A talk about Nothing

based on work with:

Eva Silverstein; Allan Adams, Xiao Liu, Alex Saltman

hep-th/0502021, hep-th/0506130

What do I mean by 'Nothing'?

A possible phase of quantum gravity where $\langle ds^2 \rangle = 0$.

'unbroken phase'.

An old idea:

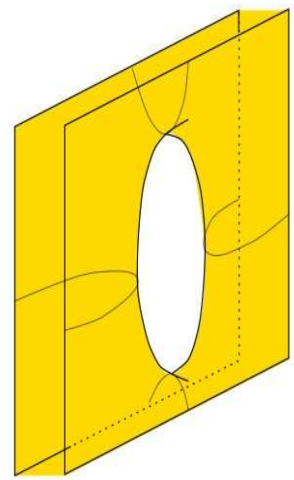
• prerequisite for Sakharov's Machian 'induced gravity' idea:

Elasticity of space $M_P^2 \sqrt{g} R$ as a result of quantum fluctuations of matter fields.

• 'The vacuum' of canonical quantum gravity, CS gravity.

• Witten Commun.Math.Phys.117:353,1988 makes generally covariant theories not by integrating over metrics, but by not introducing one.

• The inside of Bubbles of Nothing. (from Fabinger and Horava)

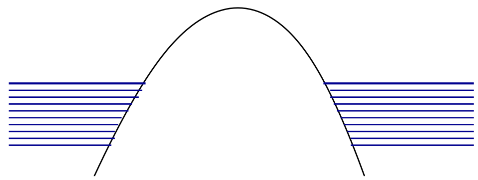


This is a nonperturbative (Euclidean QG) instability of the Kaluza-Klein vacuum of GR, and of Scherk-Schwarz vacua of supergravity. (Witten)

• ∃ attempts to construct Nothing in String Field Theory.

(Horowitz, Lykken, Rohm, Strominger; Yang, Zwiebach)

- Our best examples are still in $d = 2$.



The presence of the Fermi sea spontaneously breaks general covariance;

Closed string excitations are ripples.

This 'spacetime substance' is made of D-branes (JM, H. Verlinde).

Other states, different from the perturbative vacuum, have different numbers of fermions,

and are described by configurations of the closed-string tachyon.

Why am I talking about it?

• The previous motivations

unhiggsing restores symmetries, should tell us about microphysics

• It might help with singularity resolution.

Q: how to ask questions about the nothing state?

Attach ’regions’ of it to regions of normal spacetime.

How?

Localized tachyons.

Does string theory resolve spacelike singularities?
If so, when?

$$a)\ l_s? \quad b)\ g_s^{\nu} l_s? \quad c)\ l_P? \quad d)\ \text{other}$$

Reason to hope it might sometimes be choice a):

In perturbative string theory, the metric is already an emergent quantity
in the sense that the metric is a condensate of string modes

Existence of large dimensions is a result of massless worldsheet bosons

The stiffness of (gedanken-)rulers is a consequence of the rigidity of this condensate.

Given this circumstance, we might imagine that it can be destroyed by the presence of other strings
winding tachyons: strings that want to be there more.

more specific claims:

1. When the matter sector of the worldsheet theory has a mass gap, the theory is in a Nothing phase.

In the competition between kinetic terms $G_{\mu\nu}\partial X^\mu \partial X^\nu$ and potential terms $V(X)$, potential wins.

2. There are examples where the perturbative description is self-consistent.

i.e. such phases can be perturbatively accessible.

Modes which would back-react are lifted.

vs. The bubble of nothing is a nonperturbative Euclidean QG effect.

strategy

• Take perturbative single-string worldsheet point of view.

defined by CFT, $g_s \ll 1$.

• Take seriously the worldsheet mass gap from stringy tachyons.

• Connect the gapped phase to a 'normal' phase and make the whole thing a CFT by Liouville evolution

nonlinearly realized conformal symmetry:

$$z \mapsto \lambda z, \quad X \mapsto X - \ln \lambda$$

Confession: I won't include fluctuations of the Liouville field in all examples.

Outline

II. basic example of generating a worldsheet mass gap
which can be localized:
review of RG of XY model.

III. localized in space: RS compactification

(hep-th/0502021, with A. Adams, X. Liu, A. Saltman, E. Silverstein)

IV. localized in time: the tachyon at the end of the universe

(hep-th/0506130, with E. Silverstein)

localized in a null direction?

