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Quantum Mechanics C (Physics 130C) Winter 2015
Worksheet 7

Announcements

- The 130C web site is:

  http://physics.ucsd.edu/~mcgreevy/w15/ .

  Please check it regularly! It contains relevant course information!

- This week let's generalize Grover's algorithm

Problems

1. **Continuous Search**

   Previously we've discussed Grover's algorithm implemented by some discrete set of unitary operators. Let's consider an analogous problem where $U(t) = e^{-\mathbf{i}\hat{H}t}$.

   Define the computational basis of a single qubit to be $\{|0\rangle, |1\rangle\}$ and let us suppose we have n-many qubits living in $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_n$.

   Let $|s\rangle$ be a computational basis element[1] and $|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |x\rangle$ be an equal superposition of all such elements.

   (a) What is the dimension of the subspace that $|s\rangle$ and $|\psi\rangle$ span? Note they aren't necessarily orthogonal.

   Write an orthonormal basis for this space. (Hint: Construct a state which is composed of $|s\rangle$ and $|\psi\rangle$ but is orthogonal to $|s\rangle$)

   (b) Suppose our Hamiltonian is $\hat{H} = |s\rangle\langle s| + |\psi\rangle\langle\psi|$. Rewrite $\hat{H}$ using the basis above.

   (c) In this basis write $\hat{H}$ as a matrix and expand in terms of Pauli's. What is the form of $U = e^{-\mathbf{i}\hat{H}t}$ on this subspace? Recall $e^{-\mathbf{i}\theta\vec{\sigma}\cdot\hat{n}} = \cos\theta\mathbb{1} - \mathbf{i}\sin\theta(\vec{\sigma}\cdot\hat{n})$

   (d) Suppose we initialize our quantum computer to $|\psi\rangle$ and evolve by U for a time $t = T$ at which we then measure the state in the computational basis.

   What is the probability for measuring $|s\rangle$ at T? At what time should be measure to maximize this probability?

   (Hint: You will need to evaluate $U|\psi\rangle$ which you have in terms of Pauli's which act as they normally do on the basis vectors for the subspace we've been considering.)

[1] So s would be some n-digit string of 0's and 1's