2 Effective field Theory, cont'd

eg: Gravity. [eg: $dx^2+dy^2 = r^2 d\theta^2 + dr^2$]

$$ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$$

metric

Recipe for EFT:

① what are the dofs?

② what are the syms & redundancies?

coordinate independence or diffeomorphisms

③ what is the cutoff? $M_{new} = M_{pl}$

$$G_N = \frac{\#}{M_{pl}^2}.$$

Write down $S[\text{dof}] =$ Σ(all possible terms)

in an expansion in $\left(\frac{\partial}{M_{new}}\right)^{\#}$.

$$S[g_{\mu\nu}] = \int d^4x \sqrt{g}\left[\Lambda + M_{pl}^2 R + \frac{\#}{M_{pl}^4} R^2 + \dots\right]$$

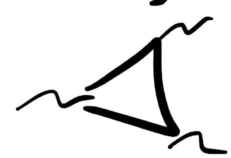
example: Standard Model of particle physics.

	$L=\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$	e_R	ν_R	$Q=\begin{pmatrix}u_L\\ d_L\end{pmatrix}$	u_R	d_R	H
$SU(3)$	—	—	—	3	3	3	—
$SU(2)$	2	—	—	2	—	—	2
$U(1)_Y$	$-1/2$	-1	0	$1/6$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Q: whence the hypercharges ?? (Q_Y's)

A: absence of gauge anomalies.

for each $\{G_1\ G_2\ G_3\}$ $\to$ a possible anomaly



$$G_{1,2,3} \in \{U(1)_Y, SU(2), SU(3)\}.$$

$$\partial_\mu j_1^{A\mu} = \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^{2B} F_{\rho\sigma}^{3C} \boxed{\sum_f (-1)^f \operatorname{tr}\{T_1^A, T_2^B\} T_3^C}$$

f: Weyl ferm.

$(-1)^f$: -1 for L, $+1$ for R.

$$U(1)_Y^3 \rightsquigarrow \sum Q_L^{Y\,3} - \sum Q_R^{Y\,3} = 0$$

$$U(1)_Y\, SU(3)^2 \rightsquigarrow \sum_{\text{quarks}} (Q_L^Y - Q_R^Y) = 0$$

$$U(1)_Y\, (\text{gravity})^2 \rightsquigarrow \sum (Q_L^Y - Q_R^Y) = 0.$$

A2: GUTs.

$$\begin{pmatrix} 3\times 3 & \\ & 2\times 2 \end{pmatrix}$$

all L

one generation of the SM $= 10 \oplus \bar{5} \oplus 1 = \underline{16}$

$$SU(3)\times SU(2)_L \times U(1)_Y \underset{\langle \Phi \rangle}{\subset} SU(5) \subset \underline{\underline{SO(10)}}$$

$10\oplus\bar{5}\oplus 1$ of $SU(5)$ is anomaly free $\Longrightarrow$ $U(1)_Y$ is compact anomaly free.

PATI-SALAM:

$SU(4)\times SU(2)_L \times SU(2)_R$

$$\begin{array}{l c c | l c c c}
 & \overbrace{SU(3)}^{3} & SU(2)_L \vee SU(2)_R & & SU(3) & SU(2)_L & \\
u_L & & \multirow{2}{*}{2} & \nu_{eL} & & \multirow{2}{*}{2} & \multirow{2}{*}{(4,2,1)} \\
d_L & & & e_L & & & \\
\bar{u}_R & & 1 \;\}\, 2 & \bar{\nu}_{eR} & & 1 \;\}\, 2 & \multirow{2}{*}{(\bar{4},1,2)} \\
\bar{d}_R & \underbrace{\qquad}_{\bar{3}} & 1 & \bar{e}_R & & 1 & \\
\end{array}$$