V. comments about other probes

II. XY model

A 2d CFT with a relevant operator whose conformal dimension we can control:

sigma model whose target is S^1. $\theta \simeq \theta + 2\pi$.

$$L_{UV} = \frac{L^2}{4\pi l_s^2} \partial\theta\bar{\partial}\theta$$

This model describes superfluid films:

$$\frac{L^2}{4\pi l_s^2} \sim \langle|\Psi|^2\rangle / T \equiv \rho_s / T,$$

$\Psi = |\Psi| e^{i\theta} \sim$ condensate wavefunction

Phase stiffness is determined by magnitude of condensate.

the main character: $\mathcal{O}_{nm} = e^{i(n\theta + m\tilde{\theta})}$ $\theta = \theta_L + \theta_R, \tilde{\theta} = \theta_L - \theta_R$

$\mathcal{O}_{nm}$ makes θ jump by $2\pi m$ (a disorder operator)

it creates a string with m units of winding around the S^1.

Winding tachyon

$$\Delta_{nm} = \left(\frac{n}{L}\right)^2 + (mL)^2$$

in $2\pi l_s^2 = 1$ units $\Longrightarrow$ For $L < L_c = \sqrt{2} l_s$, $\Delta_{0,\pm 1} = L^2 < 1$ are relevant.

Q: What happens when a gas of such insertions condenses?

Vortex condensation $\delta L = \mu \cos \tilde{\theta}$ destroys long range correlations of the θ variable:

when $\mu = 0$, correlations are algebraic:

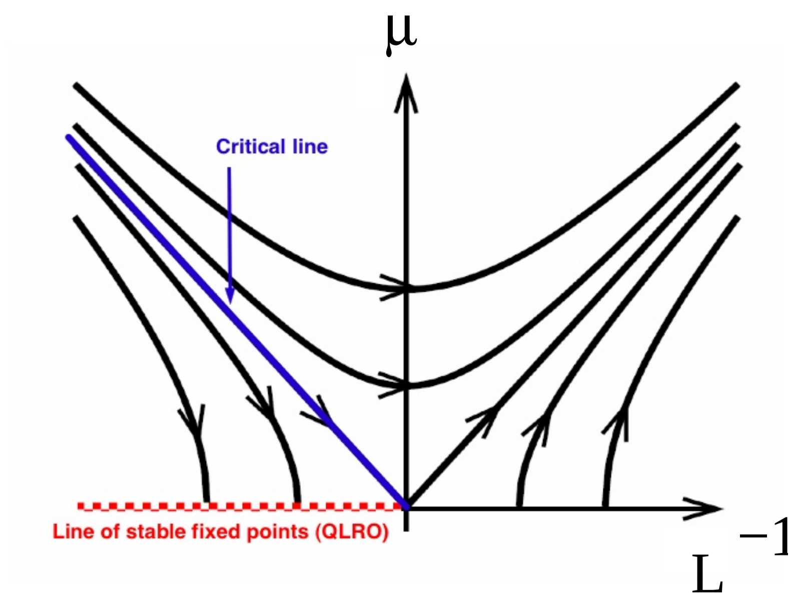
$$\langle e^{ip\theta}(z) e^{-ip\theta}(w) \rangle \sim \frac{1}{|z-w|^{l_s^2 p^2}}$$

For $\mu \neq 0$,

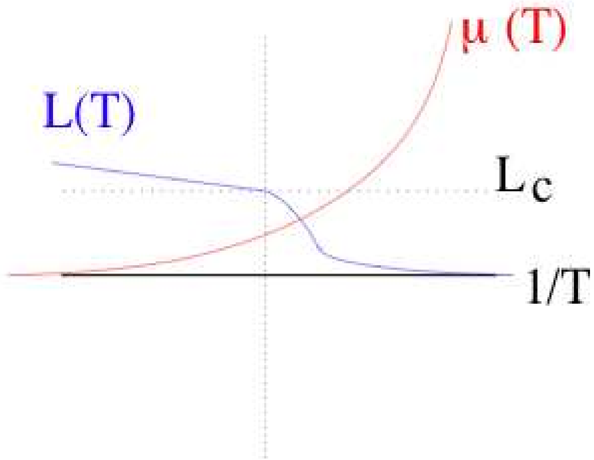
$$\langle e^{ip\theta}(z) e^{-ip\theta}(w) \rangle \sim e^{-m|z-w|}$$

To see this: fermionize.

$$\frac{d\mu}{dl} \sim (L - L_c)\mu, \quad \frac{dL}{dl} \sim \mu^2$$



The lines don't go straight up. The tachyon exerts a force on the radius.



Universal jump in phase stiffness.

Claim: supersymmetric sine-gordon is qualitatively identical with antiperiodic boundary conditions.

now let's make a string theory with this.

III. Riemann surface compactification

Make θ the coordinate along a one-cycle of a RS Σ_h

Consider IIA on Σ_h.

