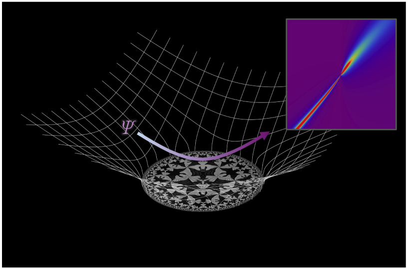
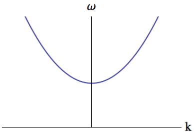
Fermi Surfaces and the Construction of Quantum Electron Stars

John McGreevy, UCSD



Hierarchy of understoodness

systems with a gap (insulators)



EFT is a topological field theory

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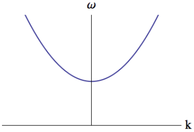

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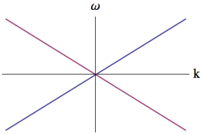
systems at critical points or topological insulators with gapless boundary dofs



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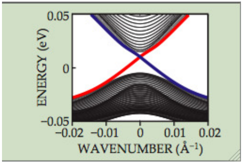

EFT is a CFT

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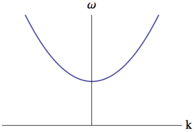

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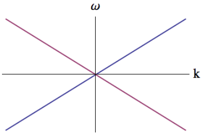

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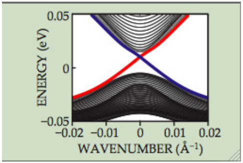
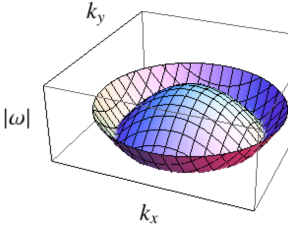
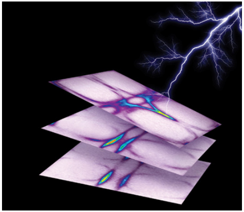


EFT is a CFT

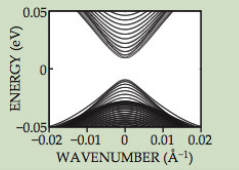
systems with a Fermi surface (metals)





??

Slightly subjective musical classification of states of matter



insulator

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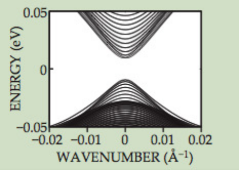

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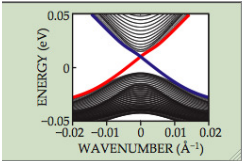

rel. critical point or TI

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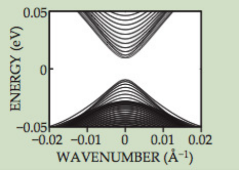

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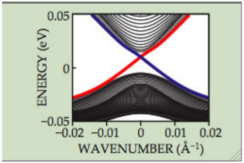

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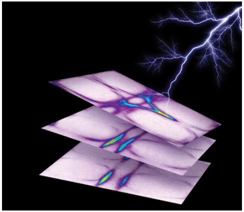

Outline

1. Introduction: ‘post-particle physics of metal’
2. Holographic Fermi surfaces from probe fermions
3. Motivation to do better
4. Electron stars, so far
5. Quantum electron stars [work in progress with Andrea Allais]

Based on:

Hong Liu, JM, David Vegh, 0903.2477;
also: Sung-Sik Lee, 0809.3402; Cubrovic, Zaanen, Schalm, 0904.1933
Tom Faulkner, HL, JM, DV, 0907.2694;
TF, Gary Horowitz, JM, Matthew Roberts, DV, 0911.3402;
TF, Nabil Iqbal, HL, JM, DV, 1003.1728, 1101.0597, in progress;
David Guarrera, JM, 1102.3908;
Andrea Allais, JM, S. Josephine Suh, 1202.5308;
Andrea Allais, JM, in progress.

Fermi Liquids

Basic question: What is the effective field theory for a system with a Fermi surface (FS)?

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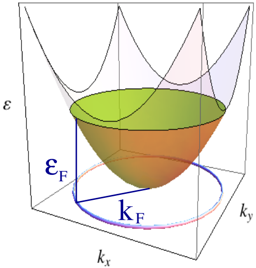
Recall [8.044, 8.06]:

if we had *free* fermions, we would fill single-particle energy levels $\epsilon(k)$ until we ran out of fermions: $\rightarrow$



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remove or add electrons near the Fermi surface ϵ_F, k_F.

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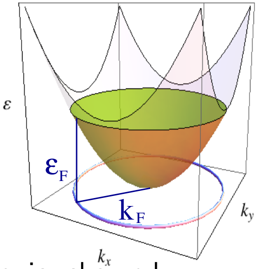
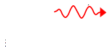


Low-energy excitations:

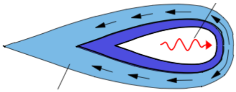
remove or add electrons near the Fermi surface ϵ_F, k_F.

Idea [Landau]: The low-energy excitations of the interacting theory are still weakly-interacting fermionic, charged 'quasiparticles'.

Elementary excitations are free fermions with some dressing:



$\overset{in\ medium}{\longrightarrow}$



The standard description of metals

The metallic states that we understand well are described by Landau's Fermi liquid theory.

Landau quasiparticles $\rightarrow$ poles in single-fermion Green function G_R

at $k_\perp \equiv |\vec{k}| - k_F = 0$, $\omega = \omega_\star(k_\perp) \sim 0$: $G_R \sim \dfrac{Z}{\omega - v_F k_\perp + i\Gamma}$

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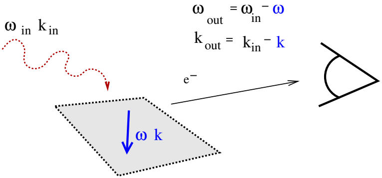
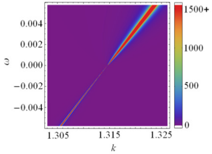
Measurable by ARPES (angle-resolved photoemission):





Intensity $\propto$ spectral density : $\quad A(\omega, k) \equiv \operatorname{Im} G_R(\omega, k) \overset{k_\perp \to 0}{\to} Z\delta(\omega - v_F k_\perp)$

Landau quasiparticles are long-lived: width is $\Gamma \sim \omega_\star^2$, residue Z (overlap with external e^-) is finite on Fermi surface.

