Disentangling quantum matter

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based on work (to appear) with
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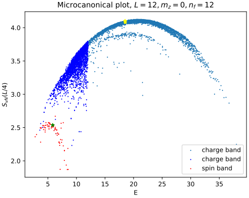


2019-09

Genetics vs biochemistry

[The Salvation of Doug, Bill Sullivan]

On a hill overlooking an automobile factory, lived Doug, a retired biochemist, and a retired geneticist (nobody knew his name). ... Having spent a life in pursuit of higher learning, both were wholly unfamiliar with how cars worked. They decided that they would like to learn about the functioning of cars and having different scientific backgrounds they each took a very different approach. Doug immediately obtained 100 cars (he is a rich man, typical of most biochemists) and ground them up. He found that cars consist of the following: 10% glass, 25% plastic, 60% steel, and 5% other materials that he could not easily identify.

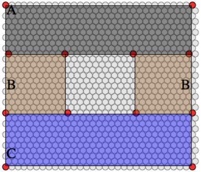
The geneticist, not being inclined toward hard work (as is true for most geneticists) pursued a less strenuous (and less expensive) approach. One day, before his morning coffee, he hiked down the hill, selected a worker at random, and tied his hands. After coffee, while the biochemist zipped up his blue jump suit, adjusted his welders goggles, and lit his blow torch to begin another day of grinding, the geneticist peered around the house, made himself another pot of coffee, and browsed through the latest issue of Genetics.

That afternoon, while the automobiles were rolling off the assembly line,

Analogs for quantum matter

Biochemistry approach to many-body entanglement:

trace everything out $\rho_A \equiv \mathrm{tr}_{\bar{A}}|\psi\rangle\langle\psi|$
compute $S(A) = -\mathrm{tr}\rho_A \log \rho_A, I(A:B), I(A:B|C)...$
e.g. $\rightarrow \gamma$, topological entanglement entropy



Idea: Take a genetics approach to many-body entanglement.

Measurements destroy entanglement.

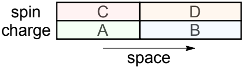
Use post-measurement properties of the many-body wavefunction as a tool for characterizing states of matter.

Focus so far on multi-component systems:
Hubbard model = spin + charge
Kondo lattice model = conduction electrons + local moments

QDL protocol

[Grover-Fisher 2014, Marvian 2013]

$\mathcal{H} = A \otimes B \otimes C \otimes D$ for example:



Input: a state ρ_{ABC} (trace out D) and a choice of operator X_C on C.

1. Measure X_C and obtain outcome c with probability $p_c = \mathrm{tr}_{AB}\langle c|\rho_{ABC}|c\rangle$. (Assume $|c\rangle$ is a basis for C.)
2. In the resulting state, $\rho^c_{AB} = \langle c|\rho_{ABC}|c\rangle / p_c$, make measurements.
   e.g. find the entanglement entropy of A, $S(\rho^c_A)$.
3. Average over the distribution p_c to obtain the QDL diagnostic

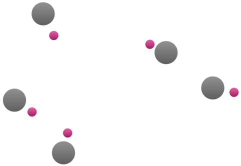
$$\sum_c p_c S(\rho^c_A) \equiv \mathrm{QDL}.$$

Also useful: $S(\rho^c_A) \mapsto I_{\rho^c_{AB}}(A:B)$. $\mathrm{IQDL} \equiv \sum_c p_c I_{\rho^c_{AB}}(A:B)$
($I(A:B) \equiv S_A + S_B - S_{AB}$ quantifies correlations between A and B.)