In large-volume ('$\alpha' \to 0$') limit, worldsheet beta functions agree with supergravity: $\beta_{\mu\nu} = R_{\mu\nu}$

constant negative curvature is a local minimum.

classically, complex structure moduli are flat directions.

tadpole for volume V_Σ:

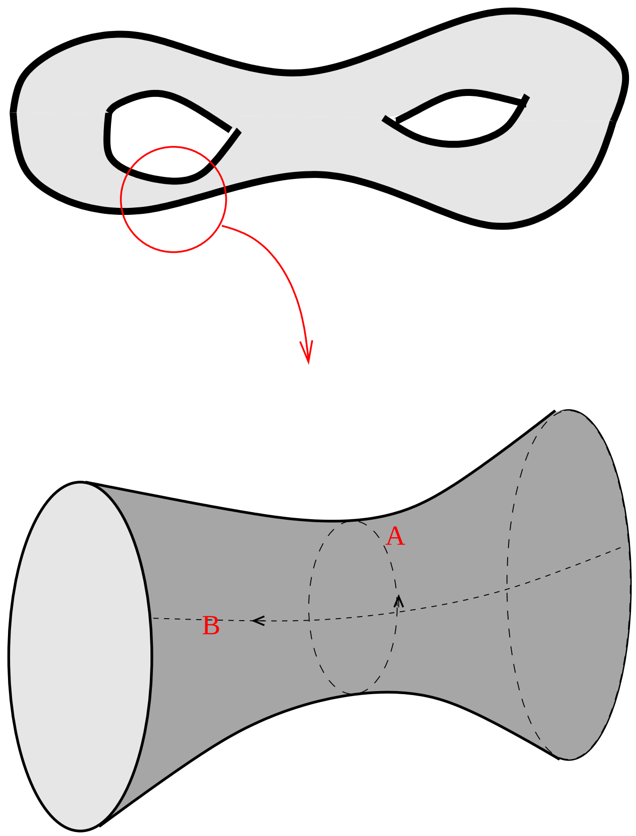
$$V_{\text{eff},8d} \propto \left(\frac{g_s}{V_\Sigma}\right)^{2/3} (2h-2)$$

rolls towards $V_\Sigma \to 0, g_s \to 0$, slowly if $V_\Sigma \gg l_s^2$.

There are 2^{2h} choices of spin structure in the target space.

Periodic BCs for the target fermions project out winding tachyons.

Consider a neighborhood of a handle that has antiperiodic boundary conditions (APBCs).



If the curvature l_s^2/V_Σ is small enough,

$$ds^2 \sim dx^2 + (L_0^2 + \mathcal{O}(1/V_\Sigma))d\theta^2 + ...$$

XY model varying adiabatically with x and t.

Spectrum of wound strings is as in flat space plus perturbations.

$$\alpha' m^2 = -1 + L_0^2/2l_s^2 + p^2 + \text{osc}...$$

If complex structure moduli are such that the length of the minimal geodesic on the A-cycle has $L_0 < L_c$, there's a winding tachyon.

It is localized to the region where $L < L_c$.

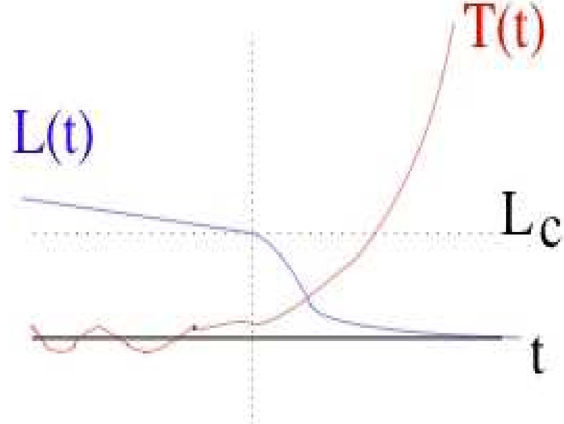
Note that the restriction to $\frac{dL}{dx} \ll 1$ is important: *e.g.* flat space in polar coordinates.

Q: what happens when it condenses?

Claim: The handle pinches off, leaving Nothing in its place.

This is why I emphasized the 'universal jump' in

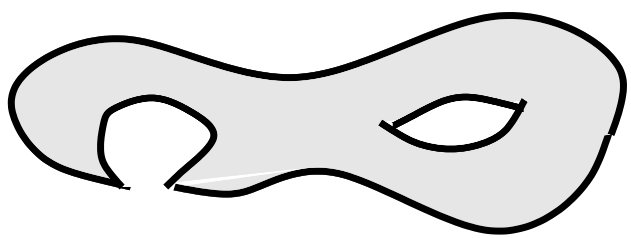
$$\frac{\rho_s}{T} = L^2/4\pi l_s^2.$$



Some disclaimers:

1. The proximate result of the tachyon gets only part of the way to $\langle ds^2 \rangle = 0$.
There's a region of 8d type 0 (plus radiation) with bulk tachyons which peacefully condense....

