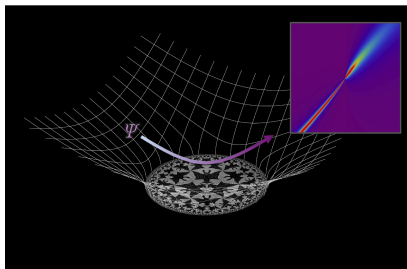


Strong Correlations with String Theory

John McGreevy



Bold claim: string theory is useful.

My goal for today is to convince you that string theory can be useful for physics.

The physical systems about which we can hope to say something have in common **strong coupling** or **strong correlations**.

This feature is a big problem for our usual techniques.

This opportunity comes about in a very sneaky way, and to explain it, we have to back up a bit.

Unity of purpose in hep-th and cond-mat

A string theorist's instruction manual for doing theoretical physics:

Step 1: Identify the quantum field theory (QFT) that describes your system.

e.g. there's one for QED, QCD, ferromagnets, high- T_c , ...

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Step 1: Identify the quantum field theory (QFT) that describes your system.

e.g. there's one for QED, QCD, ferromagnets, high- T_c , ...

Step 2: Figure out what happens:

What is the groundstate?

What are the low-energy excitations above the groundstate?

(In favorable cases: 'elementary particles' or 'quasiparticles')

Sometimes we can answer these questions using ordinary tricks.

Basically, perturbation theory around a 'solvable' theory.

When the interactions are not a small perturbation, this fails.

Plan for this talk

Here's the sneaky way of using string theory:

We can answer these questions about some field theories using an **auxiliary** string theory, some ground state of string theory that looks nothing like ours:

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This relation is called 'the AdS/CFT Correspondence' or 'Holographic Duality'. I'm going to explain its origins.

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Then I'll talk about three classes of real physical systems (which involve strong interactions between the constituents) where usual techniques have been having a hard time and where we've been trying to use these ideas to learn something about physics.

Universality and coarse-graining in field theory

and other systems with *extensive* degrees of freedom.

Old-school universality

Experimental universality [1960s]:

same critical exponents from (microscopically) very different systems.

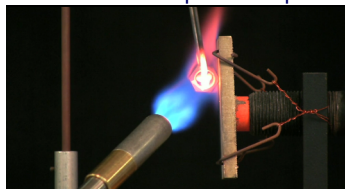
Near a (continuous) phase transition (at $T = T_c$), scaling laws:
observables depend like power laws on the deviation from the critical point.

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e.g. ferromagnet near the Curie transition

$$\text{(let } t \equiv \frac{T_c - T}{T_c}\text{)}$$

$$\text{specific heat: } c_V \sim t^{-\alpha}$$

$$\text{magnetic susceptibility: } \sim t^{-\gamma}$$

[MIT TSG]

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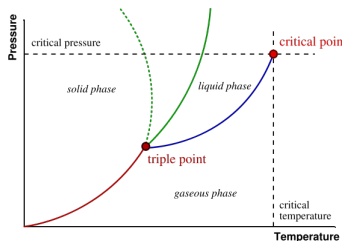
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Water near its liquid-gas critical point:

specific heat: $c_V \sim t^{-\alpha}$

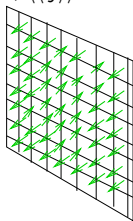
compressibility: $\sim t^{-\gamma}$

with the same α, γ !

Renormalization group idea

This phenomenon is explained by the Kadanoff-Wilson idea:

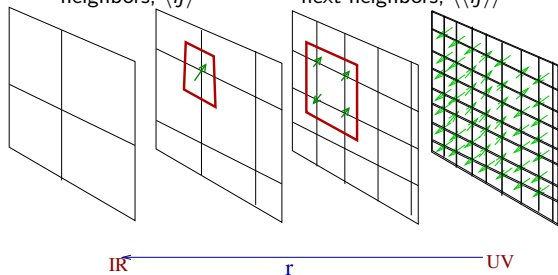
$$\text{eg : } S_i = \pm 1 \quad H = \sum_{\text{neighbors, } \langle ij \rangle} J_{ij} S_i S_j + \sum_{\text{next neighbors, } \langle\langle ij \rangle\rangle} K_{ij} S_i S_j + \dots$$



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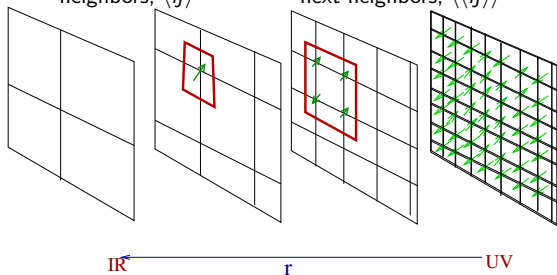
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Let 'block spin' = average value of spins in block.

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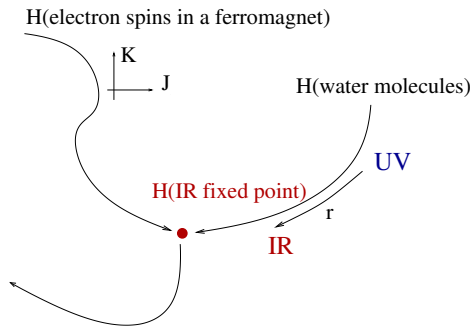
Idea: measure the system with coarser and coarser rulers.

Let 'block spin' = average value of spins in block.

Define a Hamiltonian $H(r)$ for block spins so long-wavelength observables are the same.

→ a 'renormalization group' (RG) flow on the space of hamiltonians: $H(r)$

RG fixed points give universal physics

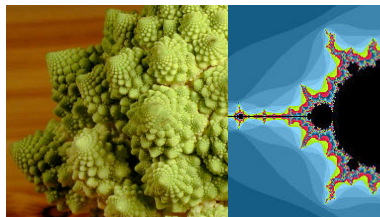


Universality: fixed points are rare.
Many microscopic theories will
flow to the same fixed-point.

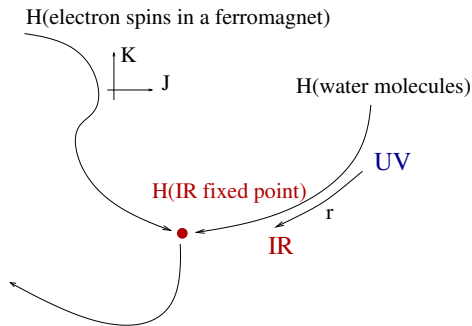
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The fixed point theory is
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if you change your resolution you get the
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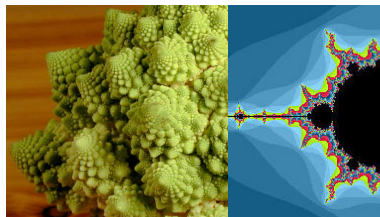
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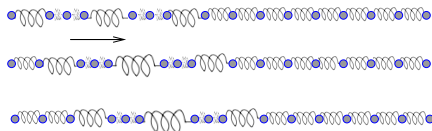
Sometimes the fixed point theory is also
'Conformally invariant'.