Quantum disentangled liquid

[Grover-Fisher 2014]

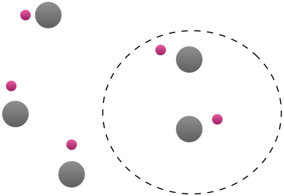
Failure of ergodicity in systems of heavy particles $(\vec{R}_i)$+ light particles $(\vec{r}_i)$



Heavy particles are ergodic,
produce random potential localizing
light particles.

$$\Psi_{\rm QDL}(\vec{R}_i, \vec{r}_i) \sim \det e^{\mathbf{i}\vec{k}_i \cdot \vec{R}_i} \det \Phi_j^R(\vec{r}_j)$$

MBL is the $M_{\rm heavy} \to \infty$ limit.



The whole state is volume-law, $S(A) \sim L^d$.
But if we measure the positions of the heavy particles:
each $\det \Phi_j^R(\vec{r}_j)$ is an area-law wavefunction:
$S(\rho_A^R) \sim L^{d-1}$.

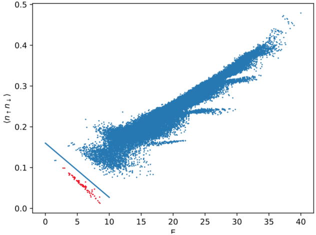
$\Longrightarrow \qquad {\rm QDL} = \sum_{\vec{R}} p(\vec{R}) S(\rho_A^R) \sim L^{d-1}$

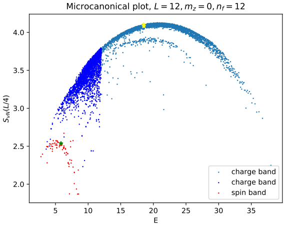
QDL in the Hubbard model

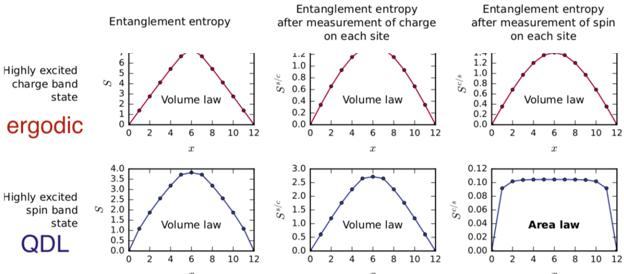
[Veness-Essler-Fisher, Garrison-Mishmash-Fisher 2017]

$$\mathbf{H} = -t\sum_{i\sigma} c^\dagger_{i\sigma} c_{i+1\sigma} + U\sum_i n_{i\uparrow}n_{i\downarrow} + V\sum_i n_i n_{i+1}$$

where $n_{i\sigma} \equiv c^\dagger_{i\sigma}c_{i\sigma}$ and $n_i \equiv \sum_\sigma n_{i\sigma}$. ‘Spin band’ = QDL states



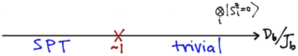




Apply QDL protocol to groundstate questions

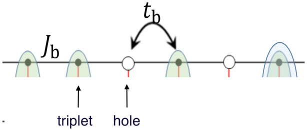
Apply to groundstate questions: gapless SPT

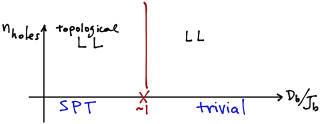
Spin-1 chain
$H = +J_b \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + D_b \sum (S_i^z)^2$ has Haldane phase. Spin-$\frac{1}{2}$ edge states, doubly-degenerate ent. spectrum [Pollmann et al 2010].



Dope it! $\mathbf{H} = -t_b \sum_{i\sigma} b^\dagger_{i\sigma} b_{i+i\sigma} + h.c. + J_b \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + D_b \sum (S_i^z)^2$.
Spin-charge separation: charges form LL.
Topological Luttinger Liquid should have spin-half edge states.

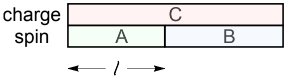
[Jiang-Li-Seidel-D-H Lee 2017]





How to tell?