Reliable calculation of thermodynamics and transport relies on this.

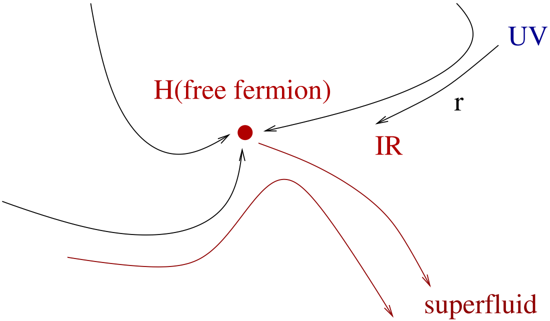
Ubiquity of Landau Fermi liquid

Physical origin of lore:

1. Landau FL successfully describes ^{3}He, metals studied before $\sim 1980s$, ...

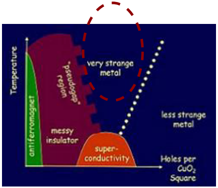
2. RG: Landau FL is stable under almost all perturbations.

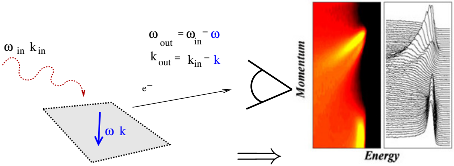
[Shankar, Polchinski, Benfatto-Gallivotti 92]



Non-Fermi liquids exist but are mysterious

e.g.: 'normal' phase of optimally-doped cuprates: ('strange metal')





among other anomalies: ARPES shows gapless modes at finite k (FS!)
with width $\Gamma(\omega_\star) \sim \omega_\star$, vanishing residue $Z \overset{k_\perp \to 0}{\to} 0$.

Working defintion of NFL:

Still a sharp Fermi surface
but no long-lived quasiparticles.

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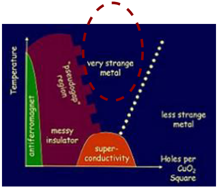

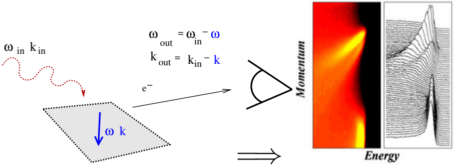

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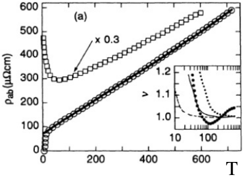
Most prominent

mystery of the strange metal phase:

e-e scattering: $\rho \sim T^2$, e-phonon: $\rho \sim T^5$, ...

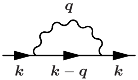
no known robust effective theory: $\rho \sim T$.



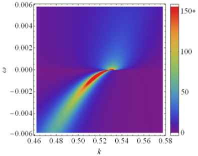
Non-Fermi Liquid from non-Holography

- Luttinger liquid in 1+1 dimensions. ✓
- loophole in RG argument:

couple a Landau FL perturbatively to a bosonic mode
(e.g.: magnetic photon, slave-boson gauge field, statistical gauge field, ferromagnetism, SDW, Pomeranchuk order parameter...)



→ nonanalytic behavior in $G^R(\omega) \sim \frac{1}{v_F k_\perp + c\omega^{2\nu}}$ at FS:
NFL.



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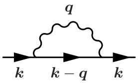

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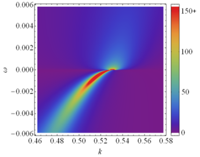


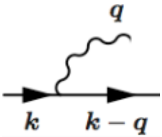
Not strange enough:

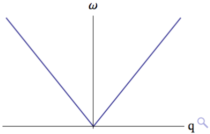
These NFLs are not strange metals in terms of transport.

FL killed by gapless bosons:

small-angle scattering dominates

$\Longrightarrow$ 'transport lifetime' $\neq$ 'single-particle lifetime'



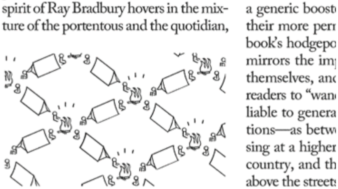


Can string theory be useful here?

It would be valuable to have a non-perturbative description of such a state in more than one dimension.

Gravity dual?

We're not going to look for a gravity dual of the whole material.



[an un-doped Cu-O plane, from the New Yorker]

Rather: lessons for universal physics of "non-Fermi liquid".

Lightning review of holographic duality

Holographic duality (AdS/CFT)

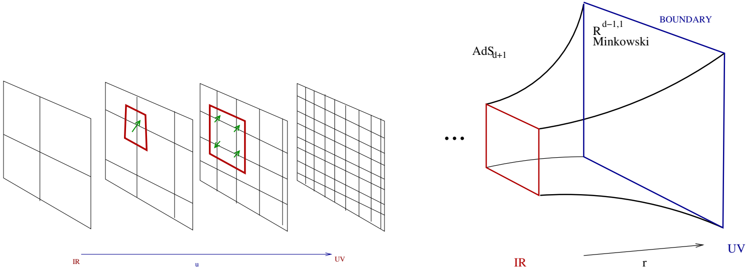
[Maldacena; Witten; Gubser-Klebanov-Polyakov]

gravity in AdS_{d+1} = d-dimensional Conformal Field Theory

(many generalizations, CFT is best-understood.)

$$AdS: \quad ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + R^2\frac{dr^2}{r^2}$$

isometries of AdS_{d+1} $\leftrightsquigarrow$ conformal symmetry



The extra ('radial') dimension is the resolution scale.

