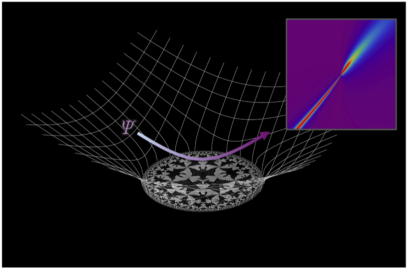
Fermions coupled to gauge fields

with cond-mat motivations

John McGreevy, UCSD



slightly more specific title:

Construction of quantum electron stars

with cond-mat motivations

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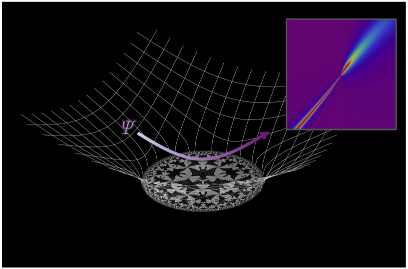


Outline

1. Brief Introduction: ‘post-particle physics of metal’
2. Limit 1: Holographic fermions with too little back-reaction
3. Limit 2: Holographic fermions with too much back-reaction
4. Limit 3: Quantum electron stars in *AdS*
5. How to make a covariant stress tensor from lattice fermions

   [work in progress with Andrea Allais]

Based on:

Andrea Allais, JM, S. Josephine Suh, 1202.5308;
Andrea Allais, JM, in progress.

A goal for holography

Can we formulate a tractable effective description of the low-energy physics of a system
with a Fermi surface*,
but without long-lived quasiparticles?

* Multiple possible definitions:

1. In terms of transport: *e.g.* $\rho(T) \sim T^{\alpha<2}$.

2. In terms of scaling of entanglement entropy of regions: $S(L) \sim L^{d-1} \ln k_F L$.

3. In terms of single-particle response:
Fermi surface $\equiv \{k \mid G^{-1}(k, \omega) = 0 \text{ at } \omega = 0\}$
Here $G = \langle c^\dagger c \rangle$ is a correlator of a gauge-invariant fermion operator, like an electron, effectively.
The kind of function I have in mind is $G \sim \frac{1}{c\omega^{2\nu} + |k| - k_F}$.

This will be worthwhile even if the toy model has exotic short-distance physics.

Benefit of holography: 0th-order approx far from weakly-coupled particles.

Lightning Review of Holographic Duality

Holographic duality (AdS/CFT)

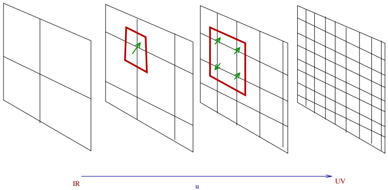
[Maldacena; Witten; Gubser-Klebanov-Polyakov]

gravity in AdS_{d+1} = d-dimensional Conformal Field Theory

(many generalizations, CFT is best-understood.)

$$AdS: \quad ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + R^2\frac{dr^2}{r^2}$$

isometries of AdS_{d+1} $\leftrightsquigarrow$ conformal symmetry



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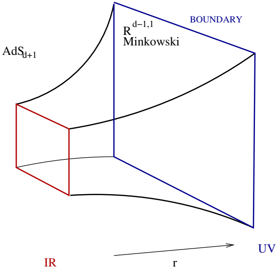


The extra ('radial') dimension is the resolution scale.

(The bulk picture is a hologram.)

when is it useful?

$$Z_{QFT}[\text{sources}] \quad = Z_{\text{quantum gravity}}[\text{boundary conditions at } r \to \infty]$$
$$\approx e^{-N^2 S_{\text{bulk}}[\text{boundary conditions at } r\to\infty]}|_{\text{extremum of } S_{\text{bulk}}}$$

classical gravity (sharp saddle) $\leftrightsquigarrow$ many degrees of freedom per point, $N^2 \gg 1$

fields in AdS_{d+1} $\leftrightsquigarrow$ operators in CFT
mass $\leftrightsquigarrow$ scaling dimension

boundary conditions on bulk fields $\leftrightsquigarrow$ couplings in field theory

e.g.: boundary value of bulk metric $\lim_{r\to\infty} g_{\mu\nu}$
$=$ source for stress-energy tensor $T^{\mu\nu}$

different couplings in bulk action $\leftrightsquigarrow$ different field theories

large AdS radius R ($\Lambda = -\frac{6}{R^2} \ll M_p^2$) $\leftrightsquigarrow$ strong coupling of QFT