Treat lepton # as a fourth color

$$G_{PS} = \underbrace{SU(4)}_{} \times \underbrace{SU(2)_L \times SU(2)_R}_{}$$

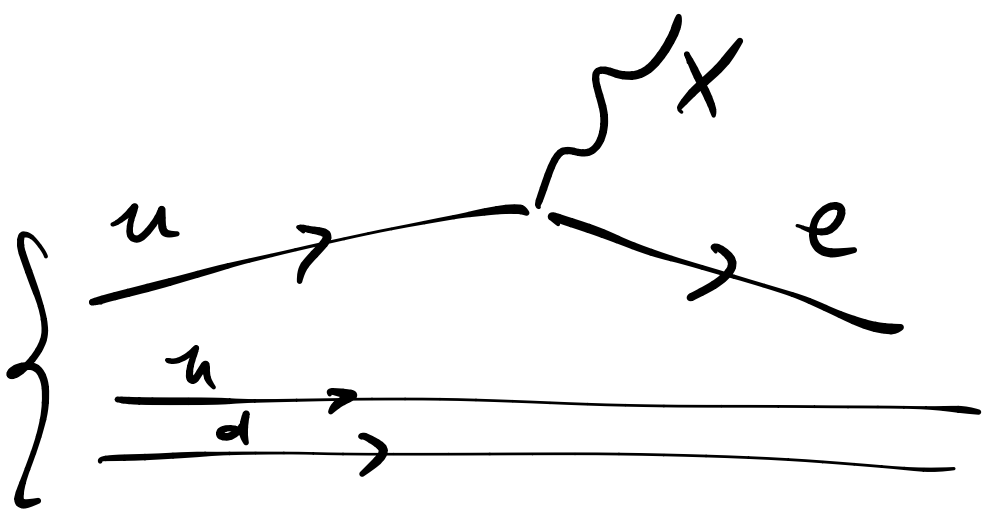
$$= SO(6) \times SO(4)$$

$$\begin{pmatrix} 6\times 6 & \\ & 4\times 4 \end{pmatrix} \subset SO(10)$$

$$10 \otimes 10 = \overset{\text{TRACE}}{\underline{1}} \oplus \overset{\text{AS}}{45} \oplus \overset{\text{SYM}}{54}$$

$$\begin{tikzcd} & SO(10) \arrow[dl, "\underline{45}"'] \arrow[dr, "\underline{54}"] & \\ SU(5) \arrow[dr] & & SO(6)\times SO(4) \arrow[dl] \\ & G_{SM} & \end{tikzcd}$$

$$SU(5) \cap SO(6)\times SO(4) = G_{SM}.$$



$\not\Downarrow$

... proton decay!

Beyond the SM $\Rightarrow$ EFT:

① 3 generations of SM fermions
+ Higgs + gauge bosons.

② Poincaré, gauge invariance, CPT

③ ?

$$S = \int \underbrace{m^2 H^2}_{\text{dim } 2} + \underbrace{\lambda (H^2)^2 + Y \bar{L} H e_R + Y \bar{Q} H d_R + Y \bar{Q} (i\tau^2 H^*) u_R}_{\text{dim } 4}$$

$$+ \operatorname{tr} F_{\mu\nu} F^{\mu\nu}$$

$$+ \bar{\Psi} i \not{D} \Psi$$

$$+ \theta F \wedge F \qquad + \text{dim} > 4.$$

eg: $\delta \mathcal{L} = \frac{1}{M^2} \bar{\Psi} A \Psi \cdot \bar{\Psi} B \Psi$

Dirac / flavor matrices

$M > 10$ TeV

$$L = L_0 + \frac{1}{\Lambda_{new}} \underline{\underline{\mathcal{O}^{(5)}}} + \frac{1}{\Lambda_{new}^2} \sum_i c_i \mathcal{O}_i^{(6)} + \dots$$

$$\mathcal{O}^{(5)} = c_5 \, \epsilon_{ij} (\bar{L}^c)^i H^i \epsilon_{kl} L^k H^l$$

$$H^i = (h^+, h^0)^i \qquad \bar{L}^c = L^T C$$

$$L^i = (\nu_L, e_L)^i$$

If $\langle H \rangle = (0, v) \longrightarrow$ neutrino mass.

$$m_\nu \sim \frac{v^2}{\Lambda_{new}}$$

$\mathcal{O}^{(5)}$ violates lepton #

$$\begin{cases} L \to e^{i\alpha_L} L \\ Q \to Q \\ H \to H \end{cases}$$

$$\text{'}qqq\ell\text{'} \equiv \underbrace{\epsilon_{\alpha\beta\gamma} \overbrace{(\bar{u}_R)^c_\alpha (u_R)_\beta}^{\text{Lorentz singlet}} \overbrace{(\bar{u}_R)^c_\gamma e_R}^{\text{Lorentz singlet}}}_{\text{color singlet}}$$