(The bulk picture is a hologram.)

when is it useful?

$$Z_{QFT}[\text{sources}] \quad = Z_{\text{quantum gravity}}[\text{boundary conditions at } r \to \infty]$$
$$\approx e^{-N^2 S_{\text{bulk}}[\text{boundary conditions at } r\to\infty]}|_{\text{extremum of } S_{\text{bulk}}}$$

classical gravity (sharp saddle) $\leftrightsquigarrow$ many degrees of freedom per point, $N^2 \gg 1$

fields in AdS_{d+1} $\leftrightsquigarrow$ operators in CFT
mass $\leftrightsquigarrow$ scaling dimension

boundary conditions on bulk fields $\leftrightsquigarrow$ couplings in field theory

e.g.: boundary value of bulk metric $\lim_{r\to\infty} g_{\mu\nu}$
$=$ source for stress-energy tensor $T^{\mu\nu}$

different couplings in bulk action $\leftrightsquigarrow$ different field theories

large AdS radius R $\leftrightsquigarrow$ strong coupling of QFT

confidence-building measures

- 1. **Many** detailed checks in special examples
  examples: relativistic gauge theories (fields are $N \times N$ matrices), with extra symmetries (conformal invariance, supersymmetry)
  checks: 'BPS quantities,' integrable techniques, some numerics
- 2. sensible answers for physics questions
  rediscoveries of known physical phenomena: *e.g.* color confinement, chiral symmetry breaking, thermo, hydro, thermal screening, entanglement entropy, chiral anomalies, superconductivity, ...
  Gravity limit, when valid, says who are the correct variables.
  Answers questions about thermodynamics, transport, RG flow, ...
  in terms of geometric objects.
- 3. applications to quark-gluon plasma (QGP)
  benchmark for viscosity, hard probes of medium, approach to equilibrium

A goal for holography

Can we formulate a tractable effective description of the low-energy physics of a system
with a Fermi surface*,
but without long-lived quasiparticles?

* Multiple possible definitions:

1. In terms of single-particle response:
Fermi surface $\equiv \{k \mid G^{-1}(k,\omega) = 0 \text{ at } \omega = 0\}$
(Here $G = \langle c^\dagger c\rangle$ is a correlator of a gauge-invariant fermion operator, like an electron, effectively.)

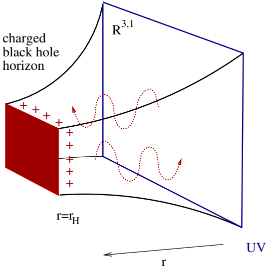
2. In terms of transport: *e.g.* $\rho(T) \sim T^{\alpha<2}$.

Minimal ingredients for a holographic Fermi surface

Consider any relativistic CFT with a gravity dual $\to g_{\mu\nu}$
a conserved $U(1)$ symmetry — proxy for fermion number $\to A_\mu$
and a charged fermion — proxy for bare electrons $\to \psi$.

$\exists$ many examples. Any $d > 1+1$, focus on $d = 2+1$.

Holographic CFT at finite density*:
charged black hole (BH) in AdS.



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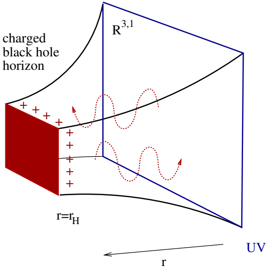

*: If we ignore the back-reaction of other fields. More soon.

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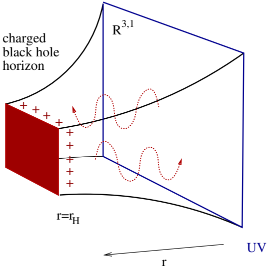
charged black hole (BH) in AdS.

To find FS: look for sharp features in fermion Green functions G_R at finite momentum and small frequency. [S-S Lee]



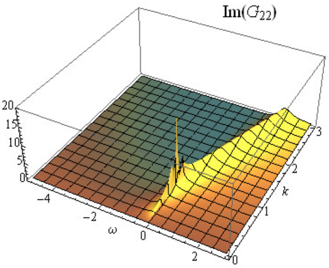
To compute G_R: solve Dirac equation in charged BH geometry.
'Bulk universality': for two-point functions, the interaction terms don't matter.
Results only depend on q, m.

*: If we ignore the back-reaction of other fields. More soon.

Fermi surface!

The system is rotation invariant, G_R depends on $k = |\vec{k}|$.

At $T = 0$, we find numerically [H. Liu-JM-D. Vegh] :



For $q = 1, m = 0$: $k_F \approx 0.92$

But it's not a Fermi liquid:

The peak has a nonlinear dispersion relation $\omega \sim k_\perp^z$ with

$$z = 2.09 \text{ for } q = 1, \Delta = 3/2$$

$$z = 5.32 \text{ for } q = 0.6, \Delta = 3/2.$$

and the residue vanishes.

Note: all frequencies measured from chemical potential. FS at $\omega = 0$.

Emergent quantum criticality

Whence these exponents?

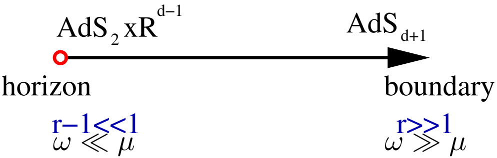
$$\text{with} \quad \mathcal{L}_{d+1} = \mathcal{R} + \frac{d(d-1)}{R^2} - \frac{2\kappa^2}{g_F^2} F^2 + \bar{\psi} i \left(\not{D} - m \right) \psi$$

the near-horizon geometry of black hole is $AdS_2 \times \mathbb{R}^{d-1}$.

The conformal invariance of this metric is **emergent**.

(We broke the microscopic conformal invariance with finite density.)

$t \to \lambda t, x \to \lambda^{1/z} x$ with $z \to \infty$.



AdS/CFT $\implies$ the low-energy physics governed by dual **IR CFT**.