Holographic Fermi surfaces

Minimal ingredients for a holographic Fermi surface

Consider any relativistic CFT with a gravity dual $\rightarrow g_{\mu\nu}$

a conserved $U(1)$ symmetry proxy for fermion number $\rightarrow A_\mu$

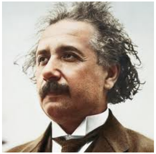
and a charged fermion proxy for bare electrons $\rightarrow \psi$.

$\exists$ many examples. Any $d > 1 + 1$, focus on $d = 2 + 1$.

The problem we really want to solve

Wilson tells us to use the following action in the bulk:

$$\mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} + \kappa \bar{\psi} i \left(\not{D} - m \right) \psi$$



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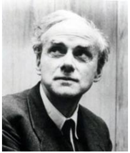
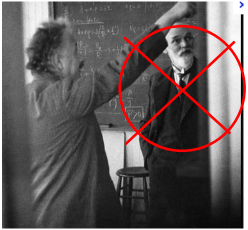




(with AdS boundary conditions,

and a chemical potential: $A_t \equiv \Phi \to \mu$ at the boundary.)

Limit 1:

Completely ignore bulk matter fields in constructing the geometry

$$\mathcal{L}_{d+1} = \mathcal{R} + \frac{d(d-1)}{R^2} - \frac{2\kappa^2}{g_F^2} F^2 + \bar{\psi} i \left(\not{D} - m \right) \psi + \ldots$$

Then the solution of the bulk EoM with the right boundary conditions is the extremal charged black hole in AdS ('Reissner-Nördstrom'):

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{R^2} \left(dx^2 + dy^2 \right),$$

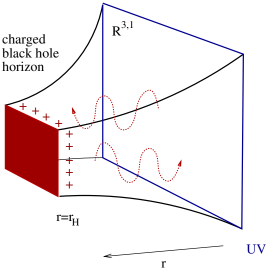
$$f(r) = \frac{r^2}{R^2} \left(1 + \frac{Q^2}{r^3} - \frac{M}{r^3} \right), \quad \Phi = \mu \left(1 - \left(\frac{r_H}{r} \right) \right).$$

'Extremal' means $T = 0$. $f \sim (r - r_H)^2$ near the horizon.

Extremal black hole in AdS

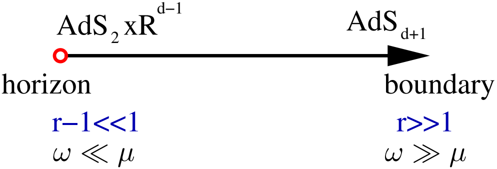
Near-horizon geometry is $AdS_2 \times \mathbb{R}^{d-1}$.

$$ds^2 \sim \frac{-dt^2 + du^2}{u^2} + d\vec{x}^2 \qquad u \equiv \frac{1}{r - r_H}$$



The conformal invariance of this metric is **emergent**.

$t \to \lambda t, x \to \lambda^{1/z} x$ with $z \to \infty$.



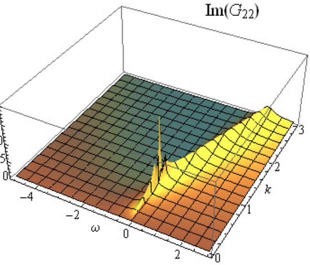
AdS/CFT $\implies$ the low-energy physics governed by dual **IR CFT**.