2.



'Pseudopods' of excess positive curvature, shrink back to constant negative curvature.

3. If h changes, the worldsheet Witten index

$$\text{tr}_{\text{ ws}}(-1)^F = \chi(\Sigma_h) = 2 - 2h$$

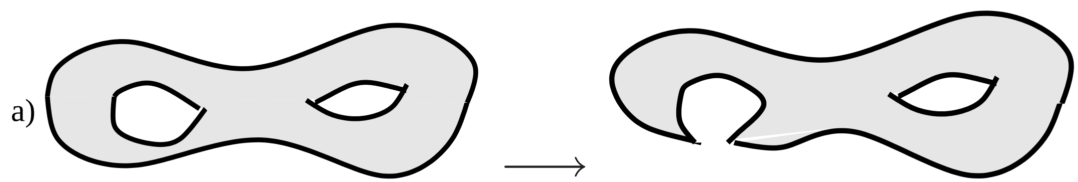
seems to jump!

resolution: some vacua are left behind in the Nothing region. 'dust'.

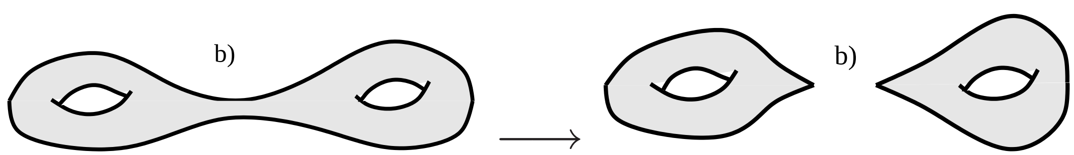
I will give evidence for each of these from LSM.

Note: two possibilities

lose a handle.



disconnect.



Consistency checks

In oriented string theory on a RS, there are $2h$ massless vector fields from $A_\gamma = \int_{\gamma \in H_1(\Sigma_h)} B^{\rm NSNS}$.
Changing h changes this number. How?

A-cycle is easy:

The winding tachyon around A is charged under A_A. $\Longrightarrow$ Higgsed.

B-cycle:

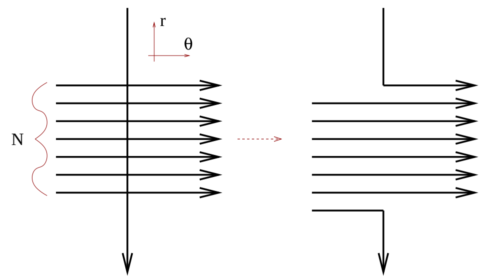
$$\int_{10} H \wedge \star H = \ldots + \int d^8x \frac{1}{\tau_2^2} |F_A + \tau F_B|^2$$

$$\tau_2 \propto g_{\theta\theta} \quad \Longrightarrow g_{YM}^B \to \infty$$

"classical confinement" (Kogut, Susskind, PRD9, 3501(1974))

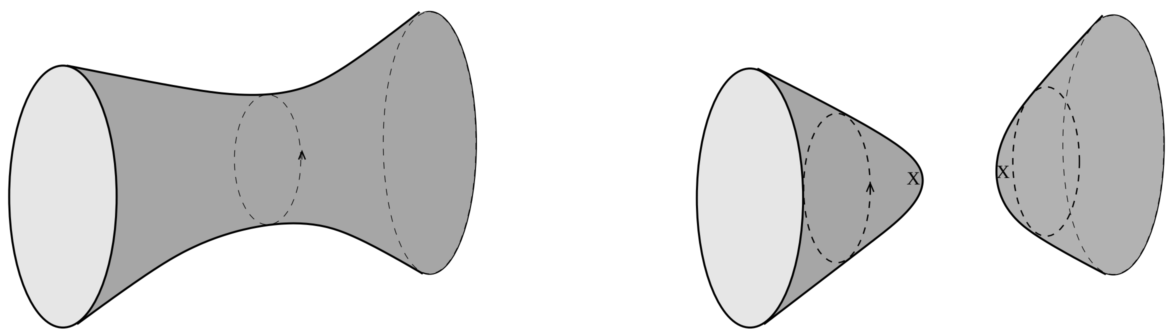
(familiar from COM $U(1)$ of unstable branes) A pair of strings wound around B oppositely develops a flux line between them in R^7 along which $\langle T \rangle = 0$.

Microscopically, This is what happens to a string on the B-cycle:



If you put RR flux through A: $q = \int_{A\times Q} F^{(1+q)}$

Q is some q-sphere in the other dims



D-brane sources appear (if there's enough energy for the process to happen).