This is the 'C' in AdS/CFT.



Quantum field theory (QFT)

Questions about long-wavelength modes, wavelength \gg lattice spacing:



lattice details absorbed in couplings between long-wavelength modes

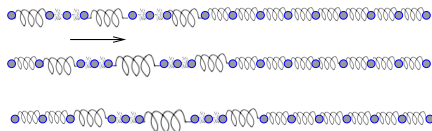
→ continuum description: this is a QFT.

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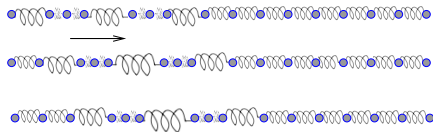
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Not everything is harmonic oscillators with small nonlinearities!

Strongly coupled QFTs are at the heart of central problems of modern physics.

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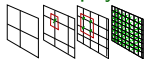
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Wouldn't it be nice if the picture satisfied some nice equation?



Some remarks on
the curious scientific status
of string theory

What is string theory?

String theory is an alien artifact, discovered in the wreckage of hadronic resonances.

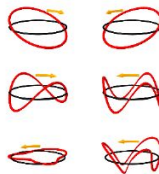
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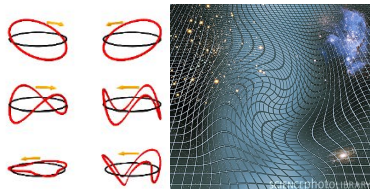
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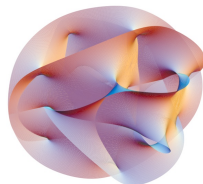


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- 1980s: people realized that it has vacua that look like the Standard Model of particle physics, coupled to gravity.

→ much rejoicing.



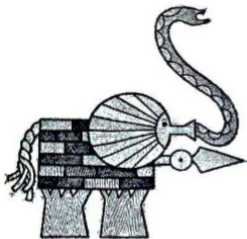
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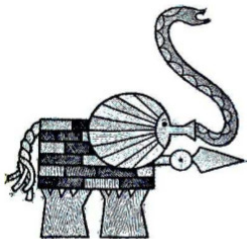


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We have a machine doing perturbation theory around free strings
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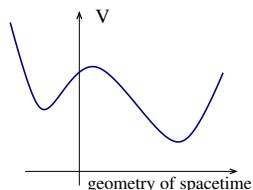
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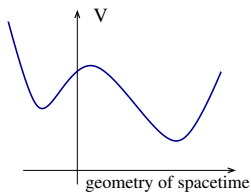
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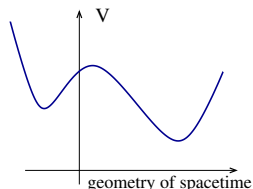
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These vacua ARE NOT the subject of today's talk.)

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most of which don't.

The holographic principle

Gravity is different.

Black hole thermodynamics

Gravity \implies black holes. (regions of no escape)

There are close parallels between black hole (BH) mechanics and the Laws of Thermodynamics. [70s]

Consistent laws of thermo require BH has entropy: ($k_B = 1$)

$$S_{\text{BH}} = \frac{\text{area of horizon}}{4\ell_p^2}. \quad \ell_p \equiv \sqrt{\frac{G_N \hbar^2}{c^3}}.$$

'Generalized 2d Law': $S_{\text{total}} \equiv S_{\text{ordinary stuff}} + S_{\text{BH}}$
 $\Delta S_{\text{total}} \geq 0$ in processes which *happen*. [Bekenstein]

Holographic principle

Recall: In an ordinary d -dim'l system without gravity
(a chunk of stuff, the vacuum...) DoFs at each point \implies
max entropy in some region of space \sim volume L^d

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Holographic Principle: In a gravitating system,
max entropy in a region of space $V =$
entropy of the biggest black hole that fits.

$$S_{max} = S_{BH} = \frac{1}{4\pi G_N} \times \text{horizon area}$$

\propto area of ∂V in planck units. [t Hooft, Susskind 1990s]

Why: suppose the contrary, a configuration with
 $S > S_{BH} = \frac{A}{4G_N}$ but $E < E_{BH}$ (biggest BH fittable in V)
Then: throw in junk (increases S and E) until you make a BH.
 S decreased, violating 2d law.

Punchline: Gravity in $d + 1$ dimensions

has the same number of degrees of freedom as

a QFT in *fewer* (d) dimensions.



Questions:

- Who is the QFT on the boundary?
- From its point of view, what is the extra dimension?
- Where do I put the boundary?

Holographic duality (AdS/CFT)

gravity in some spacetime AdS_{d+2}



$$\left(ds^2 = \frac{R^2}{r^2} (-dt^2 + d\vec{x}^2) + \frac{R^2 dr^2}{r^2} \right)$$

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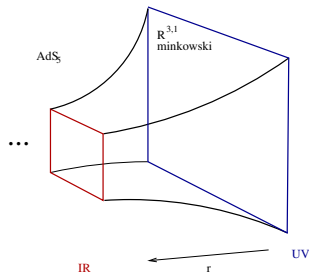
EQUALS

[Maldacena]

a CFT in $d + 1$ spacetime dimensions

- No proof yet. A zillion checks.
- LHS: 'bulk' RHS: 'boundary'. You'll see why on next slide.

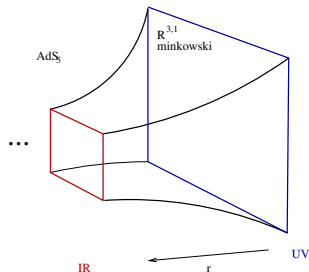
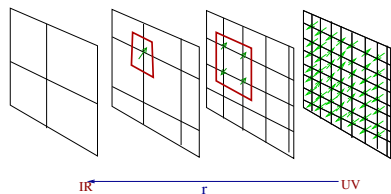
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First check: symmetries of AdS = the relativistic conformal group.
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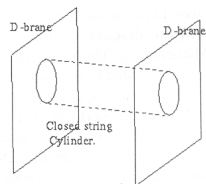
The extra ('radial') dimension is the resolution scale!

The bulk picture is a hologram:

Things with different wavelengths get put in different places.

Boundary at $r = 0$: UV data is 'initial' conditions for RG flow.