Fermi surfaces

To find FS: look for sharp features in fermion Green functions G_R at finite momentum and small frequency. [S-S Lee]

To compute G_R: solve Dirac equation in charged BH geometry.
'Bulk universality': results only depend on q, m.

$$G_R(\omega, k) \sim \frac{1}{\mathcal{G}(k, \omega) + k_\perp}$$



The location of the Fermi surface is determined by short-distance physics (analogous to band structure – $a, b \in \mathbb{R}$ from normalizable sol'n of $\omega = 0$ Dirac equation in full BH)
but the low-frequency scaling behavior near the FS is universal (determined by near-horizon region – IR CFT correlator $\mathcal{G} = c(k)\omega^{2\nu}$).

In hindsight: "semi-holographic" interpretation [FLMV, Polchinski-Faulkner]
quasiparticle decays by interacting with $z = \infty$ IR CFT d.o.f.s dual to $AdS_2 \times \mathbb{R}^2$ region.

Drawbacks of this construction

1. The Fermi surface degrees of freedom are a small part ($o(N^0)$) of a large system ($o(N^2)$). (More on this in a moment.)

2. *Too much* universality! If this charged black hole is inevitable, how do we see the myriad possible dual states of matter (*e.g.* superconductivity...)?

3. The charged black hole violates the 3rd Law of Thermodynamics (Nernst's version): $S(T = 0) \neq 0$ – it has a groundstate degeneracy.

   This is a manifestation of the black hole information paradox: classical black holes seem to eat quantum information.

Problems 2 and 3 solve each other: degeneracy $\implies$ instability.

The charged black hole describes an intermediate-temperature phase.

Idea: make the bulk fermions more important (solves problem 1).

They will back-react on the geometry (solves problems 2 and 3).

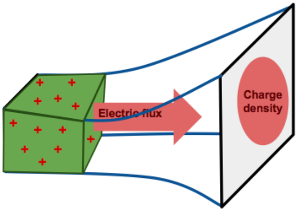
[Hartnoll-Polchinski-Silverstein-Tong 09]

Problem: it's hard.

Limit 2:

Very heavy fermions in the bulk

Electron stars



[Hartnoll and collaborators, de Boer-Papadodimas-Verlinde]

Choose q, m to reach a regime where the bulk fermions can be treated as a (gravitating) fluid

(Oppenheimer-Volkov aka Thomas-Fermi approximation).

$\longrightarrow$ "electron star"

But:

• Because of parameters (large mass) required for fluid approx, the dual Green's function exhibits *many* Fermi surfaces.

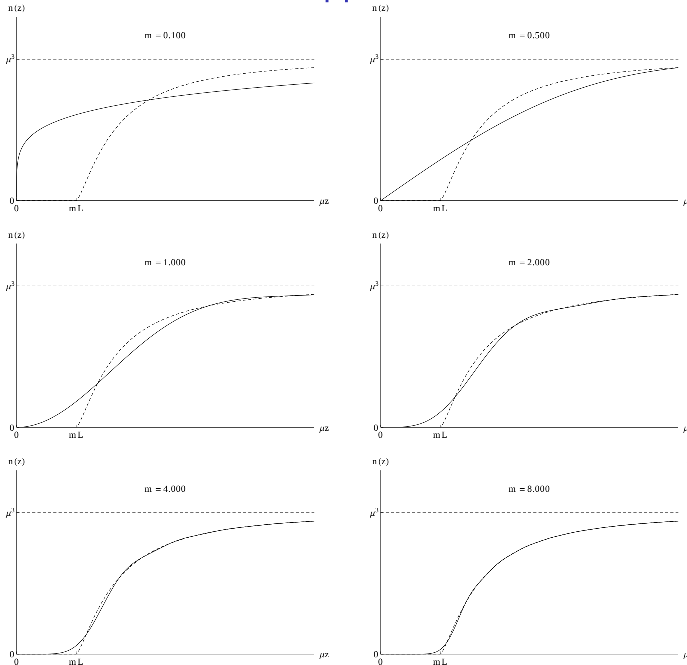
[Hartnoll-Hofman-Vegh, Iqbal-Liu-Mezei 2011]

• Large mass $\implies$ lots of backreaction $\implies$ kills IR CFT

$\implies$ stable quasiparticles at each FS.

