Recall: $SPT_D \equiv$ a phase of matter in D dims characterized by its edge states.

$\iff$ $D-1$ dim'l edge theory is anomalous.

anomaly: 0) symmetry of $S[\text{fields}]$
but not of $\int D[\text{fields}]$

1) obstruction to a lattice realization w/ on-site action of G.

$$\left[\begin{array}{ll} \rho : \bigotimes_x \mathcal{H}_x \to \bigotimes_x \mathcal{H}_x & \rho = \bigotimes_x \rho_x . \\ \text{by: } \rho_x : \mathcal{H}_x \to \mathcal{H}_x . & \end{array} \right]$$

2) obstruction to gauging G.

(Dirac = $\text{Weyl}_L + \text{Weyl}_R$.)

3) symmetry violation in the presence of BG fields

i.e. $\delta_{\text{gauge}} S_{\text{eff}}[A] \notin 2\pi\mathbb{Z}$

$\xRightarrow{\text{LSMOH}}$ g.s. is not boring.

SPT$_D^G$ is an abelian group.

[classification (in terms of generalized cohomology) does not build representatives.

SPTs of fermions & bosons (or spins) are different.

fermions $\leftarrow$ free-fermion top. insulators (TIs)

bosons $\leftarrow$ requires interactions

<u>why study SPTs?</u> ⓪ they exist.

Q: "how can symmetries be realized on quantum many-body systems?"

① <u>gauge</u> them. [<u>Q</u>: Is every T.O. a gauged SPT?]

$c = \psi_1\psi_2 + \psi_3\psi_4$

② each SPT_D $\longleftrightarrow$ some D-1 dim'l theory that can't be regularized

(a Nielsen Ninomiya thm.)

the Standard Model is a chiral gauge theory w/ <u>no</u> <u>anomalies</u>.

③ emergence of SUSY at SPT edge. ($N=1$ in $D=2+1$.)

3.1 EM response for $SPT_{G \supset U(1)}$

- $D = 2+1$, $G = U(1)$.

$$S_{eff}[A] = \frac{\nu}{4\pi}\int A \wedge dA + \cdots$$

$$\rightarrow \sigma^{xy} = \nu \frac{e^2}{h} = \frac{\nu}{2\pi}. \quad \underline{\nu \in \mathbb{Z}}.$$

labels phases of <u>fermions</u>

No T.O.

for <u>bosons</u>: $\nu \in 2\mathbb{Z}$.

<u>K-matrix construction</u>:

$$S[a_I] = \sum_{IJ} \frac{K_{IJ}}{4\pi}\int a^I \wedge da^J + \int A \, t_I \frac{da^I}{2\pi}$$

<u>Q</u>: for which $\{K, t\}$ is there no T.O?

$$\sigma^{xy} = \frac{1}{2\pi} t K^{-1} t \overset{!}{\in} \mathbb{Z}$$

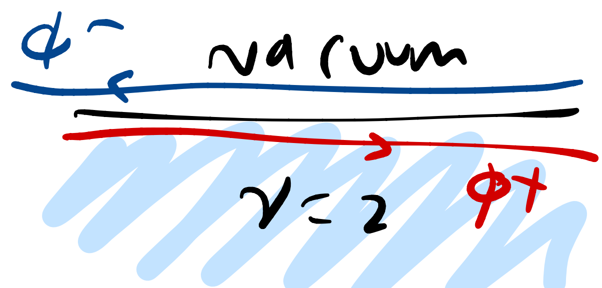
GSD on Σ_g $= |\det K|^g$

$$\Rightarrow |\det(K)| \overset{!}{=} 1 .$$

statistics of qps :
$$\begin{cases} \theta = \pi \, l^T K^{-1} l & (l_I \in \mathbb{Z}) \\ \theta_{12} = 2\pi \, l_1^T K^{-1} l_2 \\ Q = t^T K^{-1} l \end{cases}$$

eg: Boson IQH .
$$\begin{cases} K = \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ t = (1,1) \end{cases}$$

$$\rightarrow \sigma^{xy} = 2 \frac{e^2}{h} .$$



$$S_{cs}[a^I = \tilde{d}\phi^I] = \frac{1}{4\pi} \int_{bdy} dt\,dx \Big[\partial_t \phi^+ \partial_x \phi^+ - \partial_t \phi^- \partial_x \phi^- + v(\partial_x \phi^+)^2 + v(\partial_x \phi^-)^2 \Big]$$

$$\phi^{\pm} = \frac{1}{\sqrt{2}} (\phi^1 \pm \phi^2)$$
evecs of K

$$L \ni A^\mu \epsilon_{\mu\nu\rho} \partial_\nu (t^I a_{I\rho}) / 2\pi \qquad a_a^I = \epsilon_{ab} \partial_b \phi^I$$

$a, b = 0, 1$

$$= A_\mu^{a=0,1} \partial_\mu (\phi^1 + \phi^2) / 2\pi$$

$$= A_\mu \partial^\mu \phi^+$$

$\Rightarrow$ only the right mover is charged!

$$\Delta S = \int g \cos(\phi^+ + \phi^-)$$

is not $U(1)$ invariant.