Best-understood example (of infinitely many)

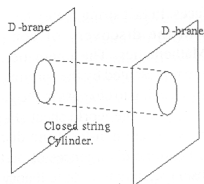


Role of string theory: identify precise dual pairs.

“ $\mathcal{N} = 4$ SYM” is a CFT.

(a supersymmetric, relativistic gauge theory).

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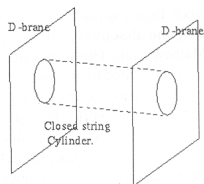
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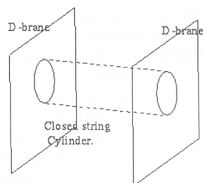
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- large N makes gravity classical
(suppresses splitting and joining of strings)
- strong coupling (large λ) makes the geometry big.

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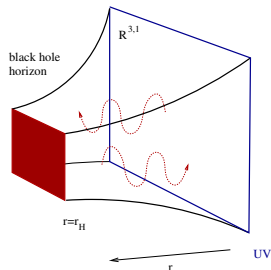
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- strong/weak duality:** hard to check, very powerful

Holographic duality at finite temperature

Black holes radiate like blackbodies [Hawking].

gravity in AdS_{d+2} with a black hole



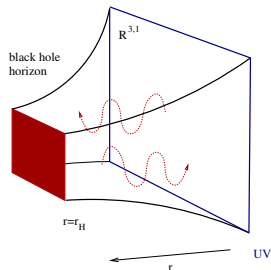
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EQUALS [Witten]

CFT in $d + 1$ spacetime dimensions at finite temperature



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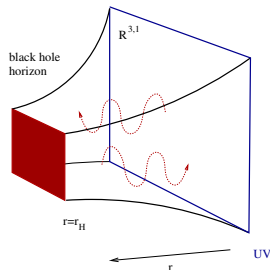
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CFT in $d + 1$ spacetime dimensions at finite temperature

GR 'no-hair theorem':
black holes labelled by few parameters
(mass, charge)



Thermal equilibrium states labelled by few parameters
(temperature, chemical potential)



What can be computed

[Gubser-Klebanov-Polyakov, Witten]

fields in the bulk \longleftrightarrow local operators in the QFT

Compute correlation functions by solving classical wave equations.

New perspective on the structure of QFT: access to otherwise uncalculable things in uncalculable situations.

$G(\omega, k, T)$

potentials for moving probes

entanglement entropy

at strong coupling
far from equilibrium
in real time

with a finite density of fermions

Applications of holographic duality to quantum liquids

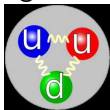
Next I'll discuss three example systems to which we can apply these ideas.

Example 1: The Strong Interactions

The theory of the Strong Interactions (QCD) is also a gauge theory.

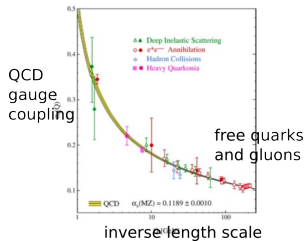
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For length scales longer than 1GeV^{-1} :



CONFINEMENT.

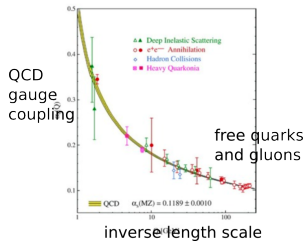
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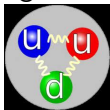
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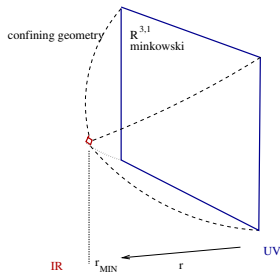


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Holography provides a useful image:

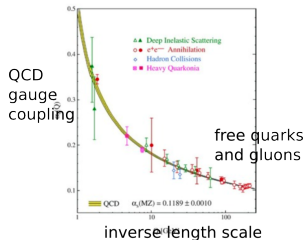
Spectrum of hadrons: resonances in this cavity.



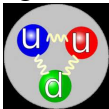
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The theory of the Strong Interactions (QCD) is also a gauge theory.

Unlike $\mathcal{N} = 4$ SYM, it's not a CFT; the coupling runs.



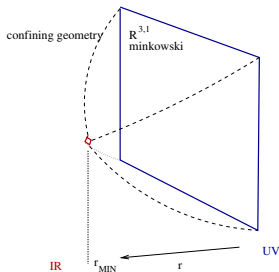
For length scales longer than 1GeV^{-1} :



CONFINEMENT.

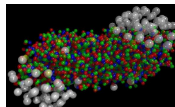
We still lack a quantitative theoretical understanding of this phenomenon.

Holography provides a useful image:



Spectrum of hadrons: resonances in this cavity.

A new state of condensed matter [RHIC, LHC]:



Quark-gluon plasma is strongly coupled

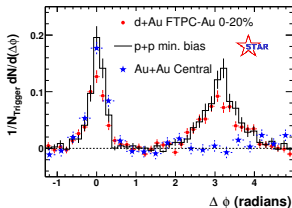
QGP is strongly coupled: a liquid, not a gas.

(RHIC, LHC not in asymptotically free regime.)

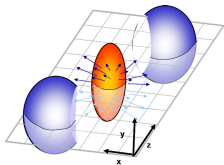
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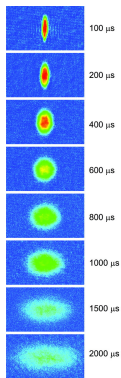
1. It is opaque:



2. It exhibits rapid thermalization,
rapid hydro-ization to a fluid with very low viscosity.
It exhibits collective motion ('elliptic flow'):



[O'Hara et al]



Holographic gauge theory plasma

Positive outcomes of approximating QCD in this regime by a QFT with a gravity dual:

- ▶ String theorists have learned lots of physics.
- ▶ The holographic plasma provided a proof of principle that low viscosity $\eta/s \sim \frac{1}{4\pi}$ was possible
(vs: perturbation theory prediction of $\frac{\eta}{s} = \frac{1}{g^4 \ln g}$ with $g \ll 1$).
- ▶ Beautiful studies of hydrodynamics
by BH horizon fluctuations

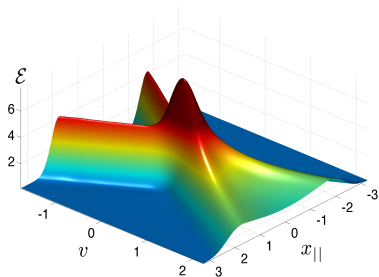
Where's the dissipation? Energy falls into BH. [Horowitz-Hubeny, 99]

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but:

RHIC and LHC unwieldy.

The QGP lasts for a time of order a few light-crossing times of a nucleus.