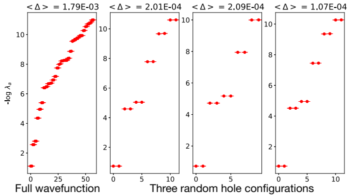
Apply to groundstate questions: gapless SPT

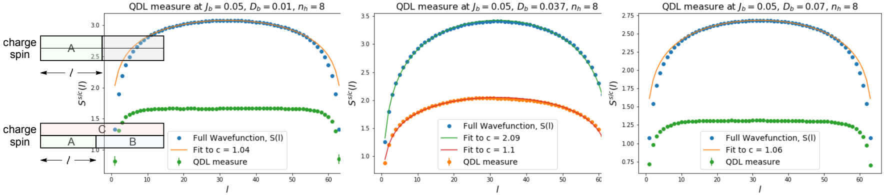


Freeze the charge dofs by measurement.

In topological LL phase: $\rightarrow$

The projected wavefunction has a $c = 1$ transition: $\downarrow$

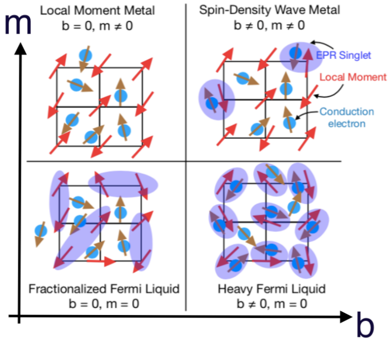




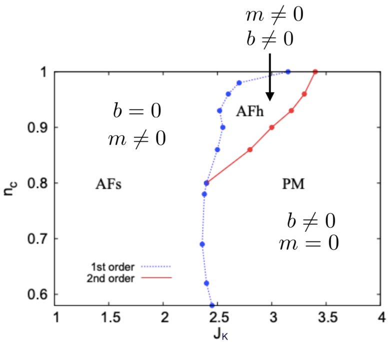
SPT critical trivial

Disentangling heavy fermions

$$\mathbf{H}_K = \sum_k \epsilon(k) c_k^\dagger c_k + J_K \sum_i \vec{S}_i \cdot \vec{s}_i + J_H \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad \vec{s}_i \equiv c_i^\dagger \vec{\sigma} c_i$$



from Tarun Grover



[from 1302.4662]

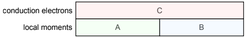
$\vec{S}_i = f_i^\dagger \vec{\sigma} f_i \to$ parton gauge field.

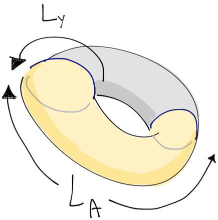
Phase diagram in terms of "$b \sim \langle c^\dagger f \rangle$" gauge-non-invariant Higgs field.

What is a sharp distinction for the Higgs (large Fermi surface) phase?

(Messy bandstructure makes k_F hard to interpret.)

Disentangling heavy fermions





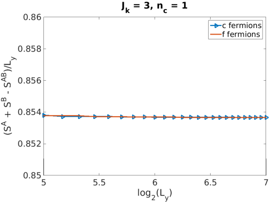
System is $2L_y \times L_y$.

If we measure all the conduction electron positions in the HFL phase, we are left with a Fermi surface groundstate: $S^{\text{local moments}}_{\text{fixed } c}(l) \sim l^{d-1} \log l$.

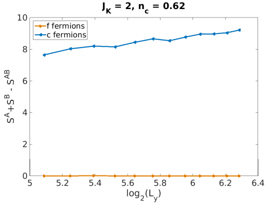
If we don't: $S^{\text{local moments}}_{\text{trace out } c}(L_A) \sim l^d + l^{d-1} \log l$
$= a_{\text{volume}} L_A L_y + a_{\text{Fermi surface}} L_y \log \min(L_A, L_{\bar{A}}) + \ldots$

$I(A:B|C) = S_{AC} + S_{BC} - S_{ABC} - S_C =$
$S_A + S_B - S_{AB}$ cancels volume-law term.

