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Quantum Mechanics C (Physics 130C) Winter 2015
Worksheet 3

Announcements

- The 130C web site is:

  http://physics.ucsd.edu/~mcgreevy/w15/ .

  Please check it regularly! It contains relevant course information!

- Today we'll look at Bloch's theorem and look at the harmonic oscillator.

  Remember my office hour is from 1430-1530!

Problems

1. **Building Bloch's Theorem**

   Consider a 1D Hamiltonian with a periodic potential $V(x) = V(x + na)$ for $n \in \mathbb{Z}$ and a the lattice spacing.

   (a) Define the operator T^n by $T^n|x\rangle = |x + na\rangle$. Show this is a symmetry.

   (b) Assuming H is *non-degenerate*, show that any eigenfunctions of this system can be chosen to obey
   $$\psi_k(x - a) = e^{-\mathbf{i}ka}\psi_k(x) \tag{1}$$
   Recall that $T|k\rangle = e^{-\mathbf{i}ka}|k\rangle$ and $\langle x|k\rangle \equiv \psi_k(x)$.

   (c) Infer from 1 that one can then write $\psi_k(x) = e^{\mathbf{i}kx}u_k(x)$ where $u_k(x) = u_k(x + a)$

2. **Harmonic Oscillator**

   A particle of mass m is in a one dimensional harmonic potential and has been prepared (at $t = 0$) in an *equal* superposition state of energies $\frac{\omega}{2}$ and $\frac{3\omega}{2}$.

   (a) What is the most general expression for the wavefunction at $t = 0$? (Hint: One should include also the possibility of a relative phase between states)

   (b) Suppose I find that the average value of the momentum $\langle\hat{p}\rangle$ at $t = 0$ is $\sqrt{\frac{m\omega}{2}}$ What constraint does this impose on my general $|\psi_0\rangle$?
   Recall that $p = \mathbf{i}\sqrt{\frac{m\omega}{2}}(\hat{a}^\dagger - \hat{a})$ where $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ and $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$

   (c) What is the time evolved $|\psi_t\rangle$? What about $\langle\hat{p}\rangle_t$?

   (d) Compute the uncertainty $\Delta p_t = \sqrt{\langle\hat{p}^2\rangle_t - \langle\hat{p}\rangle_t^2}$