Wouldn't it be nice if we could do a quantum gravity experiment on a table top...

Example 2:

Galilean CFT liquid from holography

(towards cold atoms at unitarity)

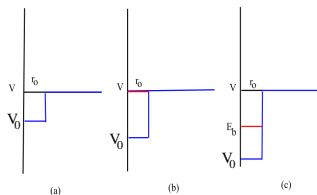
Cold atoms at unitarity

Most of the work on AdS/CFT involves relativistic CFTs.

Strongly-coupled Galilean-invariant CFTs exist, even experimentally.

[Zwierlein et al, Hulet et al, Thomas et al]

Consider nonrelativistic fermionic particles ('atoms') interacting via a short-range attractive two-body potential $V(r)$, e.g.:



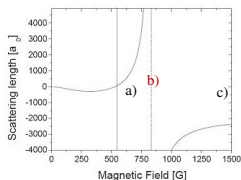
Case (b): σ saturates bound on scattering cross section from unitarity

Range of interactions $\rightarrow 0$, scattering length $\rightarrow \infty \implies$ no scale.

Lithium atoms

have a boundstate with a different magnetic moment.

Zeeman effect \implies scattering length can be controlled using an external magnetic field:



Strongly-coupled NRCFT

The fixed-point theory (“fermions at unitarity”) is a strongly-coupled nonrelativistic CFT (‘Schrödinger symmetry’)

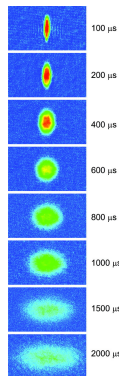
[Nishida-Son].

Universality: it also describes neutron-neutron scattering [Mehen-Stewart-Wise]
Two-body physics is completely solved.

Many body physics is mysterious.

Experiments: very low viscosity, $\frac{\eta}{s} \sim \frac{5}{4\pi}$ [Thomas, Schafer]

→ strongly coupled.



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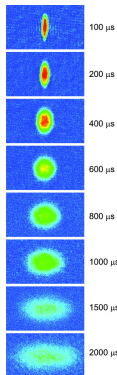
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→ strongly coupled.

AdS/CFT?

Clearly we can’t approximate it as a *relativistic* CFT.

Different hydro: conserved particle number.



A holographic description?

Method of the missing box

AdS : relativistic CFT

: Galilean-invariant CFT

A holographic description?

Method of the missing box

AdS : relativistic CFT

“Schrödinger spacetime” : Galilean-invariant CFT

A spacetime whose isometry group is the Schrödinger group:

[Son; K Balasubramanian, JM]

$$L^{-2} ds^2 = \frac{2d\xi dt + d\vec{x}^2 + dr^2}{r^2} - \frac{dt^2}{r^2}$$

This metric solves reasonable equations of motion.

Holographic prescription generalizes naturally.

But: the vacuum of a Galilean-invariant field theory is extremely boring:
no antiparticles! no stuff!

How to add stuff?

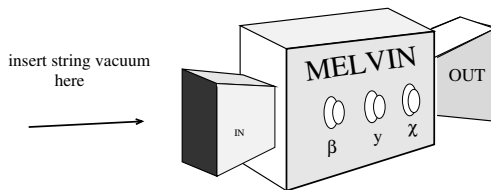
A holographic description of more than zero atoms

A black hole in Schrödinger spacetime.

[A. Adams, K. Balasubramanian, JM; Maldacena et al; Rangamani et al]

Here, string theory was extremely useful:

A solution-generating machine named Melvin: [Ganor et al]



IN: $AdS_5 \times S^5$

OUT: Schrödinger $\times S^5$

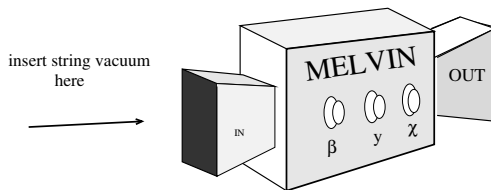
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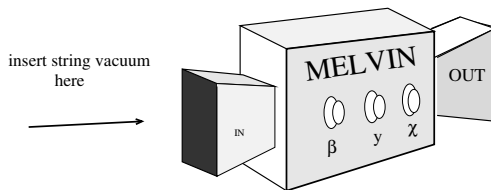
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OUT: **Schrödinger BH** \times squashed S^5

This black hole gives the thermo and hydro of some NRCFT.

Not unitary fermions: $F \sim -\frac{T^4}{\mu^2}$, $\mu < 0$.

Unnecessary assumption: all of Schröd must be realized geometrically.

We now know how to remove this assumption, can seek more realistic models.

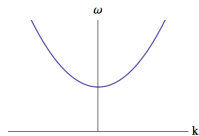
Example 3:

Strange metals from holography

Towards universal physics of interacting Fermi surfaces

Hierarchy of understoodness

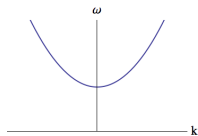
systems with a
gap (insulators)



Effective field
theory (EFT) is a
topological field
theory

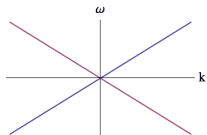
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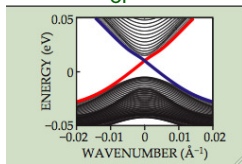


Effective field theory (EFT) is a topological field theory

systems at critical points or topological insulators with gapless boundary DoFs



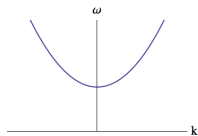
or



EFT is a CFT

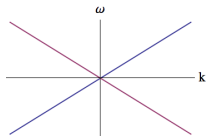
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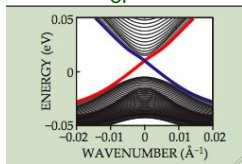


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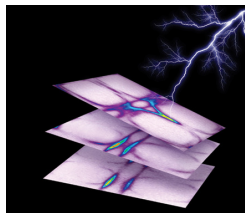
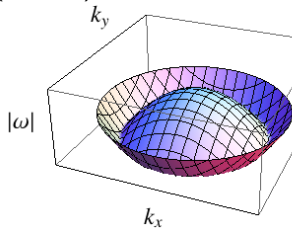


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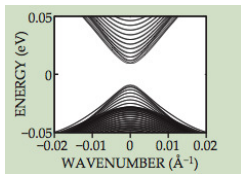


EFT is a CFT

systems with a Fermi surface (metals)



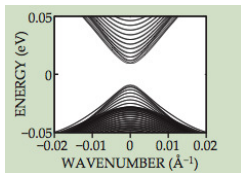
Slightly subjective musical classification of states of matter



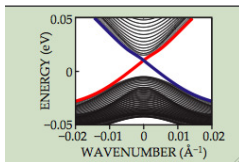
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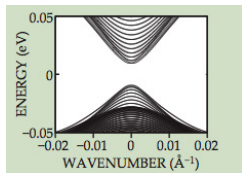
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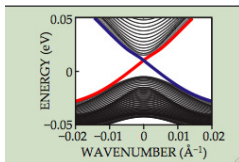
rel. critical point or TI



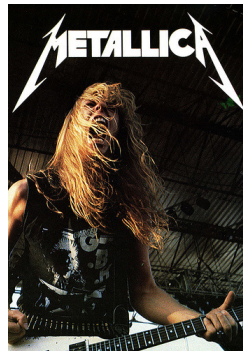
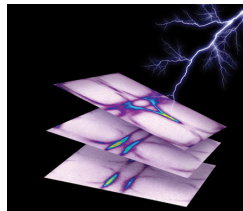
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Fermi Liquids

Basic question: What is the effective field theory for a system with a Fermi surface (FS)?