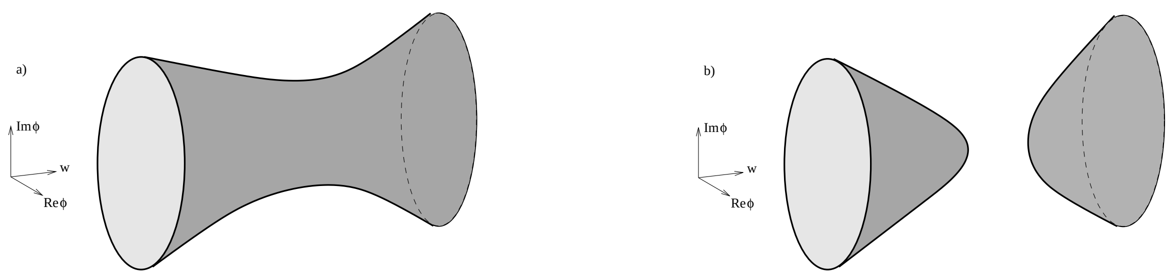
Why?

1. Stringy modes don't penetrate the handle.

localized mass gap = big potential barrier.

other probes?

2. GLSM: $\{w^2 - |\phi|^2 = \xi,\ w \in R\}$



$\longrightarrow \xi$

note: FI term is usually a kahler modulus...

ξ actually determines both volume and tachyon.

why the FI parameter controls the vortex density

Recall Buscher trick: the dual circle coordinate is a dynamical theta angle.

$$S = \int d^2z \left(L^2(\partial\theta + A)^2 + \tilde{\theta}F + \frac{1}{e^2}F^2 \right)$$

gauge symmetry acts as $A \mapsto A - d\lambda, \theta \mapsto \theta + \lambda$.

At long distance, vortex configuration has $F \sim \delta(z - z_0)$
its contribution is $e^{-S_{cl}} e^{i\tilde{\theta}(z_0)}$

with $(2,2)$ susy, this is e^{-t}, $\quad t \equiv \xi + i\tilde{\theta}$.

this is why this LSM is better than $W = P(XY - \mu)$.

first attempt

one $U(1)$ with chirals $\phi_+, \eta_+, \phi_{-2}, P_{-2}$

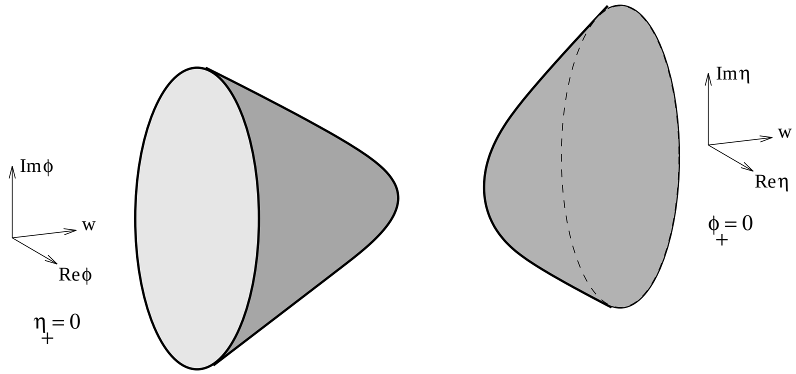
$$D = |\phi_+|^2 + |\eta_+|^2 - 2|P_{-2}|^2 - 2|\phi_{-2}|^2 - \xi$$

$$\beta_\xi = \sum_i Q_i \equiv Q_T = -2$$

$\xi \to +\infty$ in IR. add $W = mP_{-2}\phi_+\eta_+$ take $m \sim e$ large, mass of fluctuations off vacuum manifold

large $|\xi|$ semiclassical.

at $\xi \to +\infty$ (IR): either ϕ_+ or η_+ must be nonzero
branches of $\phi_+\eta_+ = 0$ are disconnected
if $\phi_+ \neq 0$, use $U(1)$ to set $\phi_+ = w \in R_+$.
$w^2 - 2|\phi_{-2}|^2 = \xi \quad \longrightarrow$ a cap.

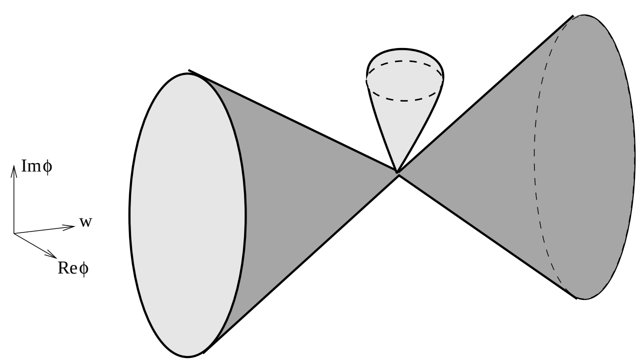


at $\xi \to -\infty$ (UV):, either ϕ_{-2} or P_{-2} must be zero

if $P_{-2} = 0$, $\phi_+ \eta_+ = 0$, if not, $\phi_+ = \eta_+ = 0$.

if $t \equiv \xi + i\theta \neq 0$, still nonsingular

An extra $\mathbb{P}^1$ is attached at $\phi_+ = \eta_+ = 0$.