The bulk geometry is a picture of the RG flow from the CFT_d to this NRCFT.

Idea for analytic understanding of FS behavior:

solve Dirac equation by matched asymptotic expansions.

In the QFT, this is RG matching between UV and IR CFTs.

Analytic understanding of Fermi surface behavior

$$\boxed{G_R(\omega, k) = K \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}}$$

The location of the Fermi surface is determined by short-distance physics (analogous to band structure –

$a, b \in \mathbb{R}$ from normalizable sol'n of $\omega = 0$ Dirac equation in full BH)

but the low-frequency scaling behavior near the FS is universal

(determined by near-horizon region – IR CFT $\mathcal{G}$).

Analytic understanding of Fermi surface behavior

$$\boxed{G_R(\omega,k) = K\frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}}$$

The location of the Fermi surface is determined by short-distance physics (analogous to band structure –
$a, b \in \mathbb{R}$ from normalizable sol'n of $\omega = 0$ Dirac equation in full BH)
but the low-frequency scaling behavior near the FS is universal
(determined by near-horizon region – IR CFT $\mathcal{G}$).

In hindsight: "semi-holographic" interpretation [FLMV, Polchinski-Faulkner]
quasiparticle decays by interacting with $z = \infty$ IR CFT d.o.f.s

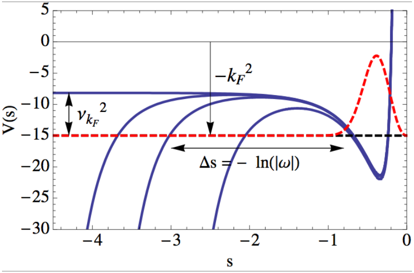
$\mathcal{G} = c(k)\omega^{2\nu}$: IR CFT correlator.

Death of the quasiparticles

Rewrite spinor equation as Schrödinger equation (with $E = 0$)
$(-\partial_s^2 + V(r(s)))\,\Psi(z(s)) = 0.$

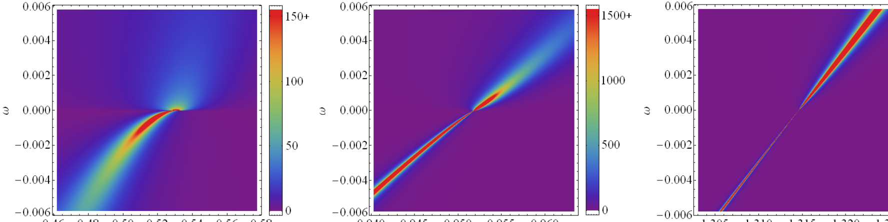
Spinor boundstate at $\omega = 0$ tunnels into AdS_2 region with rate

$$\Gamma \propto e^{-2\int ds\sqrt{V(s)}} \sim e^{2\nu \ln\omega} = \omega^{2\nu}$$



(WKB approx good at small ω)

FT interpretation: quasiparticle decays by interacting with IR CFT.



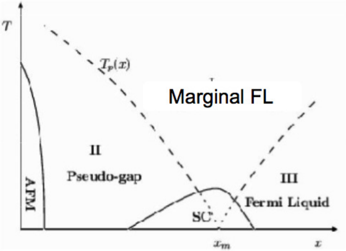
$\nu < \frac{1}{2}$ $\quad$ $\nu = \frac{1}{2}$ $\quad$ $\nu > \frac{1}{2}$

$\nu = \frac{1}{2}$: Marginal Fermi liquid

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \ln \omega + c_1 \omega}, \quad \tilde{c}_1 \in \mathbb{R}, \quad c_1 \in \mathbb{C}$$

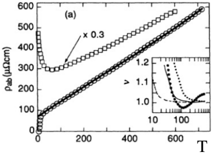
$$\frac{\Gamma(k)}{\omega_\star(k)} \stackrel{k_\perp \to 0}{\to} \mathrm{const}, \qquad Z \sim \frac{1}{|\ln \omega_\star|} \stackrel{k_\perp \to 0}{\to} 0.$$

A well-named phenomenological model of strange metal regime [Varma et al, 1989].



Charge transport by holographic Fermi surfaces

Most prominent mystery
of strange metal phase: $\rho_{\rm DC} \sim T$ $\rightarrow$



We can compute the contribution
to the conductivity from the Fermi surface

[Faulkner-Iqbal-Liu-JM-Vegh, 1003.1728 and to appear (???)]:

$$\rho_{\rm FS} \sim T^{2\nu}$$

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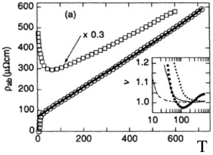

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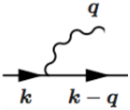
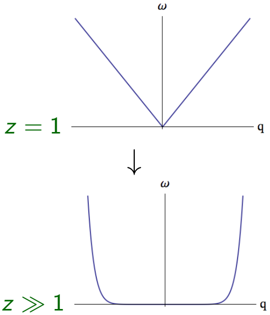


Dissipation of current is controlled by
the decay of the fermions into the AdS_2 DoFs.
$\Longrightarrow$ single-particle lifetime controls transport.



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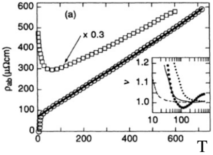

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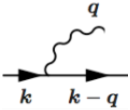
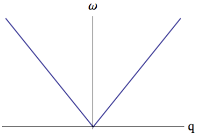
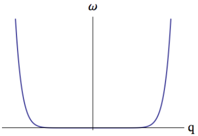


$z = 1$



$\downarrow$

$z \gg 1$



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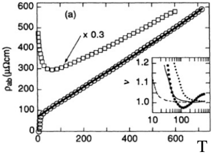

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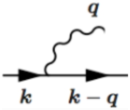

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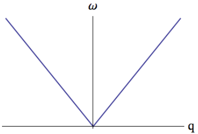

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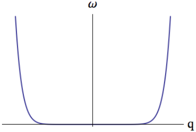

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[Important disclaimer: this is NOT the leading contribution to $\sigma_{\rm DC}$!]