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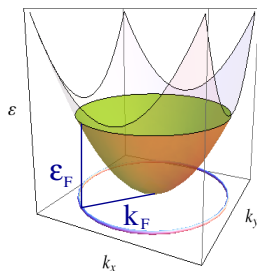
Lore: must be Landau Fermi liquid [Landau, 50s].

Recall [8.044, 8.06]:

if we had *free* fermions, we would fill single-particle energy levels $\epsilon(k)$ until we ran out of fermions: \rightarrow

Low-energy excitations:

remove or add electrons near the Fermi surface ϵ_F, k_F .



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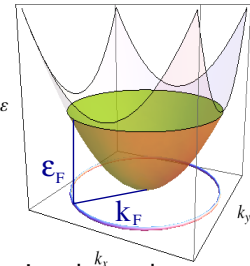
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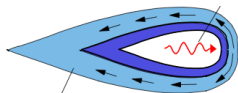
remove or add electrons near the Fermi surface ϵ_F, k_F .

Idea [Landau]: The low-energy excitations of the interacting theory are still weakly-interacting fermionic, charged 'quasiparticles'.

Elementary excitations are free fermions with some dressing:



in medium
 \rightarrow



The standard description of metals

The metallic states that we understand well are described by **Landau's Fermi liquid theory**.

Landau quasiparticles \rightarrow poles in single-fermion Green function G_R

at $k_{\perp} \equiv |\vec{k}| - k_F = 0$, $\omega = \omega_*(k_{\perp}) \sim 0$: $G_R \sim \frac{Z}{\omega - v_F k_{\perp} + i\Gamma}$

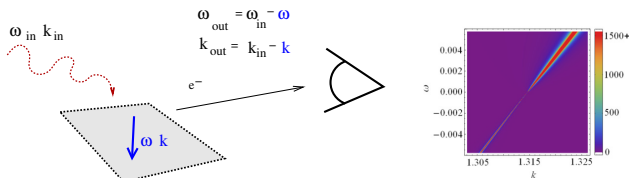
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Measurable by ARPES (angle-resolved photoemission):



Intensity \propto
spectral density :
$$A(\omega, k) \equiv \text{Im } G_R(\omega, k) \xrightarrow{k_{\perp} \rightarrow 0} Z \delta(\omega - v_F k_{\perp})$$

Landau quasiparticles are long-lived: width is $\Gamma \sim \omega_{\star}^2$,
residue Z (overlap with external e^-) is finite on Fermi surface.

Reliable calculation of thermodynamics and transport relies on this.

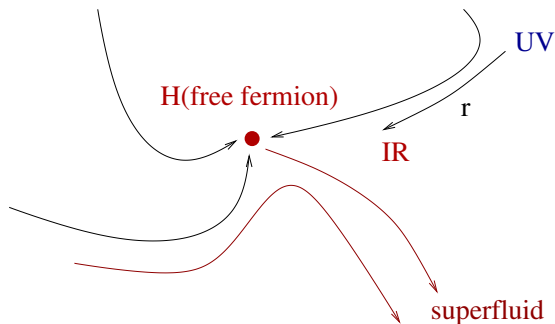
Ubiquity of Landau Fermi liquid

Physical origin of lore:

1. Landau FL successfully describes ^3He ,
metals studied before $\sim 1980\text{s}$, ...

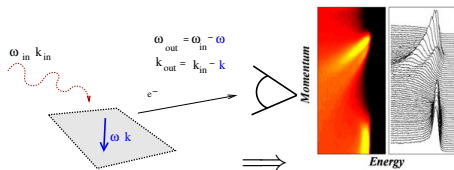
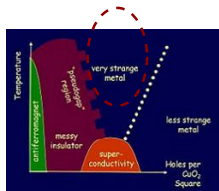
2. RG: Landau FL is stable under almost all perturbations.

[Shankar, Polchinski, Benfatto-Gallivotti 92]



Non-Fermi liquids exist but are mysterious

e.g.: 'normal' phase of optimally-doped cuprates: ('strange metal')



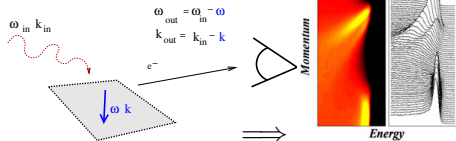
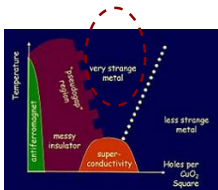
among other anomalies: ARPES shows gapless modes at finite k (FS!) with width $\Gamma(\omega_*) \sim \omega_*$, vanishing residue $Z \xrightarrow{k_{\perp} \rightarrow 0} 0$.

Working definition of NFL:

Still a sharp Fermi surface
but no long-lived quasiparticles.

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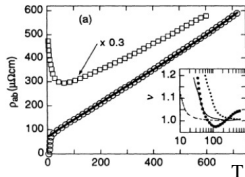
Still a sharp Fermi surface
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Most prominent

mystery of the strange metal phase:

e-e scattering: $\rho \sim T^2$, e-phonon: $\rho \sim T^5$, ...

no known robust effective theory: $\rho \sim T$.



Non-Fermi Liquid from non-Holography

- Luttinger liquid in 1+1 dimensions. ✓

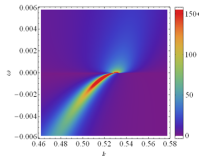
- loophole in RG argument:

couple a Landau FL **perturbatively** to a bosonic mode

(e.g.: magnetic photon, slave-boson gauge field, statistical gauge field, ferromagnetism, SDW, Pomeranchuk order parameter...)



→ nonanalytic behavior in
 $G^R(\omega) \sim \frac{1}{v_F k_{\perp} + c\omega^{2\nu}}$ at FS:
NFL.

