University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 230 Quantum Phases of Matter, Spr 2024
Assignment 6

Due 11pm Thursday, May 16, 2024

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1. **Hall plateaux as a crazy manifestation of quantum oscillations.** Check the claim that the hierarchy states at fillings $\nu = \frac{\nu^\star}{2\nu^\star \pm 1}$ for $\nu^\star \in \mathbb{Z}$ can be regarded as an extreme version of quantum oscillations in the HLR state at $\nu = \frac{1}{2}$.

2. **Quantum Hall states of quasiparticles.** In lecture we explained how to find incompressible states with filling fraction $\nu = \frac{1}{k - \frac{1}{\tilde{k}}}$ by placing the quasiparticle excitations of a $\nu = 1/k$ FQH state in a $\tilde{\nu} = 1/\tilde{k}$ FQH state. Check this relation. When $\tilde{k} = 2$, this reproduced one branch of the composite fermion states we found previously. Explain how to get the other branch.

3. **Excitations of hierarchy states.** Find the torus groundstate degeneracy, and the charges and statistics of the quasiparticle excitations of the abelian incompressible FQH state at $\nu = \frac{2}{5}$ (for example, using the description in terms of the K-matrix CS theory).

4. **Boson Integer Quantum Hall State from Partons.** Consider a system made from two species of bosons, $b_\uparrow, b_\downarrow$. They could be distinguished by living in two layers. We'll assume that only the total boson number, acting by $(b_\uparrow, b_\downarrow) \to e^{\mathbf{i}\alpha}(b_\uparrow, b_\downarrow)$ is conserved (so that if the label is a layer label, the particles are able to tunnel between layers), and couple to a background field $\mathcal{A}$ for that symmetry.

   (a) Consider the parton ansatz:

   $$b_\uparrow = f_0 f_\uparrow, \quad b_\downarrow = f_0 f_\downarrow f_1 f_2$$

   where all the fs are fermionic partons. There are three $\mathsf{U}(1)$ gauge fields that glue these partons back together, and the charge assignments are as follows:

   Also in the table are the Chern numbers of the bands filled by each of the partons in three distinct phases. (Only the Chern number of f_2 changes.) Identify the three phases, and describe the critical theories separating them.

   Hint: I recommend describing the parton currents in terms of dynamical gauge fields $j_\mu^{(\alpha)} = \frac{1}{2\pi}\epsilon_{\mu\nu\rho}\partial_\nu b_\rho^{(\alpha)}$, where $\alpha = \uparrow, \downarrow, 0, 1, 2$.

	a_1	a_2	a_3	$\mathcal{A}$	Chern # in Phase 1	Chern # in Phase 2	Chern # in Phase 3
$f_\uparrow$	1	0	0	1	1	1	1
$f_\downarrow$	1	1	0	1	1	1	1
f_0	-1	0	0	0	-1	-1	-1
f_1	0	-1	1	0	-1	-1	-1
f_2	0	0	-1	0	-1	0	1

(b) For this part of the problem, let's retreat to the continuum. Consider the simpler parton ansatz:

$$b_\uparrow = f_0 f_\uparrow, \quad b_\downarrow = f_0 f_\downarrow$$

where all the fs are fermionic partons. Choose the $\mathsf{U}(1)_{\mathcal{A}}$ to be charges $q_0 = 2, q_\uparrow = -1, q_\downarrow = -1$.

Consider an equal number N of $b_\uparrow$ and $b_\downarrow$ particles, so that the total filling fraction is $\nu = 2$. How many f_0 particles are there, and how many $f_\downarrow, f_\uparrow$ particles are there?

Write a candidate groundstate wavefunction $\Psi(r_i^\uparrow, r_i^\downarrow)$ for the bosons.

(c) Bonus question: why does the simpler ansatz of the previous part produce a wavefunction in the same phase as one of the phases of the first part?

(d) Actually, here is a simpler description of the same phase diagram, closer to what I said in lecture. Consider a single species of boson, with the simple parton ansatz with $b = d_1 d_2$ in terms of two fermions. Let d_1 and d_2 fill Chern bands with total Chern number c_1 and c_2. Fix $c_1 = -1$. Consider what happens when $c_2 = 2$.

Describe the effective field theory of d_2 filling two bands with chern number 1 by introducing two gauge fields each with CS term $\frac{1}{4\pi} b_a d b_a$.