University of California at San Diego – Department of Physics – Prof. John McGreevy

Quantum Mechanics C (130C) Winter 2014
Final exam — frontmatter

Please remember to put your name on your exam booklet. This is a closed-book exam. There are 6 problems, each with several parts, of varying levels of difficulty; make sure you try all of the parts. None of the problems require very extensive calculation; if you find yourself involved in a morass of calculation, step back and think. Good luck.

Possibly useful information:

$$\mathbf{U}(t) = e^{-\mathbf{i}\mathbf{H}t/\hbar} \quad \text{satisfies} \quad \mathbf{i}\hbar\partial_t \mathbf{U} = [\mathbf{H}, \mathbf{U}].$$

$$\boldsymbol{\sigma}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}^y = \begin{pmatrix} 0 & -\mathbf{i} \\ \mathbf{i} & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|\uparrow_{\hat{n}}\rangle = e^{-\mathbf{i}\varphi/2}\cos\frac{\theta}{2}|\uparrow_{\hat{z}}\rangle + e^{+\mathbf{i}\varphi/2}\sin\frac{\theta}{2}|\downarrow_{\hat{z}}\rangle \quad \text{satisfies} \quad \vec{\boldsymbol{\sigma}}\cdot\hat{n}|\uparrow_{\hat{n}}\rangle = |\uparrow_{\hat{n}}\rangle$$

$$e^{-i\alpha\hat{n}\cdot\vec{\boldsymbol{\sigma}}} = \mathbb{1}\cos\alpha - i\hat{n}\cdot\vec{\boldsymbol{\sigma}}\sin\alpha.$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{H}_{\text{SHO}} = \hbar\omega\left(\mathbf{a}^\dagger\mathbf{a} + \frac{1}{2}\right) = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega^2\mathbf{q}^2$$

$$\mathbf{q} = \sqrt{\frac{\hbar}{2m\omega}}\left(\mathbf{a} + \mathbf{a}^\dagger\right), \quad \mathbf{p} = \frac{1}{\mathbf{i}}\sqrt{\frac{\hbar m\omega}{2}}\left(\mathbf{a} - \mathbf{a}^\dagger\right); \quad [\mathbf{q}, \mathbf{p}] = \mathbf{i}\hbar \implies [\mathbf{a}, \mathbf{a}^\dagger] = 1.$$

In Coulomb gauge, in vacuum ($\vec{\nabla}\cdot\vec{A} = 0, \Phi = 0$): $\vec{E} = -\partial_t\vec{A}$, $\vec{B} = \vec{\nabla}\times\vec{A}$.

$$\vec{\mathbf{A}}(\vec{r}) = \sum_{\vec{k}}\sqrt{\frac{\hbar}{2\epsilon_0\omega_k L^3}}\sum_{s=1,2}\left(\mathbf{a}_{\vec{k},s}\vec{e}_s(\hat{k})e^{\mathbf{i}\vec{k}\cdot\vec{r}} + \mathbf{a}^\dagger_{\vec{k},s}\vec{e}^\star_s(\hat{k})e^{-\mathbf{i}\vec{k}\cdot\vec{r}}\right),$$

$$\vec{\mathbf{E}}(\vec{r}) = \mathbf{i}\sum_{\vec{k}}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 L^3}}\sum_{s=1,2}\left(\mathbf{a}_{\vec{k},s}\vec{e}_s(\hat{k})e^{\mathbf{i}\vec{k}\cdot\vec{r}} - \mathbf{a}^\dagger_{\vec{k},s}\vec{e}^\star_s(\hat{k})e^{-\mathbf{i}\vec{k}\cdot\vec{r}}\right)$$

Chain of coupled springs:

$$\mathbf{q}(x) = \sqrt{\frac{\hbar}{2m}}\sum_k\frac{1}{\sqrt{\omega_k}}\left(e^{\mathbf{i}kx}\mathbf{a}_k + e^{-\mathbf{i}kx}\mathbf{a}_k^\dagger\right), \qquad \mathbf{p}(x) = \frac{1}{\mathbf{i}}\sqrt{\frac{\hbar m}{2}}\sum_k\sqrt{\omega_k}\left(e^{\mathbf{i}kx}\mathbf{a}_k - e^{-\mathbf{i}kx}\mathbf{a}_k^\dagger\right).$$

1. **Short answers and conceptual questions** [4 points each, except as noted]

   For true or false questions: if the statement is false, you must explain what is wrong or correct it or give a counterexample; if the statement is true, you can simply say 'true'.