4-fermi theory of proton decay:

$$\mathcal{L} \ni \frac{1}{M_{planck}} p \mathcal{O} \rightsquigarrow \Gamma_p \sim \frac{m_p^3}{M_{pe}^2} \sim 10^{-13}\, s^{-1}$$

vs in the SM:

$$\Delta \mathcal{L}_B = \frac{1}{M_{pe}^2} qqq\ell \rightsquigarrow \Gamma_p < \text{observed.} \quad \checkmark$$

$$\frac{1}{M_{GUT}^2} qqq\ell \qquad M_{GUT} \sim 10^{16}\, GeV \qquad \times.$$

$\Delta\mathcal{L}_B$ violates $\underbrace{\text{Baryon}}$ # & lepton #

$$\begin{cases} L \to L \\ Q \to e^{i\alpha B/3} Q \\ H \to H \end{cases}$$

anomalous

But $U(1)_{B-L}$ is not anomalous $\subset SU(2)_R$

$\subset G_{PS}$.

- <u>Theorem</u>: You can't break CPT in a Lorentz-inv't QFT.

- Discrete Syms: $U(1)_B$ is anomalous

$$\partial^\mu j_\mu^B = \underbrace{N_g}_{3} \underbrace{\left(\frac{F \wedge F}{16\pi^2} \right)}_{\int (\;) \in \mathbb{Z}}$$

$$\Rightarrow U(1)_B \supset \mathbb{Z}_{N_g} \text{ not anomalous}$$

$$z \to e^{i\alpha A} z$$

$A \in 3\mathbb{Z}$

[Koren]

if $\alpha \in \frac{2\pi}{3}$

$$\Rightarrow \underline{z \to z},$$

7.6 Superconductors & Superfluids.

① A_i, Φ of charge 2.

② Rot. sym, gauge inv.

③ ?

$$\Rightarrow \mathcal{F}_{SC}[\Phi] = \frac{1}{4}F_{ij}F^{ij} + |D_i\Phi|^2 + a|\Phi|^2 + b|\Phi|^4 + \dots$$

$$D_i\Phi = (\partial_i - 2ieA_i)\Phi \, .$$

Abelian Higgs model. Φ has charge 2.

<u>who is Φ?</u>

<u>New IR dofs</u>

microscopic theory of electrons

$\rightsquigarrow$

L.G. EFT of superconductor

<u>dofs</u>

charge 1 spinor fermion ψ_α

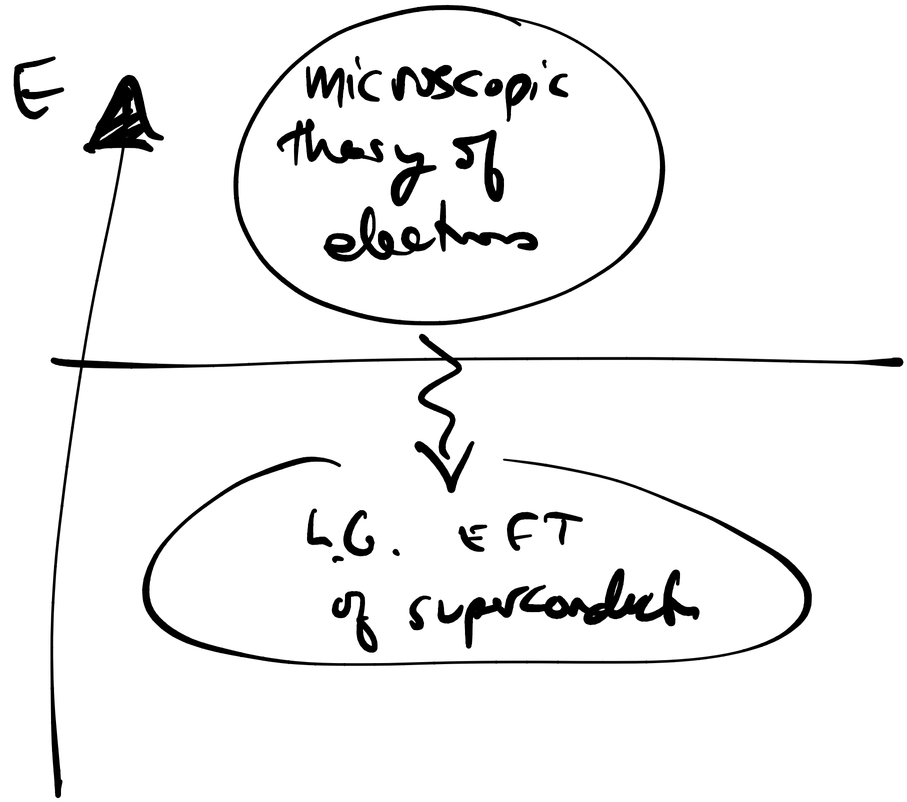
$\alpha = \uparrow, \downarrow$

charge 2 scalar boson Φ.