To do better, we need to take into account the wavefunctions of the bulk fermion states: a *quantum* electron star.

The Thomas-Fermi approximation matters



Exact charge density in *AdS* with $\Phi(z) = \mu$.

Does change IR physics ($z \to \infty$).
$\mu_{\text{local}}(z) = \frac{\Phi}{\sqrt{g_{tt}}} = \mu z$.

A (warmup) quantum electron star

Limit 3:

Find back-reaction of fermions on the gauge field, but ignore gravitational back-reaction of both fermions *and* gauge fields.

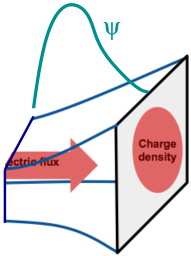
$$\mathcal{L}_{d+1} = \frac{\mathcal{R} + \Lambda}{G_N} - \frac{1}{g^2} F^2 + \kappa \bar{\psi} i \left(\not{D} - m \right) \psi$$

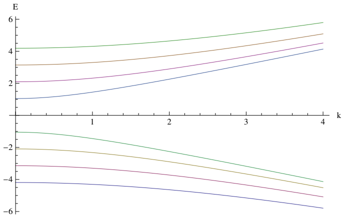
Probe limit: $G_N \to 0$ [like HHH 0803]

QFT Interpretation: most CFT dofs are neutral. $(\underbrace{c \sim \frac{L^2}{G_N}}_{\propto \langle TT \rangle} \gg \frac{1}{g^2} \propto \langle jj \rangle)$

A solution of QED in AdS [A. Allais, JM, S. J. Suh].

Towards a quantum electron star





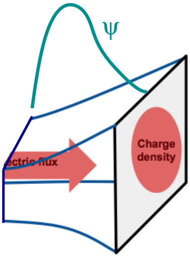
[Sachdev, 2011]: A holographic model of a Fermi liquid.

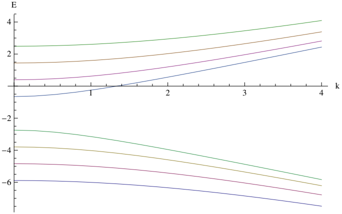
Like AdS/QCD: a toy model of the groundstate of a confining gauge theory from a hard cutoff in AdS.

Add chemical potential.
Compute spectrum of Dirac field, solve for backreaction on A_μ. Repeat as necessary. (Hartree-Fock)

The system in the bulk *is* a Fermi liquid (in a box determined by the dual gauge dynamics).

Towards a quantum electron star





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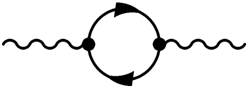
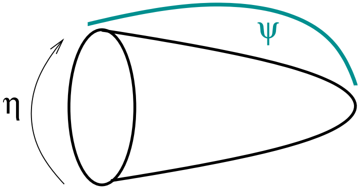
Towards a fermion-driven deconfinement transition

Lots of low-E charged dofs screen gauge interactions.

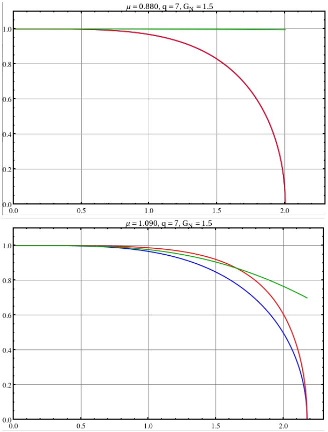




Effect of fermions on the gauge dynamics = gravitational backreaction.

A real holographic model of confinement: AdS soliton

first attempt: $\rightarrow$



What's the endpoint of this transition?

A quantum electron star in *AdS*

Zeroth-order problem: what can the state of the bulk fermions be if the geometry has a horizon?