Frameworks for non-Fermi liquid

- a Fermi surface coupled to a critical boson field

[Recent work: S-S Lee, Metlitski-Sachdev, Mross-JM-Liu-Senthil, 1003.0894]

$$L = \bar{\psi}\left(\omega - v_F k_\perp\right)\psi + \bar{\psi}\psi a + L(a)$$

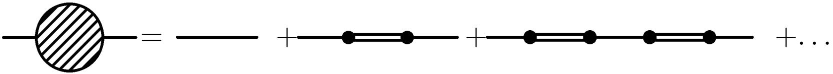
small-angle scattering dominates $\implies$ transport is not that of strange metal.

- a Fermi surface mixing with a bath of critical fermionic fluctuations with large dynamical exponent

[FLMV 0907.2694, Faulkner-Polchinski 1001.5049, FLMV+Iqbal 1003.1728]

$$L = \bar{\psi}\left(\omega - v_F k_\perp\right)\psi + \bar{\psi}\chi + \psi\bar{\chi} + \bar{\chi}\mathcal{G}^{-1}\chi$$

χ: IR CFT operator



$$\langle\bar{\psi}\psi\rangle = \frac{1}{\omega - v_F k_\perp - \mathcal{G}} \qquad \mathcal{G} = \langle\bar{\chi}\chi\rangle = c(k)\omega^{2\nu}$$

$\nu \leq \frac{1}{2}$: $\bar{\psi}\chi$ coupling is a relevant perturbation.

Drawbacks of this construction

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2. *Too much* universality! If this charged black hole is inevitable, how do we see the myriad possible dual states of matter (*e.g.* superconductivity...)?

3. The charged black hole we are studying violates the 3rd Law of Thermodynamics (Nernst's version): $S(T=0) \neq 0$ – it has a groundstate degeneracy.

   This is a manifestation of the black hole information paradox: classical black holes seem to eat quantum information.

Problems 2 and 3 solve each other: degeneracy $\implies$ instability.
The charged black hole describes an intermediate-temperature phase.

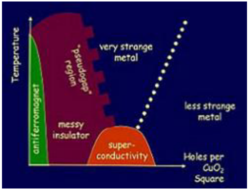
Stability of the groundstate

Often, $\exists$ charged bosons.

At small T, the dual scalar can condense

spontaneously breaking the $U(1)$ symmetry;

BH acquires hair [Gubser, Hartnoll-Herzog-Horowitz].



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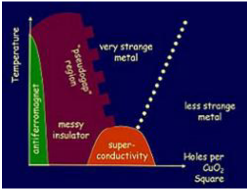
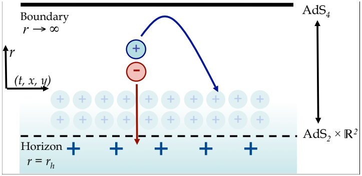
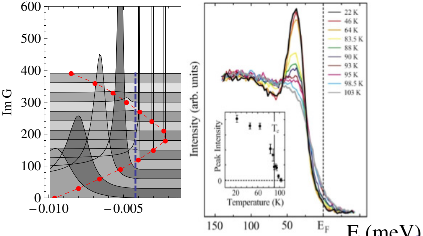


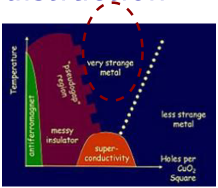


Why: black hole *spontaneously* emits charged particles [Starobinsky, Unruh, Hawking].
AdS is like a box: they can't escape.
Fermi: negative energy states get filled.
Bose: the created particles then cause *stimulated emission* (superradiance).
A holographic superconductor is a black hole laser.

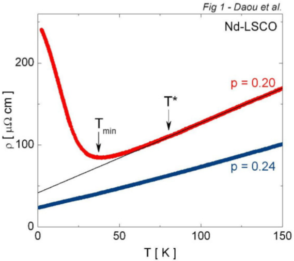
Photoemission 'exp'ts' on holographic superconductors:
[Faulkner-Horowitz-JM-Roberts-Vegh]
In SC state: a sharp peak forms in $A(k, \omega)$.
The condensate lifts the IR CFT modes into which they decay.



Superconductivity is a distraction



Look 'behind' superconducting dome by turning on magnetic field:

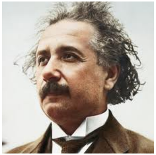


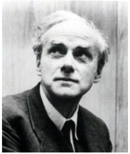
Strange metal persists to $T \sim 0$!

So we want to look for a theory of this intermediate-scale physics (like Fermi liquid theory).

The problem we really want to solve

$$\mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} + \kappa \bar{\psi} i \left(\not{D} - m \right) \psi$$





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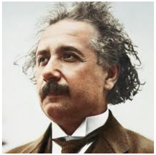
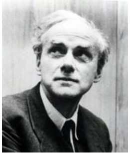
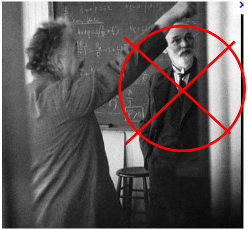
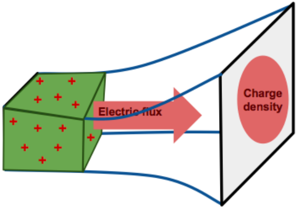






(with AdS boundary conditions, with a chemical potential.)

Electron stars



[Hartnoll and collaborators, 2010-2011]

Choose q, m to reach a regime where the bulk fermions can be treated as a (gravitating) fluid

(Oppenheimer-Volkov aka Thomas-Fermi approximation).

$\longrightarrow$ "electron star"

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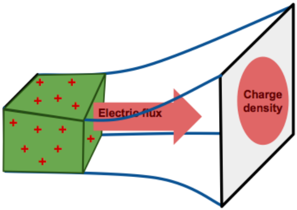

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But:

- Because of parameters (large mass) required for fluid approx, the dual Green's function exhibits *many* Fermi surfaces.

