University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215C QFT Spring 2022
Assignment 1

Due 11:59pm Monday, April 4, 2022

- Please hand in your homework electronically via Canvas. The preferred option is to typeset your homework. It is easy to do and you need to learn to do it anyway as a practicing scientist. A LaTeX template file with some relevant examples is provided here. If you need help getting set up or have any other questions please email me. I am happy to give TeX advice.

- To hand in your homework, please submit a pdf file through the course's Canvas website, under the assignment labelled hw 01.

Thanks in advance for following these guidelines. Please ask me by email if you have any trouble.

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1. **Brain-warmer.** [optional] Find the constant a in

$$a \prod_{\mu=0}^{D-1} \gamma^\mu = \gamma^5$$

so that $(\gamma^5)^2 = 1$.

2. **Chiral anomaly in two dimensions.**

Consider a massive relativistic Dirac fermion in 1+1 dimensions, with

$$S = \int \mathrm{d}x\mathrm{d}t \bar{\psi} \left(\mathbf{i}\gamma^\mu \left(\partial_\mu + eA_\mu \right) - m \right) \psi .$$

By heat-kernel regularization of its expectation value, show that the divergence of the axial current $j^5_\mu \equiv \mathbf{i}\bar{\psi}\gamma_\mu\gamma^5\psi$ is

$$\partial_\mu j^5_\mu = 2\mathbf{i}m\bar{\psi}\gamma^5\psi + \frac{e}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} .$$

3. **Chiral anomaly in six dimensions.** [optional] Find the divergence of the axial current in QED in $D = 5 + 1$.

4. **An application of the anomaly to a theory without gauge fields.**

Consider a 1+1d theory of Dirac fermions coupled to a background scalar field θ as follows:

$$\mathcal{L} = \bar{\Psi}\left(\mathbf{i}\partial\!\!\!/ + me^{\mathbf{i}\theta\gamma^5}\right)\Psi.$$

We wish to ask: if we subject the fermion to various configurations of $\theta(x)$ (such as a domain wall where $\theta(x > 0) = \pi + \theta(x < 0)$) what does the fermion number do in the groundstate?

(a) Convince yourself that when θ is constant

$$\langle j^\mu \rangle = 0$$

where $j^\mu = \bar{\Psi}\gamma^\mu\Psi$ is the fermion number current.

(b) Minimally couple the fermion to a *background* gauge field A_μ. Let $e^{\mathbf{i}\Gamma[A,\theta]} = \int [d\Psi]e^{\mathbf{i}S}$. Convince yourself that the term linear in A in $\Gamma[A, \theta] = \text{const} + \int A_\mu J^\mu + \mathcal{O}(A^2)$ is the vacuum expectation value of the current $\langle j^\mu \rangle = J^\mu$ (at $A = 0$).

(c) Show that by a local chiral transformation $\Psi \to e^{-\mathbf{i}\theta(x)\gamma^5/2}\Psi$ we can remove the dependence on θ from the mass term.

(d) Where does the theta-dependence go? Use the 2d chiral anomaly to relate $\langle j^\mu \rangle$ to $\partial\theta$. Notice that the result is independent of m. [This relation was found by Goldstone and Wilczek. The associated physics is realized in Polyacetylene.]

(e) Show that a domain wall where θ jumps from 0 to π localizes *fractional* fermion number.

(f) Consider the Dirac hamiltonian in the presence of such a domain wall. Show that there is an exponentially-localized mode of zero energy.