Probe limit ($G_N \to 0$):

Fix the geometry to be *AdS* with an IR cutoff.

$$\psi =: (-g g^{zz})^{-\frac{1}{4}} e^{-i\omega t + i k_i x^i} \chi$$

Normalizable BCs at $z = 0$,

hard-wall BC at $z = z_m$

$$\not{D}_\Phi \psi = 0$$

$$\begin{pmatrix} \Phi(z) + k & \frac{\partial}{\partial z} - \frac{m}{z} \\ -\frac{\partial}{\partial z} - \frac{m}{z} & \Phi(z) - k \end{pmatrix} \chi_n = \omega_n \chi_n$$

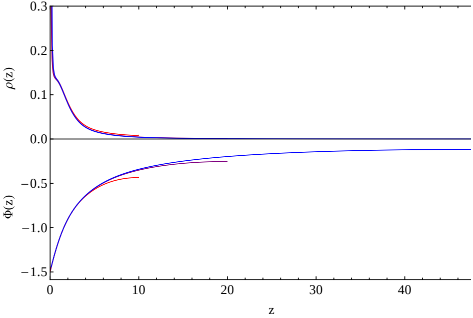
$$\Phi'' = -q^2 \rho$$

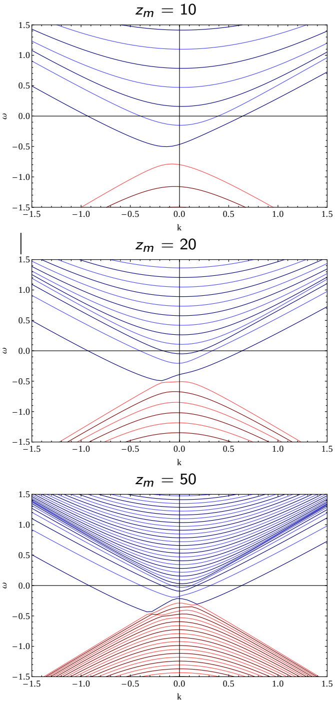
$$\Phi''(z) = -q^2 \left(\rho(z) - \rho(z)|_{\Phi=0} \right),$$

$$\rho(z) \equiv \sum_{n, \omega_n < 0} \psi_n^2(z)$$

A quantum electron star in *AdS*

The limit $z_m \to \infty$ exists! :





Electrostatic and Pauli repulsion supports the fermions against falling into the AdS gravitational well.

The padded room

Compute charge density:

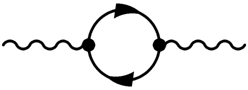
$$\langle n(z)\rangle = \langle \psi^\dagger(z)\psi(z)\rangle = \sum_k n_k(z) \sim \int^\Lambda d^2k \frac{1}{k^2}\Phi''(z) + \text{finite}$$

Cutoffs everywhere: UV cutoff on AdS radial coordinate, bulk UV cutoff (lattice), UV cutoff on k integral, IR cutoff on AdS radial coordinate: z_m.

Charge renormalization.

Define charge susceptibility by linear response:

$$\chi \equiv \sum_k \chi(k), \;\; \chi(k) = \frac{\Delta\rho_k(z_\star)}{\Phi''(z_\star)}$$



$$q_R^2 = q_0^2 \frac{1}{1 - q_0^2\chi}$$

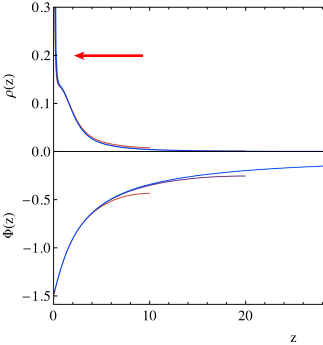
Two physics checks:

1) Surface charge. Our bulk charges are not mobile in the AdS radial direction.

(Like metal of finite extent along one axis.)

An electric field applied to an insulator polarizes it.

This results in a surface charge $\sigma_b = \hat{n} \cdot \vec{P}$.



2) Chiral anomaly.

Each k mode is a 1+1 fermion field

$S_k = \int drdt\ i\bar{\psi}_k \left(\not{D} + m + i\gamma^5 k\right) \psi_k$

$\overset{?}{\Longrightarrow}\ \partial_r n_k \to 0$ when $m, k \to 0$.