[Hartnoll-Hofman-Vegh, Iqbal-Liu-Mezei 2011]

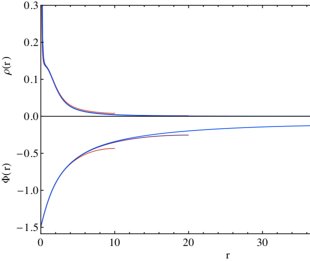
- Large mass $\Longrightarrow$ lots of backreaction $\Longrightarrow$ kills IR CFT

$\Longrightarrow$ stable quasiparticles at each FS.

To do better, we need to take into account the wavefunctions of the bulk fermion states: a *quantum* electron star.

A (warmup) quantum electron star

$$\mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F^2 + \kappa \bar{\psi} i \left(\not{D} - m \right) \psi$$



A solution of QED in AdS [A. Allais, JM, S. J. Suh].

In retrospect, the dual system describes

a Fermi Surface coupled to relativistic CFT.

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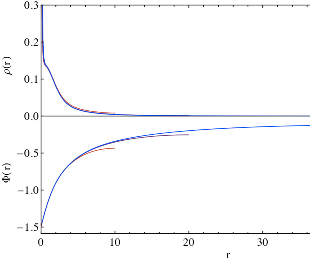


a Fermi Surface coupled to relativistic CFT.

• FS quasiparticles survive this:

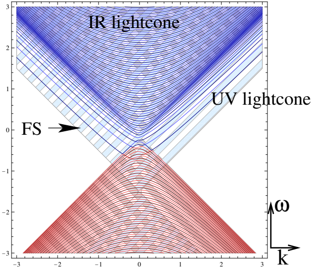
FS at $\{\omega = 0, |\vec{k}| = k_F \neq 0\}$

is outside IR lightcone $\{|\omega| \geq |\vec{k}|\}$.

Interaction is kinematically forbidden.

[Landau: minimum damping velocity in superfluid;

Gubser-Yarom; Faulkner-Horowitz-JM-Roberts-Vegh]



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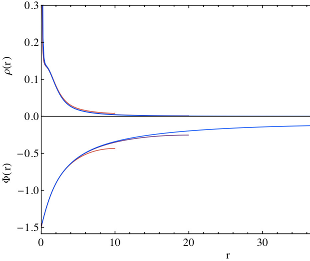

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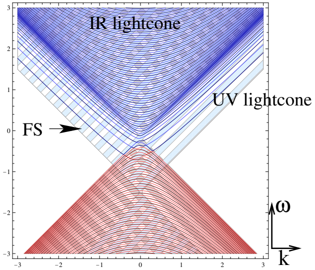


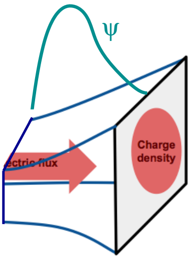
- When we include gravitational backreaction [in progress with Andrea Allais]

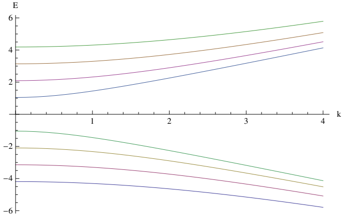
(dual to effects of FS on gauge theory dynamics)

the IR geometry will be different from AdS.

Optimism: A quantum electron star is a happy medium between AdS_2 (no fermions) and classical electron star (heavy fermions).

Towards a quantum electron star





[Sachdev, 2011]: A model of a Fermi liquid.

Like AdS/QCD: a toy model of the groundstate of a confining gauge theory from a hard cutoff in AdS.

Add chemical potential.
Compute spectrum of Dirac field, solve for backreaction on A_μ. Repeat as necessary.

The system in the bulk *is* a Fermi liquid (in a box determined by the gauge dynamics).

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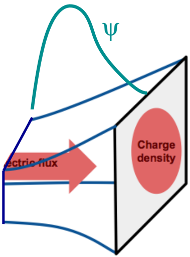

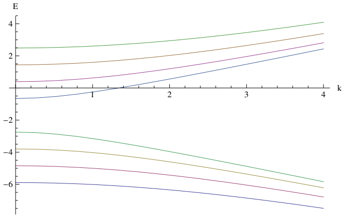

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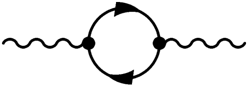
Towards a fermion-driven deconfinement transition

Lots of low-E charged dofs screen gauge interactions.



Effect of fermions on the gauge dynamics = gravitational backreaction.

A real holographic model of confinement: AdS soliton

so far: $\rightarrow$

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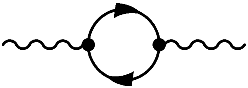

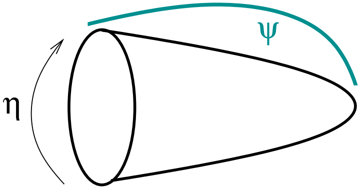

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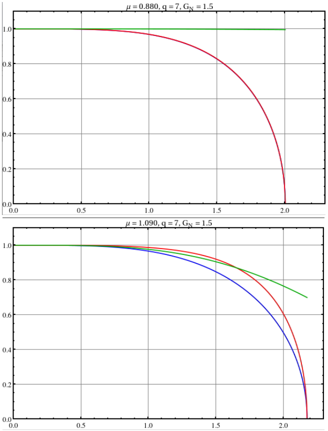

What's the endpoint of this transition?

A quantum electron star

Numerics explode before transition. Essential problem: what can the state of the bulk fermions be if the geometry has as horizon?

Probe limit: $G_N \to 0$ [like HHH 0803]

FT Interpretation: most CFT dofs are neutral. $(c \sim \frac{L^2}{G_N}(\propto \langle TT \rangle) \gg k \propto \langle jj \rangle)$

$$\not{D}_\Phi \psi = 0$$

$$\Phi'' = -q^2 \rho$$

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Numerics explode before transition. Essential problem: what can the state of the bulk fermions be if the geometry has as horizon?