In the simplest (s-wave) examples

$$\Phi \sim \psi_\alpha \psi_\beta \epsilon^{\alpha\beta}$$

Cooper pair field



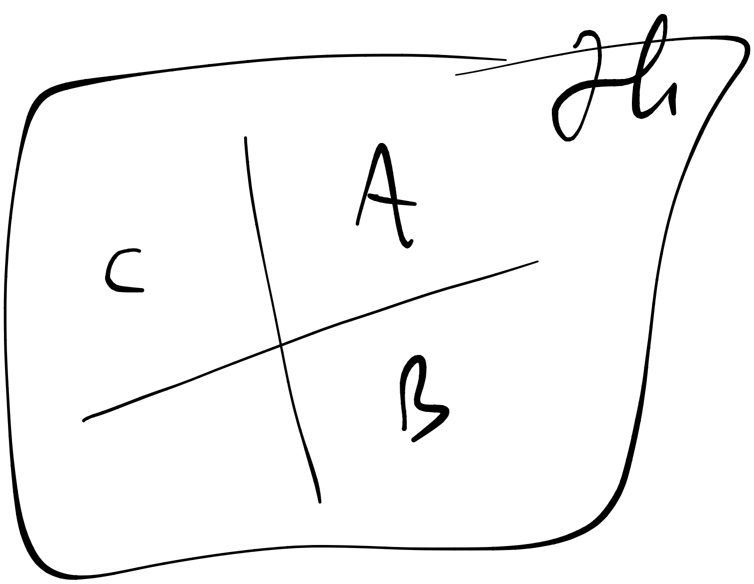
<u>Claim</u>:

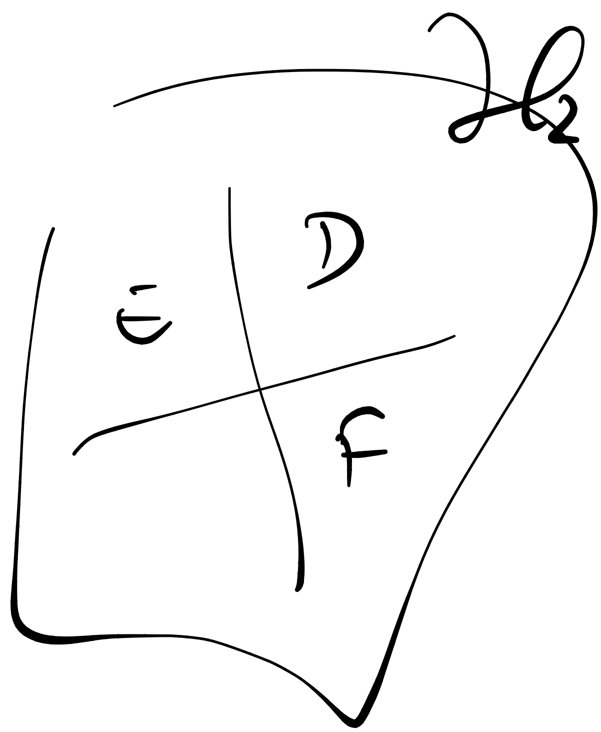
cutoff $\sim \Delta E_\psi$

$\sim m_\psi$

<u>next time</u>:

where did the fermions go?





$$S[\psi] = \int d\omega\, dk \;\; \psi^\dagger(\omega,k)\,(\omega - \epsilon(k))\,\psi(\omega,k)$$

$$\Leftrightarrow \quad H = \int dk\; c^\dagger_{\underline{k}}\; \epsilon(k)\, c_{\underline{k}}$$

$$-i\,\partial_t\, \mathcal{O} = [H, \mathcal{O}]$$