Not so in numerics:

$$\partial_\mu j_5^\mu = \frac{1}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} = -\frac{1}{\pi}\Phi' \qquad \checkmark$$

Semi-holographic interpretation

In retrospect, the dual system can be regarded as
a Fermi Surface coupled to relativistic CFT (with gravity dual)

$\Phi(z)$: how much of the chemical potential is seen by the dofs of wavelength $\sim z$.

Convergence of EOM requires $\Phi(\infty) = 0$, complete screening in far IR.

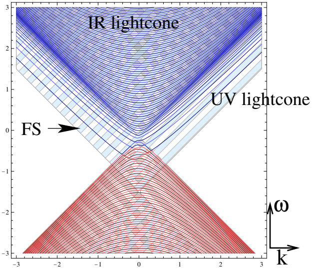
$\Phi(\infty) = 0$ means FS survives this coupling to CFT:
FS at $\{\omega = 0, |\vec{k}| = k_F \neq 0\}$ is outside IR lightcone $\{|\omega| \geq |\vec{k}|\}$.

Interaction is kinematically forbidden.
[Landau: minimum damping velocity in SF;

Gubser-Yarom; Faulkner et al 0911]

In probe limit, quasiparticles survive.

With "Landau damping," IR speed of light smaller, maybe not.



Electron stars minus zero/no limit

She knows there's no success like failure
And that failure's no success at all.

Towards gravitating quantum electron stars

When we include gravitational backreaction
(dual to effects of FS on gauge theory dynamics)
the IR geometry can be different from the UV AdS.

Optimism: happy medium between

AdS_2 (no fermions)
and
classical electron star (heavy fermions).

Obstacles

Consider a massless Dirac fermion in 1+1 dimensions, $\Phi = 0$.

$$\text{Fixed metric:} \quad ds^2 = -f_t(z)dt^2 + f_z(z)dz^2$$

For convenience take $z \simeq z + 2\pi$. WLOG $f_z = 1$ gauge.
Conformal anomaly:

$$T^\mu_\mu = \frac{1}{4\pi}\mathcal{R}(z) = \frac{1}{4\pi}\left(\frac{1}{2}\left(\frac{f_t'^2}{f_t}\right)^2 - \frac{f_t''}{f_t}\right)$$

$$H = \begin{pmatrix} 0 & -\left(\frac{f_t}{f_z}\right)^{1/4}\partial_z\left(\frac{f_t}{f_z}\right)^{1/4} \\ \left(\frac{f_t}{f_z}\right)^{1/4}\partial_z\left(\frac{f_t}{f_z}\right)^{1/4} & 0 \end{pmatrix}, \quad H\psi_a = \omega_a\psi_a.$$

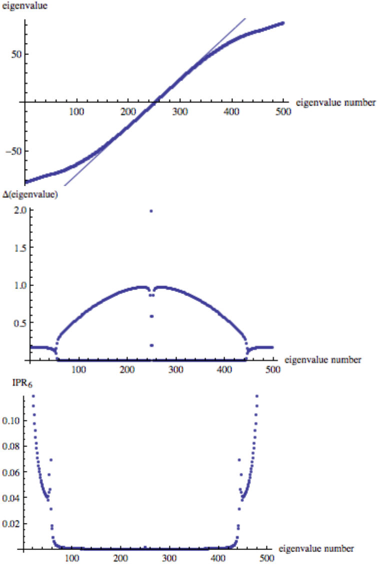
Latticize, add up:

$$T^\mu_\mu = \sum_{a\,\in\,\text{spectrum of H}} \theta(-\omega_a)\psi_a^\dagger(\ldots)\psi_a = \frac{1}{4\pi}\left(\frac{3}{4}\left(\frac{f_t'^2}{f_t}\right)^2 - \frac{f_t''}{f_t}\right)$$

Not a scalar!