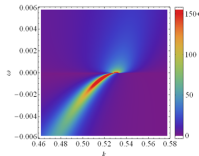


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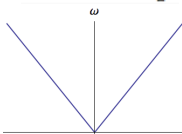
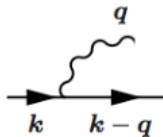
Not strange enough:

These NFLs are **not** strange metals
in terms of transport.

FL killed by gapless bosons:

small-angle scattering dominates

⇒ 'transport lifetime' \neq 'single-particle lifetime'



Can string theory be useful here?

It would be valuable to have a non-perturbative description of such a state in more than one dimension.

Gravity dual?

We're not going to look for a gravity dual of the whole material.

spirit of Ray Bradbury hovers in the mixture of the portentous and the quotidian,



a generic boost their more perri book's hodgep mirrors the im; themselves, and readers to "wan liable to genera tions—as betw sing at a higher country, and th above the street

[an un-doped Cu-O plane, from the New Yorker]

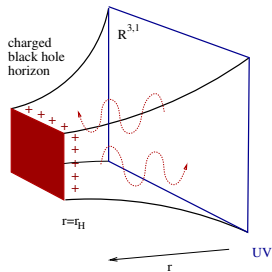
Rather: lessons for universal physics of "non-Fermi liquid".

Minimal ingredients for a holographic Fermi surface

Consider any relativistic CFT with a gravity dual $\rightarrow g_{\mu\nu}$
a conserved $U(1)$ symmetry proxy for fermion number $\rightarrow A_\mu$
and a charged fermion proxy for bare electrons $\rightarrow \psi$.

\exists many examples. Any $d > 1 + 1$, focus on $d = 2 + 1$.

Holographic CFT at finite density*:
charged black hole (BH) in AdS .

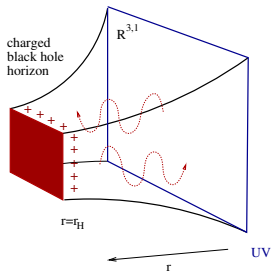


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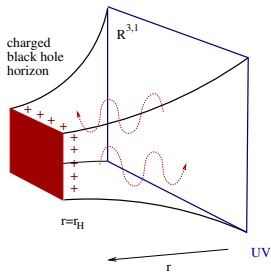
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To find FS: look for sharp
features in fermion Green functions G_R
at finite momentum and small
frequency. [S-S Lee]



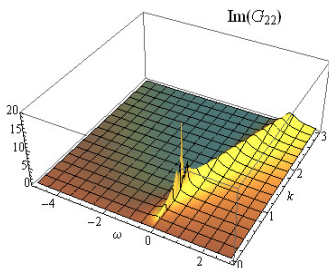
To compute G_R : solve Dirac equation in charged BH geometry.
'Bulk universality': for two-point functions, the interaction terms don't matter.
Results only depend on q, m .

*: If we ignore the back-reaction of other fields. More soon.

Fermi surface!

The system is rotation invariant, G_R depends on $k = |\vec{k}|$.

At $T = 0$, we find numerically [H. Liu-JM-D. Vegh] :



For $q = 1, m = 0$: $k_F \approx 0.92$

But it's not a Fermi liquid:

The peak has a nonlinear dispersion relation $\omega \sim k_{\perp}^z$ with

$$z = 2.09 \text{ for } q = 1, \Delta = 3/2$$

$$z = 5.32 \text{ for } q = 0.6, \Delta = 3/2.$$

and the residue vanishes.

Emergent quantum criticality

Whence these exponents? [T. Faulkner-H. Liu-JM-D. Vegh]

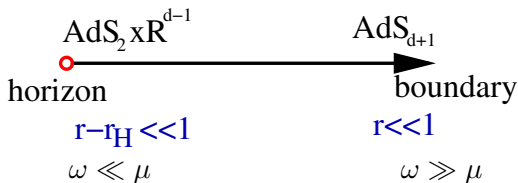
Near-horizon geometry of black hole is $AdS_2 \times \mathbb{R}^{d-1}$:

$$ds^2 \sim \frac{-dt^2 + du^2}{u^2} + d\vec{x}^2 \quad u \equiv r - r_H$$

The conformal invariance of this spacetime is **emergent**.
(We broke the microscopic conformal invariance with finite density.)

$$t \rightarrow \lambda t, x \rightarrow \lambda^{1/z} x \text{ with } z \rightarrow \infty .$$

The bulk geometry is a picture of the RG flow from the CFT_d to this NRCFT.



$AdS/CFT \implies$ low-energy physics is governed by the dual **IR CFT**.

Analytic understanding of Fermi surface behavior

$$G_R(\omega, k) = K \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + c(k)\omega^{2\nu} \left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2) \right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + c(k)\omega^{2\nu} \left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2) \right)}$$

The location of the Fermi surface ($a_+^{(0)}(k = k_F) = 0$) is determined by short-distance physics (analogous to band structure – find normalizable sol'n of $\omega = 0$ Dirac equation in full BH) but the low-frequency scaling behavior near the FS is universal (determined by near-horizon region – IR CFT \mathcal{G}).

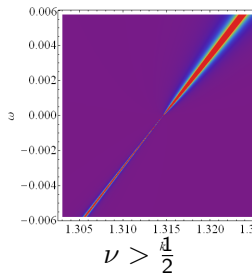
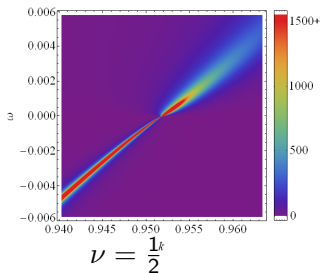
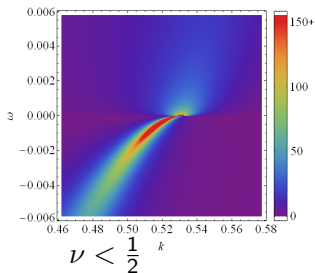
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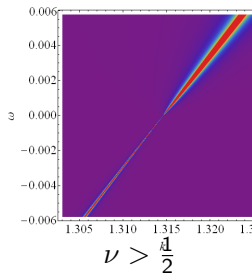
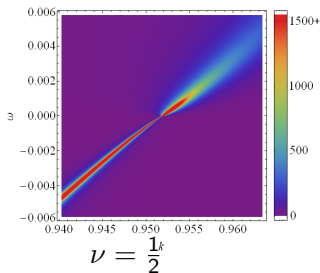
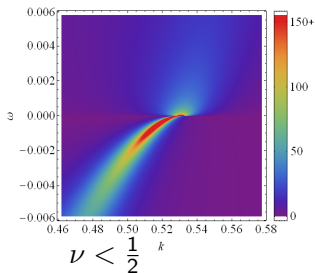
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In hindsight: “semi-holographic” interpretation [FLMV, Polchinski-Faulkner] quasiparticle decays by interacting with $z = \infty$ IR CFT degrees of freedom.