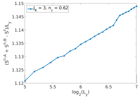
At half-filling, no Fermi surface:

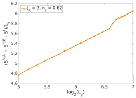


Small-Fermi-surface (AFM) phase ($b = 0$):



Large-Fermi-surface (HFL) phase ($b \neq 0$):





(In the plots $L_A = L_y$.)

Relations between genetics and biochemistry?

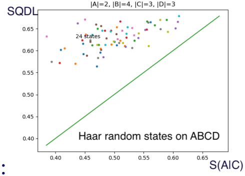
Bounds in terms of information-theoretic quantities

- Lower bound in terms of conditional entropy, $S(A|C) = S(AC) - S(C)$:

$$S_\rho(A|C) = -D(\rho_{AC}||\mathbb{1}_A \otimes \rho_C) \overset{\text{MRE}}{\leq} S_{\mathcal{E}(\rho)}(A|C) = \sum_c p_c S_{\rho^c}(A) = \text{QDL}.$$

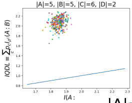
MRE $\equiv$ monotonicity of the relative entropy.

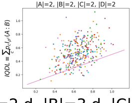
$\mathcal{E}(\rho) \equiv \sum_c p_c |c\rangle\langle c| \otimes \rho^c_{AB}$ is the result of measuring X_C and not looking at the answer.

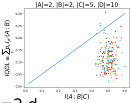


- BHS bound in terms of conditional mutual information:

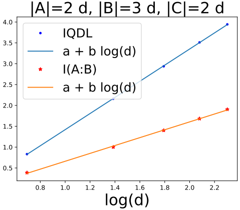
$$I_\rho(A:B|C) - X(AB,C) \leq \text{IQDL} \leq I_\rho(A:B|C) + X(A,C)$$







$X(A,C) \equiv I_\rho(A:C) - I_{\mathcal{E}(\rho)}(A:C) = I_\rho(A:C) - \chi(\rho^c_A, p_c))$ is the 'quantum discord' [Henderson-Vedral, Ollivier-Zurek 2001], a measure of how quantum is ρ_{AC}.



Genetics vs. Biochemistry

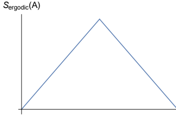
[The Demise of Bill, by Douglas R. Kellogg]

On a hill overlooking an automobile factory, lived Bill, a retired geneticist, and a retired biochemist (nobody knew his name)...

Having spent a life in pursuit of higher learning, both were wholly unfamiliar with how cars worked, and they decided that they would like to learn about the functioning of cars. Having different scientific backgrounds they each took a very different approach. Bill, not being inclined towards hard work (like most geneticists), immediately came up with a scheme that he thought would lead him to an understanding of cars... When he looked in the garage he found that the biochemist had gotten one of the cars from the factory and was already covered with grease and oil as he was doing something under the hood. When Bill asked the biochemist what he was doing, he replied, “I’m taking the car apart to see how it works.”

...As [the geneticist] sat down to his coffee, he heard an explosion in the garage. He ran out to see what had happened, and he found the biochemist picking himself up off the ground, his face black and most of his hair burned away. When Bill asked in amazement what had happened, the biochemist simply replied, “I have found that the liquid in the tank of the car is fairly explosive.” ...

Conditional entropy distinguishes QDL from ergodic



An ergodic state satisfies Page's rule: $S(A) \sim \min(A, \bar{A})$.

When $A + C$ is less than half of $\mathcal{H}$,

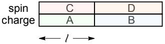
$S(A|C) = S(AC) - S(C) \sim A + C - C = A$ is volume law.