Claim: This weird UV phase is near the narrow handle universality class.

technicality: winding tachyon and 'deformation' not mutually $(2,2)$.

$$\delta L = q_+ q_- (-\mu)(P_{-2}\bar{P}_{-2} + \phi_+ \bar{\eta}_+ + \bar{\phi}_+ \eta_+)$$

$q_\pm \equiv \frac{1}{\sqrt{2}}(Q_\pm + \bar{Q}_\pm)$ are the preserved supercharges.

Claim: the fact that we've broken the worldsheet supersymmetry $(2,2) \longrightarrow (1,1)$ doesn't disturb the usual GLSM RG flow, for small μ.

the off-vacuum field space of the LSM (the embedding space) provides coords on the Nothing region.

Dust vacua!

So far, we've talked about the 'higgs branch' of the vacuum manifold.

$$\Sigma = \sigma + \theta\lambda + \theta^2(F + iD) + \dots$$

consider region of large σ.
$L \ni -|\phi|^2|\sigma|^2 \Longrightarrow \Phi$s are massive, integrate out.

$$\tilde{W} = t\Sigma + Q_T \Sigma \ln \Sigma$$

$L \ni \int d\theta_+ d\bar{\theta}_- \tilde{W} + \text{h.c.}$

R-symmetries:

$$\theta_+ \mapsto e^{i\alpha_+}\theta_+, \theta_- \mapsto e^{i\alpha_-}\theta_-,$$

$$\tilde{W} \propto \Sigma \Longrightarrow \Sigma \mapsto e^{i(\alpha_+ - \alpha_-)}\Sigma$$

The second term reflects the anomaly in the axial R-symmetry.

$$Q_T = -2 \in 2Z \Longrightarrow$$

there is a non-anomalous $Z_2 = \langle g \rangle \subset U(1)_{\text{axial}}$ by which the chiral GSO acts.

vacua appear at $0 = \frac{\partial \tilde{W}}{\partial \sigma}$

$$\sigma_{\pm} = \pm e^{t/2}$$

Reliable at large $t > 0$.

$$g : \sigma_{+} \mapsto \sigma_{-}$$

$$\{\text{two vacua}, \sigma_{\pm}\}/\text{GSO} = \text{point}/\text{diagGSO}$$

8d type zero.

The tachyon at the end of the universe

Attempt to turn the previous picture sideways.

Consider the FRW-like:

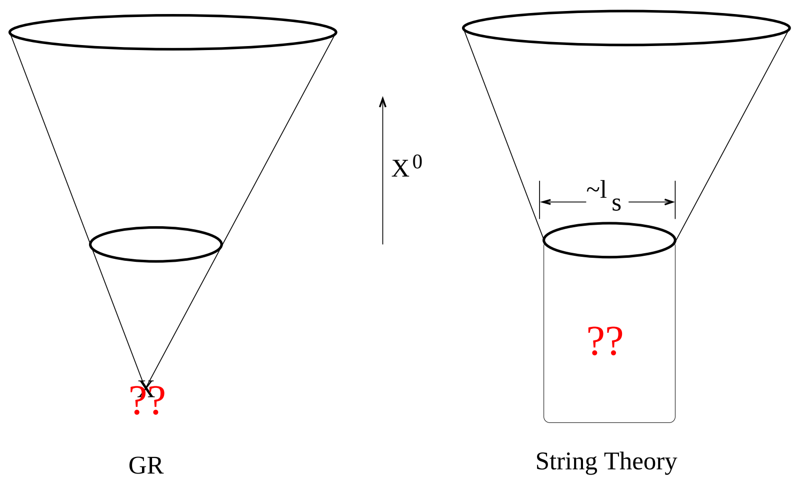
$$ds^2 = -dt^2 + L^2(t)d\theta^2 + ds_\perp^2$$

with APBCs on θ

Like inside of the BTZ black hole.

classically: $\ddot{L} = 0$.
demand $\dot{L} \ll 1$
take $\dot{L} > 0$ (bang).

Note: $\exists$ witten bubble



Consider evolving towards the past.

What does a single particle probe see?