Depending on the dimension of the operator ($\nu + \frac{1}{2}$) in the IR CFT, we find Fermi liquid behavior or non-Fermi liquid behavior:



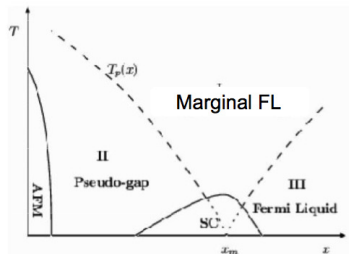
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$$\nu = \frac{1}{2} : G_R \approx \frac{h_1}{k_{\perp} + \tilde{c}_1 \omega \ln \omega + c_1 \omega}$$

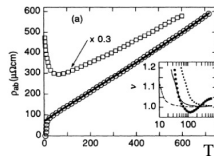
A well-named phenomenological model of high- T_c cuprates near optimal doping

[Varma et al, 1989].



Charge transport by holographic Fermi surfaces

Most prominent mystery
of strange metal phase: $\rho_{DC} \sim T$



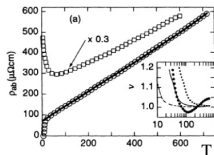
We can compute the contribution
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[Faulkner-Iqbal-Liu-JM-Vegh, 1003.1728 and to appear (???)]:

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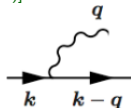
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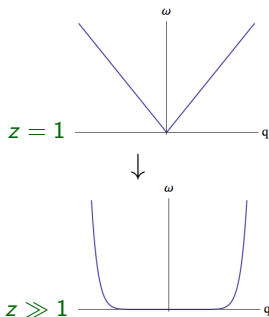
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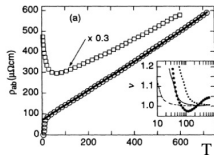


Dissipation of current is controlled by
the decay of the fermions into the AdS_2 DoFs.
⇒ single-particle lifetime controls transport.



Charge transport by holographic Fermi surfaces

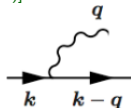
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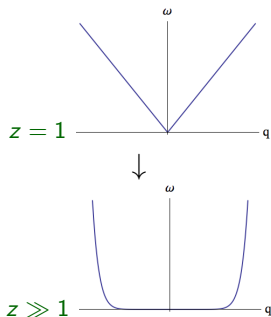
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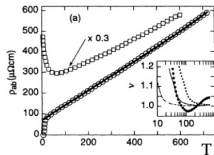
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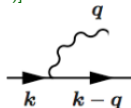
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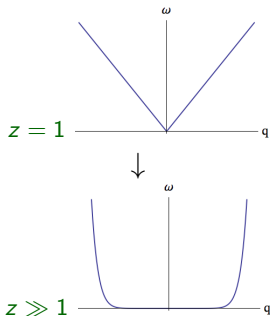
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marginal Fermi liquid: $\nu = \frac{1}{2} \implies \boxed{\rho_{FS} \sim T}$.

[Important disclaimer: this is NOT the leading contribution to σ_{DC} !]

Drawbacks of this construction

1. The Fermi surface degrees of freedom are a small part ($\mathcal{O}(N^0)$) of a large system ($\mathcal{O}(N^2)$). (More on this in a moment.)

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3. The charged black hole we are studying violates the 3rd Law of Thermodynamics (Nernst's version):
 $S(T = 0) \neq 0$ – it has a groundstate degeneracy.

This is a manifestation of the black hole information paradox:
classical black holes seem to eat quantum information.

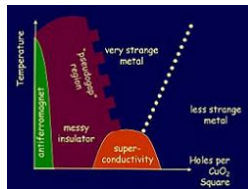
Problems 2 and 3 solve each other: degeneracy \implies instability.

The charged black hole describes an intermediate-temperature phase.

Stability of the groundstate

Often, \exists charged bosons.

At small T , the dual scalar can condense spontaneously breaking the $U(1)$ symmetry;
BH acquires hair [Gubser, Hartnoll-Herzog-Horowitz].



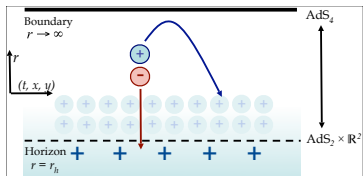
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Why: black hole *spontaneously*

emits charged particles [Starobinsky, Unruh, Hawking].

AdS is like a box: they can't escape.

Fermi: negative energy states get filled.

Bose: the created particles then cause *stimulated emission* (superradiance).

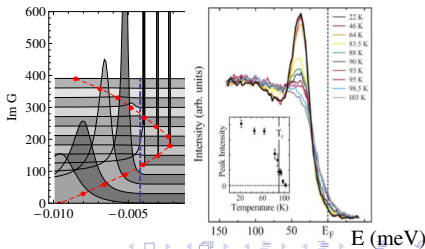
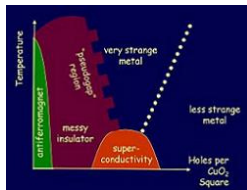
A holographic superconductor is a black hole laser.

Photoemission 'exp'ts' on holographic superconductors:

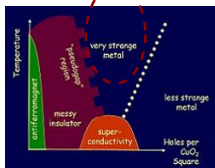
[Faulkner-Horowitz-JM-Roberts-Vegh]

In SC state: a sharp peak forms in $A(k, \omega)$.

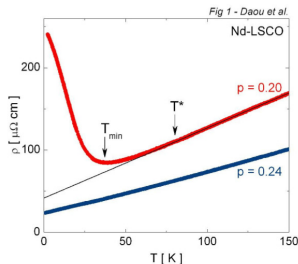
The condensate lifts the IR CFT modes into which they decay.



Superconductivity is a distraction



Look 'behind' superconducting dome by turning on magnetic field:



Strange metal persists to $T \sim 0$!

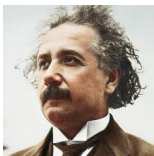
So we want to look for a theory of this intermediate-scale physics (like Fermi liquid theory).