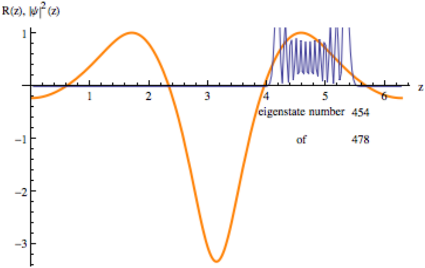
Obstacles, cont'd

Why not covariant?



e.g.: $f_t = 1 + .3\cos z + .2\cos 2z$,

$n = 249$ sites.



$$IPR_k \equiv \int dz\ |\psi|^{2k}$$

Solution, pt 1: Even more regulators!

- An additional (bulk UV) regulator is required:

$$\rho_{\text{bare}}(s) \equiv \sum_a \theta(-\omega_a)\psi_a^\dagger \psi_a \; e^{-s|\omega_a|}$$

Cuts off (exponentially) the contribution of the localized modes.

Must have: $1/s \ll 1/a$ to keep the lattice artifacts out.

(This is point-splitting in t. Not covariant.
Hamiltonian Pauli-Villars would also work in principle, and *is* covariant. But it only kills the UV bits by a power-law suppression: $1/p^2 - 1/(p^2 + M^2)$.

Not fast enough.)

- Hard wall IR cutoff at $z = z_m$ also obstructs covariant T_μ^ν.

Better (bdy) IR regulator: $dx^2 + dy^2 \quad \to \quad d\theta^2 + \sin^2\theta d\varphi^2$

Bulk geometry can end smoothly in IR when radius$(S^2) \to 0$

(with obvious boundary conditions on the spinors).

[Used by Gentle, Rangamani, Withers for holographic SF.]

Must also have: $1/s \ll 1/\ell_{\text{max}}$.

Sol'n, pt 2: Adapted spectral methods

Need: accurate *orthonormal* eigenfunctions with few lattice points.

Approximate them by orthogonal polynomials ϕ_i: $\quad \psi = \mu(r)\left(\sum_i a^i \phi_i\right)$

choose measure $\Omega \propto \sqrt{\mu}$ to factor out singularities of eigenf'ns:

$$\int_{-1}^{1} dr \Omega(r) \phi_i^\dagger(r) \phi_j(r) = \delta_{ij} \ .$$

Choose grid points by solving a QM problem:

diagonalize the position operator

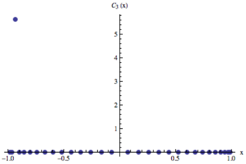
$$X_{ij} \equiv \langle i | \hat{x} | j \rangle \equiv \int_{-1}^{1} dr \Omega(r) \phi_i^\dagger(r) r \phi_j(r).$$

The eigenvectors are the *cardinal functions* C_i: $\quad (C_i, C_j) = \delta_{ij} \ , \quad (C_i, xC_j) = x_i \delta_{ij} \ .$

$$\int \Omega(r) p(r) dr = \sum_{i=1}^{n} w_i p(x_i)$$

is exact if p is a polynomial of degree less than $2n$.

Quadrature weight is $w_i = \frac{1}{C_i(x_i)^2}$.



Sol'n, pt 3: Adiabatic subtraction

$$\langle T \rangle \sim \frac{a}{s^4} + \frac{b}{s^2} + c \ln s + \text{finite}.$$

UV divergences come from local contributions. To compute the contribution at each point, Taylor expand around that point [Schwinger, de Wit, Birrell-Davies].

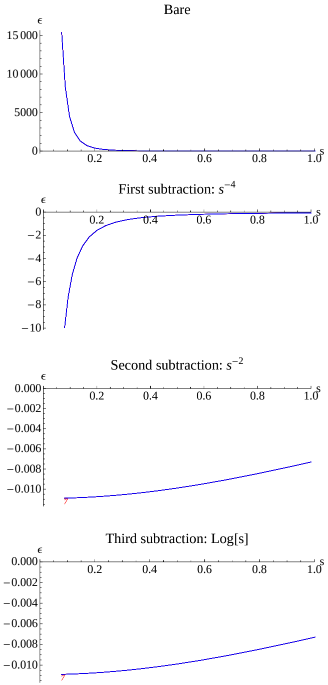
$a = \delta\Lambda \; g_{\mu\nu}$

$b = \delta\left(\frac{1}{G_N}\right)\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)$

It works!