Probe limit: $G_N \to 0$ [like HHH 0803]

FT Interpretation: most CFT dofs are neutral. $(c \sim \frac{L^2}{G_N}(\propto \langle TT \rangle) \gg k \propto \langle jj \rangle)$

$$\not{D}_\Phi \psi = 0$$

$$\begin{pmatrix} \Phi(z) + k & \frac{\partial}{\partial z} - \frac{m}{z} \\ -\frac{\partial}{\partial z} - \frac{m}{z} & \Phi(z) - k \end{pmatrix} \chi_n = \omega_n \chi_n$$

$$\psi =: (-g g^{zz})^{-\frac{1}{4}} e^{-i\omega t + i k_i x^i} \chi$$

Normalizable BCs at $z = 0$,
hard-wall BC at $z = z_m$

$$\Phi'' = -q^2 \rho$$

$$\Phi''(z) = -q^2 \left(\rho(z) - \rho(z)|_{\Phi=0} \right),$$

$$\rho(z) \equiv \sum_{n, \omega_n < 0} \psi_n^2(z)$$

The padded room

Compute charge density:

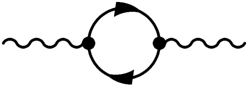
$$\langle n(r)\rangle = \langle \psi^\dagger(r)\psi(r)\rangle = \sum_k n_k(r) \sim \int d^2k \frac{1}{k^2}\Phi''(r)$$

Cutoffs everywhere: UV cutoff on AdS radial coordinate, bulk UV cutoff (lattice), UV cutoff on k integral, IR cutoff on AdS radial coordinate: z_m.

Charge renormalization.

Define charge susceptibility by linear response:

$$\chi \equiv \sum_k \chi(k), \quad \chi(k) = \frac{\Delta\rho_k(z_\star)}{\Phi''(z_\star)}$$



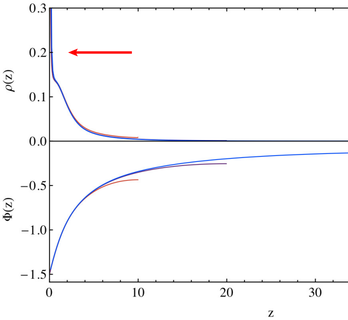
$$q_R^2 = q_0^2 \frac{1}{1 - q_0^2\chi}$$

Physics checks:

1) Surface charge. Our bulk charges are not mobile in the *AdS* radial direction. (Like metal of finite extent along one axis.)

An electric field applied to an insulator polarizes it.

This results in a surface charge $\sigma_b = \hat{n} \cdot \vec{P}$.



2) Chiral anomaly.

Each k mode is a 1+1 fermion field

$S_k = \int drdt \; i\bar{\psi}_k \left(\not{D} + m + i\gamma^5 k \right) \psi_k$

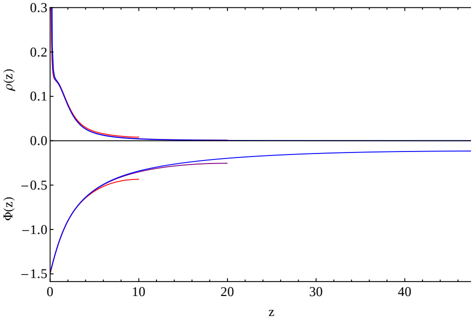
$\stackrel{?}{\Longrightarrow} \; \partial_r n_k \to 0$ when $m, k \to 0$.

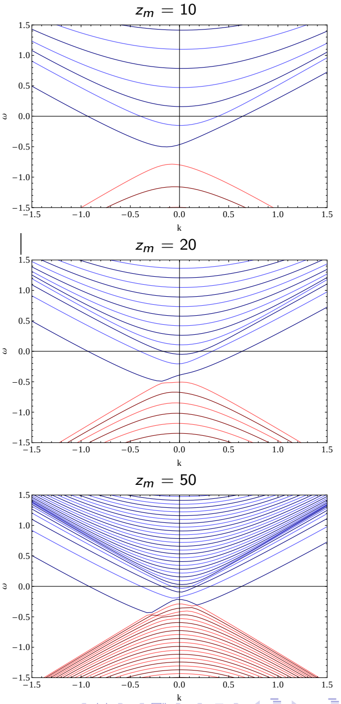
Not so in numerics:

$$\partial_\mu j_5^\mu = \frac{1}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu} = -\frac{1}{\pi} \Phi' \qquad \checkmark$$

A quantum electron star

The limit $z_m \to \infty$ exists! :





Electrostatic and Pauli repulsion supports the fermions against falling into the AdS gravitational well.

Semi-holographic interpretation

The dual system can be regarded as

a Fermi Surface coupled to relativistic CFT (with gravity dual)

$\Phi(r)$: how much of the chemical potential is seen by the dofs of wavelength $\sim r$.

Convergence of EOM requires $\Phi(\infty) = 0$, complete screening in far IR.

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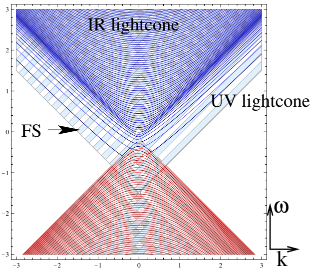
$\Phi(r)$: how much of the chemical potential is seen by the dofs of wavelength $\sim r$.
Convergence of EOM requires $\Phi(\infty) = 0$, complete screening in far IR.

$\Phi(\infty) = 0$ means FS survives this coupling to CFT:
FS at $\{\omega = 0, |\vec{k}| = k_F \neq 0\}$ is outside IR lightcone $\{|\omega| \geq |\vec{k}|\}$.
Interaction is kinematically forbidden.