Drawbacks of this construction, revisited

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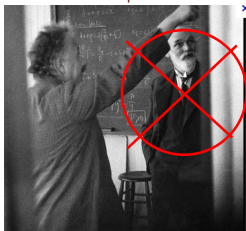
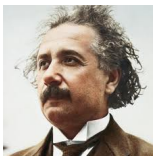
The problem we really want to solve

$$\mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} + \kappa \bar{\psi} i (\not{D} - m) \psi$$



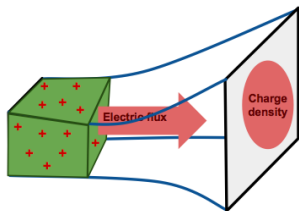
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(with AdS boundary conditions, with a chemical potential.)

Electron stars



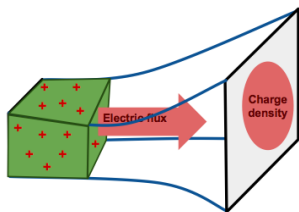
[Hartnoll and collaborators, 2010-2011]

Choose q, m to reach a regime where the bulk fermions can be treated as a (gravitating) fluid

(Oppenheimer-Volkov aka Thomas-Fermi approximation).

→ “electron star”

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[Hartnoll and collaborators, 2010-2011]

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But:

- Because of parameters (*large mass*) required for fluid approx, the dual Green's function exhibits *many* Fermi surfaces.

[Hartnoll-Hofman-Vegh, Iqbal-Liu-Mezei 2011]

- Large mass \implies lots of backreaction \implies kills IR CFT \implies stable quasiparticles at each FS.

To do better, we need to take into account the wavefunctions of the bulk fermion states: a *quantum* electron star.

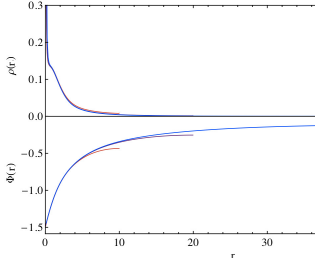
A (warmup) quantum electron star

$$\mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F^2 + \kappa \bar{\psi} i (\not{D} - m) \psi$$

A solution of QED in AdS [A. Allais, JM, S. J. Suh].

In retrospect, the dual system describes

a Fermi Surface coupled to relativistic CFT.



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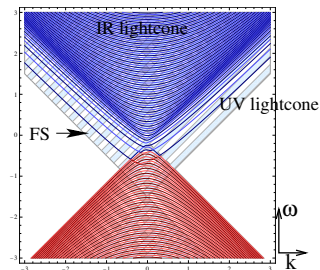
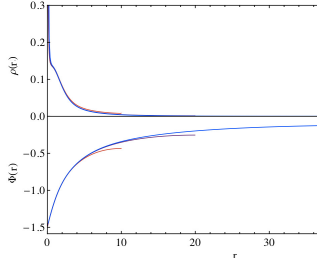
FS at $\{\omega = 0, |\vec{k}| = k_F \neq 0\}$

is outside IR lightcone $\{|\omega| \geq |\vec{k}|\}$.

Interaction is kinematically forbidden.

[Landau: minimum damping velocity in superfluid;

Gubser-Yarom; Faulkner-Horowitz-JM-Roberts-Vegh]



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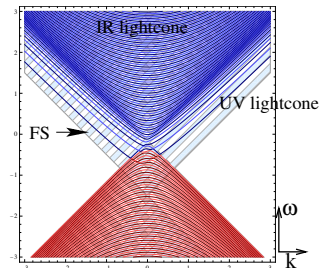
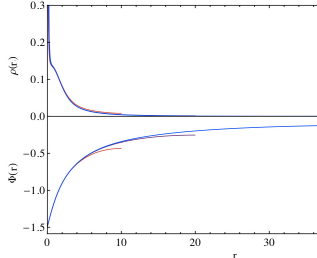
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- When we include gravitational backreaction [in progress with Andrea Allais]

(dual to effects of FS on gauge theory dynamics)

the IR geometry will be different from AdS.

Optimism: A quantum electron star is a happy medium between

AdS_2 (no fermions) and classical electron star (heavy fermions). ◀ ▶ ≡ 🔍 ↻

Concluding remarks

Two driving questions about holographic duality

1. What physics is contained in the simplest **classical gravity** version of the correspondence (**large N , strong coupling**)?

In principle, any QFT has a *quantum string theory* dual.

Not yet a practical description.

So far the gravity limit encompasses: **color confinement, relativistic gauge theory plasma, non-BCS superconductivity...**

here: it includes 'strange metal'

(the most mysterious phase of high- T_c superconductors).

2. Which quantum systems admit such a description?

'AdS/CFT' is a bad name. Holographic duality is much more general. e.g.

▶ relevant deformations of CFT

▶ here: *microscopically* non-relativistic systems.

Lessons for gravity from many-body physics

- Violation of no-hair expectations for AdS black holes.
- Information is not lost in BH evaporation.

- How does space emerge from QM?

Entanglement RG [G. Vidal]:

a real space RG which keeps track of entanglement

builds an extra dimension

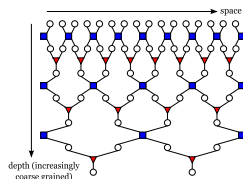
$$ds^2 = dS^2 \quad [\text{Swingle 0905.1317, Raamsdonk 0907.2939}]$$

- Basic facts about QM forbid traversable wormholes in AdS

(information can't propagate between decoupled theories) [Swingle, to appear]

even at finite N , small $\lambda \implies \exists$ "quantum horizons"

- Weak evidence for weak gravity conjecture [Arkani et al] from studies of holographic superconductors [Denef-Hartnoll, 0901]



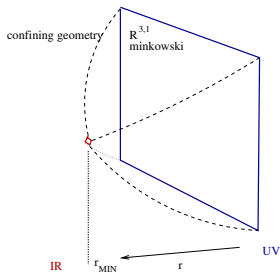
Are there new strongly-coupled states of matter?

Theoretical prediction of liquid phase? [Weisskopf]

of color confinement (70s), of superconductivity (1911 or 1956),
of FQHE (1982), of strange metal (80s)...

An old strongly-coupled state of matter from holography

If we didn't happen to be made from the excitations of a confining gauge theory (QCD),
we would have predicted color confinement using AdS/CFT
via this cartoon:

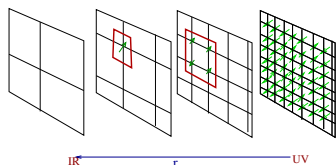


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Our ability to imagine possibilities for states of matter
so far has been limited by our weak coupling descriptions
and by our ability to build things.



For some model systems, the RG
picture satisfies a nice equation
(Einstein's equation!).

Perhaps this will help us to answer...

What other states of interacting matter may still be hidden?

The end.

Thanks to my collaborators:

Allan Adams

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Koushik Balasubramanian

Maissam Barkeshli

Tom Faulkner

Ben Freivogel

Dave Guarrera

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Thanks for listening.