Model QDL wavefunction [Grover-Fisher 2014]:

$$\Psi_{\mathrm{QDL}}(N, n) = \psi(N) \prod_{j=1}^{L} \frac{1}{\sqrt{2}} \left(\delta_{n_j,0} + e^{\mathbf{i}\pi N_j} \delta_{n_j,1} \right)$$

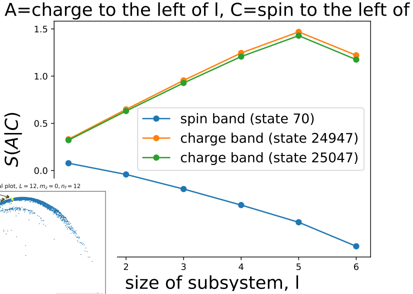
$n_j = 0, 1$ light, $N_j = 0, 1$ heavy. $\psi(N)$ is an ergodic wavefunction,

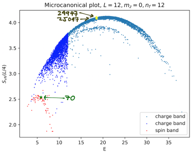
e.g. $\psi(N) = \mathrm{sgn}(\{N_j\}) 2^{-L/2}$.



When $|A| = |C| = l$, we expect $S(A|C) \propto -l2^{-L}$, a negative volume law with a coefficient that decreases rapidly with system size.

In the Hubbard model ($L = 12, n_f = 12, m_z = 0, V = 3/4, U = 4$):





(CMI also works!)

Negativity distinguishes QDL from ergodic

(Logarithmic) negativity is a useful measure of mixed-state entanglement

$E_N(\rho) \equiv -\log|\rho^{T_A}| = -\log\left(\sum_a |\lambda_a|\right)$, $\lambda_a =$ evals of $\rho^{T_A}_{ab,a'b'} \equiv \rho_{a'b,ab'}$.

In a QDL state, $\rho_{AB} = \sum_c p_c \rho^c_{AB}$ where each ρ^c_{AB} is area law due to QDL-ness.

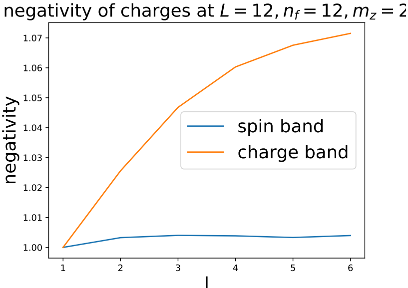
heavy	C	D
light	A	B

In thermal states $e^{-\beta H}$ for local H, negativity is also area law [Sherman-Devakul-Hastings-Singh].

But: even in an ergodic system, we have to trace out more than half the stuff to approximate a thermal state: the negativity of $\mathrm{tr}_{\text{less than half}}|\psi\rangle\langle\psi|$ is volume law [Bhosale-Tomsovic-Lakshminarayan 2012, Lu-Grover, to appear].

In the Hubbard model:

We can make the bath (heavy particles = spins) less than half the system by fixing $m_z = 2$.



Outlook

- Spatial partitions of systems with one component.

  e.g.: Marvian 2013, 'SPT entanglement'

- Systems with more than two components.

  'field-space entanglement' [Taylor, Mozaffar et al, Mollabashi et al, 2014]

- Experimental implementation?

  The mutual information between two subsystems A and B bounds their correlations $I_\rho(A:B) \geq \frac{1}{2}\frac{\langle \mathcal{O}_A \mathcal{O}_B \rangle_c^2}{\| \mathcal{O}_A \|^2 \| \mathcal{O}_B \|^2}$ [Wolf et al 2008]
  $\langle \mathcal{O}_A \mathcal{O}_B \rangle_c \equiv \mathrm{tr}\rho \mathcal{O}_A \mathcal{O}_B - \mathrm{tr}\rho \mathcal{O}_A \mathrm{tr}\rho \mathcal{O}_B$.

- Application to FL* [Senthil-Sachdev-Vojta 2003]

  Our MFT missed the most interesting phase!

  In the natural variational wavefunction, measuring the conduction electrons leaves behind a gapped topological state of the local moments. A natural target for variational MC.

- Generalized measurements?

- General understanding of gapless topological states?

The end.

Thanks for listening.