It gives a covariant answer.

Spinor energy density in empty AdS : $\longrightarrow$



Holographic UV divergences

Energy density in some *AAdS* geom: $\longrightarrow$

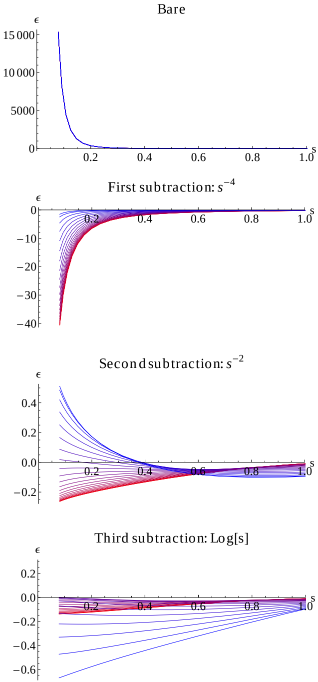
Color is bulk position:
red–blue is IR–UV

Note: divergence at UV boundary.

Finite at fixed radial cutoff.

This is a dual-QFT UV divergence,

fixed by holographic renormalization. $\longrightarrow$



Remaining step: iteration

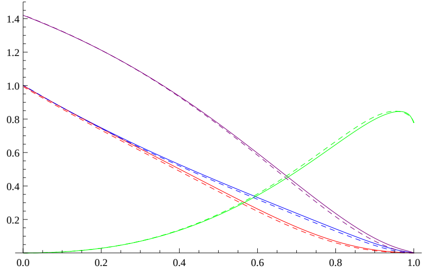
10 Given geometry, find sources

20 Given sources, find geometry

30 GOTO 10

Solve (lattice) Einstein equations using relaxation method.

Check of method: zero-temperature holographic superconductor:



Comparison with [Gubser-Nellore, Horowitz-Roberts 0908]

Concluding comments

1. This may seem like a lot of effort, but it's still a lot easier than directly solving a strongly-coupled quantum many body problem.
2. We are used to interpreting the radial dependence of the bulk fields as encoding running coupling constants in the dual QFT (along with information about the state).
   How should we interpret holographically
   the (quantum) information in bulk fermion fields?
3. Q: What do the bulk fermions do to the IR geometry?
   What other Fermi surface states can arise holographically?

   A: We'll see!

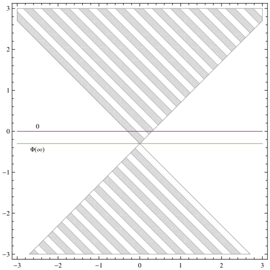
The end.

Thanks for listening.

Physics of ($G_N = 0$) quantum electron star

UV lightcone for charge-q dofs:
$\{(\omega, k) | (\omega + q\mu)^2 \leq c^2 k^2\}$

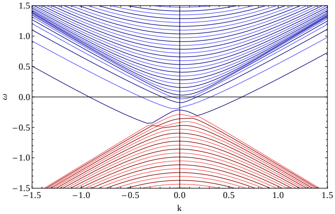
IR lightcone for charge-q dofs:
$\{(\omega, k) | (\omega + q\Phi(\infty))^2 \leq c^2 k^2\}$
FS boundstate can scatter off these dofs (recall tunneling into AdS_2).



Q: What's $\Phi(\infty)$?
A: $\Phi(\infty) = 0$. If $\Phi(\infty) \neq 0$: occupation of continuum.

$$\psi_{\text{IR LC}}(z) \overset{z\to\infty}{\to} e^{i\kappa z} \implies \rho(z) \overset{z\to\infty}{\to} \text{const}$$

$$\implies \Phi(z) \overset{z\to\infty}{\sim} z^2 \neq \Phi(\infty)$$



Q: Whence power-law?
A: The modes which skim the IR lightcone. Matching calculation?