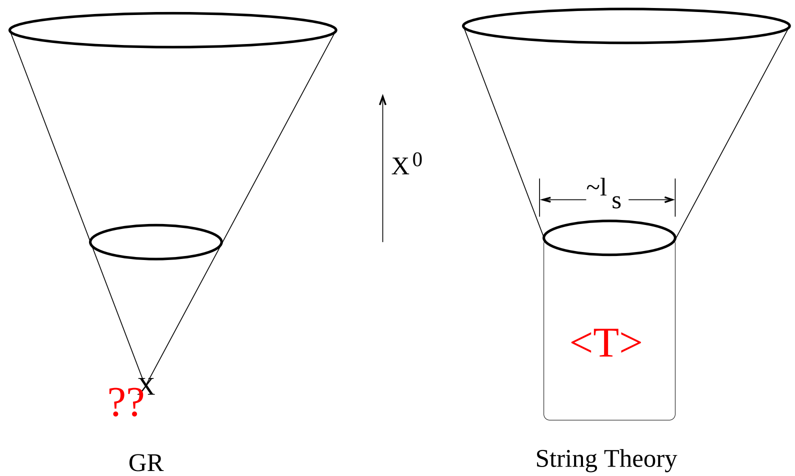
in GR or with periodic BCs

$$Z \sim \int [dX]\, e^{\frac{i}{4\pi l_s^2} \int d^2 z G_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu}$$

when $G_{\theta\theta} < l_s^2$, fluctuations are unsuppressed.

Now, if $\dot{L} \ll 1$, $\alpha' m^2_{\text{winding}} = -1 + L^2/l_s^2$: winding tachyon if $L < L_c$

$$\longrightarrow Z \sim \int [dX] \; \exp\left(\frac{i}{4\pi l_s^2} \int d^2 z [G_{\mu\nu} \partial X^{\mu} \bar{\partial} X^{\nu} - \hat{T}(X^0) \; \cos \tilde{\theta}] \right)$$



an attempt

$$\hat{T}(X^0) \sim \mu e^{-\kappa X^0}$$

where κ is the 'tachyon mass at the onset time'

determined by nonlinear dynamics.

Like sine-Liouville.

Use this integral to try to define amplitudes.

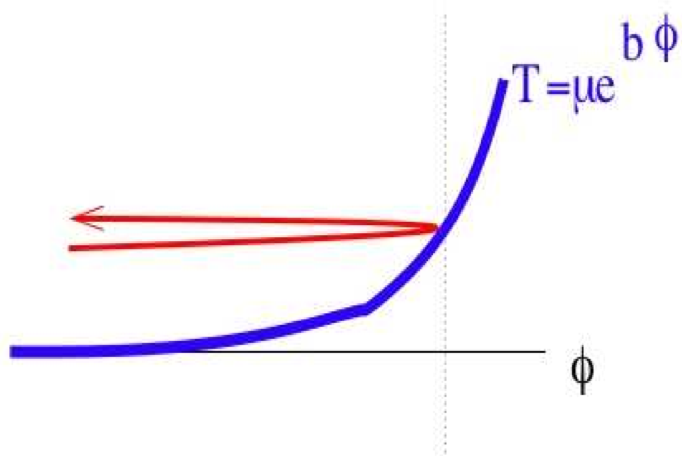
(like Strominger-Takayanagi, Schomerus)

This specifies a particular state.

Note: like in Liouville, we only know the asymptotic behavior away from $\langle T \rangle$ of $\langle T \rangle, \langle \Phi \rangle$...

Claim: results are insensitive to the behavior under the barrier.

On the worldline, T is a potential (position-dependent mass).



What do these amplitudes compute?

Coefficients of the wavefunction in the free-string basis.

sample calculation

Old trick (Gupta-Trivedi-Wise): $X^0 = X_0^0 + \hat{X}^0$.

$$\frac{\partial}{\partial\mu} Z_{T^2} = \int dX_0^0 \int [d\hat{X}^0] \int [dX_\perp]\ e^{iS_{kin}} \frac{C}{\mu} e^{-\kappa X_0^0} e^{-Ce^{-\kappa X_0^0}}$$

here $C \equiv \int d^2\sigma\ \mu e^{-\kappa \hat{X}^0} \hat{T}$ is the nonzeromode part of T.

$$= \int [d\hat{X}^0] \int [dX_\perp] \frac{C}{\kappa\mu} \left(\int_0^\infty dy\ e^{-Cy} \right) e^{iS_{kin}}$$

$$Z_{T^2} = -\frac{\ln \mu/\mu_\star}{\kappa} \hat{Z}_{T^2} = (X_\star^0 - X_\mu^0) \hat{Z}_{T^2}$$

$\mu_\star = e^{\kappa X_\star^0}$ IR cutoff in the free region, $\mu \equiv e^{\kappa X_\mu^0}$.

