u}(p')m_e$ and the Gordon identity.

3. **Symmetry is attractive.** [from Jared Kaplan] Consider a field theory in $D = 3 + 1$ with two massless (for simplicity) scalar fields which interact via the interaction Lagrangian

$$V = -\frac{g}{4!}\left(\phi_1^4 + \phi_2^4\right) - \frac{2\lambda}{4!}\phi_1^2\phi_2^2.$$

   (a) Show that when $\lambda = g$ the model possesses an $\mathsf{O}(2)$ symmetry.

   (b) Will you need a counterterm of the form $\phi_1\phi_2$ or $\phi_1\Box\phi_2$? If not, why not?

   (c) Renormalize the theory to one loop order by regularizing (for example with Pauli Villars), adding the necessary counterterms, and imposing a renormalization condition on the masses and $2 \to 2$ scattering amplitudes at some energy $\sqrt{s_0}$.

   (d) Consider the limit of low energies, *i.e.* when $s_0 \ll \Lambda^2$ where Λ is the cutoff scale. Tune the location of the poles in both propagators to $p^2 = 0$. Show that the coupling goes to the $\mathsf{O}(2)$-symmetric value if it starts nearby (nearby means $\lambda/g < 3$).

4. **The Rosenbluth formula.** [optional]

   If you wish to experience the true suffering of the field theory student, do Peskin problem 6.1. I recommend undoing the use of the Gordon identity in the parametrization of the vertex.