[Landau: minimum damping velocity in SF; Gubser-Yarom; Faulkner et al 0911]

In probe limit, quasiparticles survive.
With "Landau damping," IR speed of light smaller, maybe not.



Future quantum electron stars

• When we include gravitational backreaction

(dual to effects of FS on gauge theory dynamics)

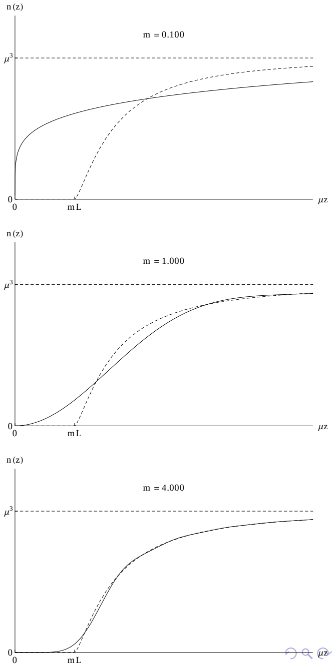
the IR geometry can be different from AdS.

Optimism: happy medium between AdS_2 (no fermions) and classical electron star (heavy fermions).

Q: What happens?

What other Fermi surface states can arise holographically?

A: We'll see!



One example of the difficulties involved

Consider a massless Dirac fermion in 1+1 dimensions.

$$ds^2 = -f_t(z)dt^2 + f_z(z)dz^2$$

For convenience take $z \simeq z + 2\pi$ and $f_z = 1$ gauge.
Conformal anomaly:

$$T^\mu_\mu = \frac{1}{4\pi}\mathcal{R}(z) = \frac{1}{4\pi}\left(\frac{1}{2}\left(\frac{f_t'^2}{f_t}\right)^2 - \frac{f_t''}{f_t}\right)$$

$$H = \begin{pmatrix} -m\sqrt{f_t} & -\left(\frac{f_t}{f_z}\right)^{1/4}\partial_z\left(\frac{f_t}{f_z}\right)^{1/4} \\ \left(\frac{f_t}{f_z}\right)^{1/4}\partial_z\left(\frac{f_t}{f_z}\right)^{1/4} & m\sqrt{f_t} \end{pmatrix}.$$

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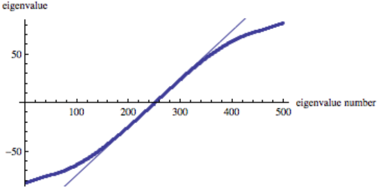
Latticeize, add up:

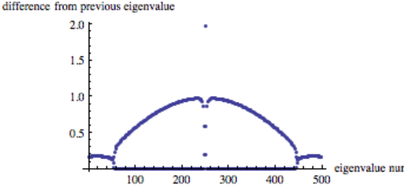
$$T^\mu_\mu = \sum_{a\,\in\,\text{spectrum of H}} \theta(\omega_a)\psi_a^\dagger\left(\ldots\right)\psi_a = \frac{1}{4\pi}\left(\frac{3}{4}\left(\frac{f_t'^2}{f_t}\right)^2 - \frac{f_t''}{f_t}\right)$$

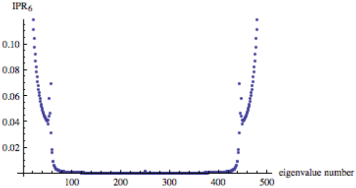
Not a scalar!

One example of the difficulties involved, cont'd

Why?

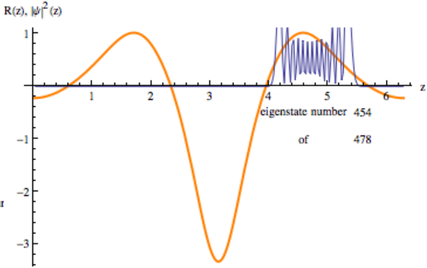






e.g.: $f_t = 1 + .3\cos z + .2\cos 2z$,

$n = 249$ sites.



$$IPR_k \equiv \int dz \, |\psi|^{2k}$$

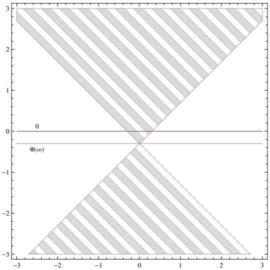
The end.

Thanks for listening.

Physics of quantum electron star

UV lightcone for charge-q dofs:
$\{(\omega, k)|(\omega + q\mu)^2 \leq c^2k^2\}$

IR lightcone for charge-q dofs:
$\{(\omega, k)|(\omega + q\Phi(\infty))^2 \leq c^2k^2\}$
FS boundstate can scatter off these dofs (recall tunneling into AdS_2).



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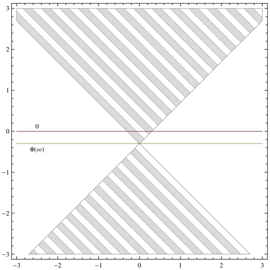

Q: What's $\Phi(\infty)$?
A: $\Phi(\infty) = 0$. If $\Phi(\infty) \neq 0$: occupation of continuum.

$$\psi_{\text{IR LC}}(z) \overset{z\to\infty}{\to} e^{i\kappa z} \implies \rho(z) \overset{z\to\infty}{\to} \text{const}$$

$$\implies \Phi(z) \overset{z\to\infty}{\sim} z^2 \neq \Phi(\infty)$$

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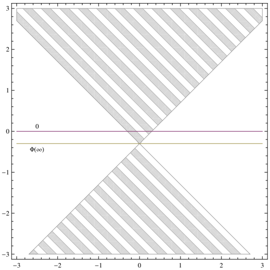

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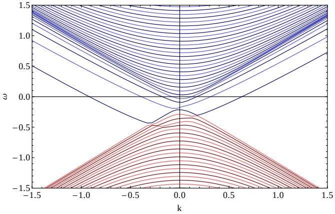

Q: Whence power-law?
A: The modes which skim the IR lightcone. Matching calculation?