Compare:

$$Z_{T^2}(\text{no tachyon}) = T \hat{Z}_{T^2}$$

$T = \delta(0) = \int_{-\infty}^{\infty} dX_0^0$

Some final comments

0. suppression of back-reaction: if indeed formerly-light string modes are made heavy by tachyon condensate, their back-reaction to the time-dependence will be suppressed.

1. reversing the process: some amount of radiation comes out.
by making some agreement with someone far away, and sending in exactly the time-reversal of the radiation that comes out (specific correlations),
you could (in principle!) create such a wormhole.

In the case of disconnected components, this is quite strange.

2. Restoration of symmetries hidden by nonlinear dynamics.

3. Q: What happens in the tails of the tachyon wavefunctions?
these are less localized than APS.

4. Effective field theory description of disconnection process?

5. Q: Do D-brane probes agree?

Polyakov (hep-th/9304146) suggests a probe of the Nothingness (diffusion dimension).

$$d_{\text{eff}} = \frac{d}{d\ln\tau}\left(\frac{\int R(x,x,\tau)}{\int 1}\right)$$

$R(x,x',\tau)$ = probability of propagating from x to x' in worldline time τ.

$$d_{\text{eff}} = \begin{cases} d, & \text{flat space} \\ 0, & \text{nothing} \end{cases}$$

In closed string theory, this is an annulus amplitude.

Between what branes? see Hikida, Tai hep-th/0510129

The End.

details about (1,1) vacuum manifold

$$F_{P_{-2}} = m\phi_+\phi_- - \mu\bar{P}_{-2} \quad (1)$$

$$F_{\phi_+} = mP_{-2}\eta_+ - \mu\bar{\eta}_+ \quad (2)$$

$$F_{\eta_+} = mP_{-2}\phi_+ - \mu\bar{\phi}_+ \quad (3)$$

$$(2) + (3) \Longrightarrow P_{-2} = \frac{\mu}{m}\frac{\overline{\phi_+ + \eta_+}}{\phi_+ + \eta_+}$$

determines P_{-2}

$$\Longrightarrow |P_{-2}| = \frac{\mu}{m}$$

$$(2)/(3) \Longrightarrow \frac{\phi_+}{\eta_+} = \frac{\bar{\phi}_+}{\bar{\eta}_+} \equiv x \in R$$

$\eta_+ = x\phi_+$ determines η_+

$$(1) \Longrightarrow x|\phi_+|^2 = \left(\frac{\mu}{m}\right)^2 \Longrightarrow x > 0$$

determines $|\phi_+| \neq 0$, fix $U(1)$ with $\phi_+ \in R_+$. D-term

$$(x + 1/x)\left(\frac{\mu}{m}\right)^2 = \xi + 2\left(\frac{\mu}{m}\right)^2 + 2|\phi_{-2}|^2$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \left(\frac{\mu}{m}\right)^2 = \xi + 2|\phi_{-2}|^2$$

This says

$$2|\phi_{-2}|^2 = w^2 - \xi,$$

$w \equiv \sqrt{x} - \frac{1}{\sqrt{x}}$.

Claim: for small enough μ, $(2, 2)$ RG is preserved.

Aside about field theory dual

In a dual gauge theory ,
winding tachyon on $\gamma \leftrightarrow$ Wilson loop operator W on γ
$\langle W \rangle \neq 0 \Leftrightarrow$ γ is contractible.

If γ is the Euclidean time circle, this is the argument Barbon-Rabinovici, Aharony et al that shows that

a vev for the Polyakov-Susskind loop

$$\Leftrightarrow$$

the dual geometry contains a BH horizon.

A slide about minisuperspace

Minisuperspace worldline theory:

$H = 0$ is a Schrodinger equation with a rapidly falling potential.

If $V(x)$ grows faster than $-x^2$, *e.g.* $V \sim e^{\kappa x}$

$x(\tau)$ reaches $x = \infty$ at finite parameter time τ_∞.

H isn't self-adjoint.

reparametrization BRST anomaly.

No on-shell poles in Green's functions.

Required: a prescription for 'bouncing off the future'.

Warning: In field theory, such a prescription is different than local Hamiltonian evolution.

WKB wavefunctions

for bang case wavefunctions look like

$$u_k(t \to \infty) \sim \frac{1}{\omega(t)} e^{\pm i \int^t dt' \omega(t')} + \ldots$$

with $\omega^2(t) = k^2 + m_0^2 + \mu e^{-\kappa t}$.

Shrinking and rapidly oscillating.

A family of choices of "self-adjoint extensions" arises if we restrict to

$$u_k^\nu(t \to \infty) \sim \frac{1}{\omega(t)} \cos\left(\int^t dt' \omega(t') + \nu \right) + \ldots$$