$U(1)$ protects the edge.

• $D = 3+1$ with $G = U(1) \rtimes \mathbb{Z}_2^T$

$\uparrow$ time reversal.

(fermions w/ <u>real</u> hopping)

$$S_{eff}[\vec{E},\vec{B}] = \int d^3x\,dt \Big[\cancel{\vec{p}\cdot\vec{E}} + \cancel{\vec{\mu}\cdot\vec{B}}$$

ferroelectric ferromagnet

$$+ \cancel{\epsilon_{ij} E^i E^j + \left(\tfrac{1}{\mu}\right)_{ij} B^i B^j} + \frac{e^2}{4\pi^2}\,\theta\,\vec{E}\cdot\vec{B} + O(E,B)^4 \Big]$$

$$\epsilon\,\vec{E}^2 - \frac{1}{\mu}\vec{B}^2$$

(assume rotation symmetry)

Flux quantization $\oint_S F \in 2\pi\mathbb{Z}$.

$$\Rightarrow \frac{e^2}{4\pi^2}\int_{M_4} \vec{E}\cdot\vec{B} = \underline{\frac{e^2}{8\pi^2}\int_{M_4} F\wedge F \in \mathbb{Z}}$$

if $\partial M_4 = \phi$ and M_4 is spin.

$\Rightarrow Z_{M_4}(\theta) = Z_{M_4}(\theta + 2\pi)$. $\Rightarrow$ spectrum on X_3 is periodic.

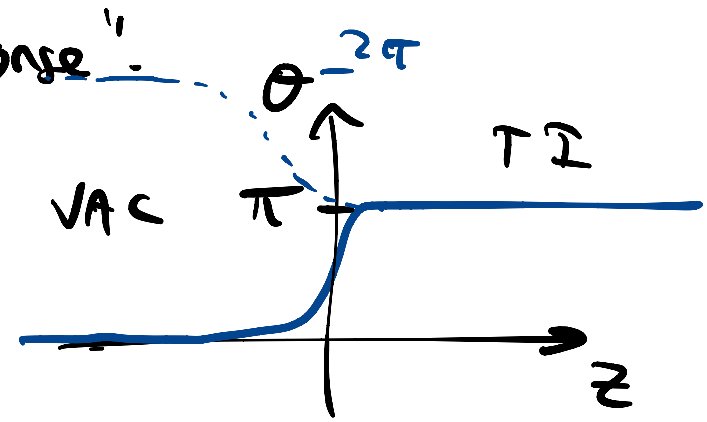
$T: (\vec{E}, \vec{B}) \to (\vec{E}, -\vec{B})$

$\theta \to -\theta$.

preserves the spectrum if $\theta = 0$ or $\underline{\underline{\pi \simeq -\pi}}$

$\Rightarrow \frac{\theta}{\pi} = 0, 1$ are distinct SPT phases.

"magnetoelectric response"



$$S_\theta = e^2 \int \frac{\theta(z)}{8\pi^2} F \wedge F \overset{IBP}{=} \frac{e^2}{\ } \int \frac{d\theta \wedge A \wedge F}{8\pi^2}$$

IF edge is gapped:

$$d\theta = \Delta\theta \, \delta(z) \, dz$$

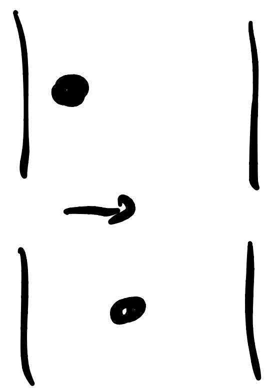
$$j^\mu_{EM} = \frac{\delta S_{eff}}{\delta A_\mu} = \frac{-e^2}{8\pi^2} \partial_\nu \theta \, F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} + \dots$$

$(h = 2\pi)$

apply $\vec{E} = \hat{y} E \Rightarrow \sigma_{xy} = \frac{e^2}{h} \frac{\Delta\theta}{2\pi} = \frac{e^2}{h} \left(\frac{1}{2} + n \right)$.

Parallel in polarization

unit cell



polarization : θ ::

surface charge : IQH layers
(n)

$$\text{polarization} \sim \langle e\,\hat{r} \rangle \sim e \oint_{BZ} \langle i \nabla_k \rangle dk$$

$$= e \oint_{BZ} A(k)\, dk$$

$D = 1+1$ in $U(1) \times \underset{\sim}{C}$

$$F \to -F.$$
$$(j_\mu \to -j_\mu)$$

$$S_\theta[A] = \int_{X_2} \frac{\theta}{2\pi} F \qquad \oint \frac{F}{2\pi} \in \mathbb{Z}.$$

$$\Rightarrow Z(\theta + 2\pi) = Z(\theta)$$

$C \Rightarrow \theta \in 0, \pi.$

$$j_\mu = \epsilon_{\mu\nu} \partial_\nu \theta / 2\pi \Rightarrow \text{wall acquires charge } \frac{\Delta\theta}{2\pi}. \text{ [SSH]}$$

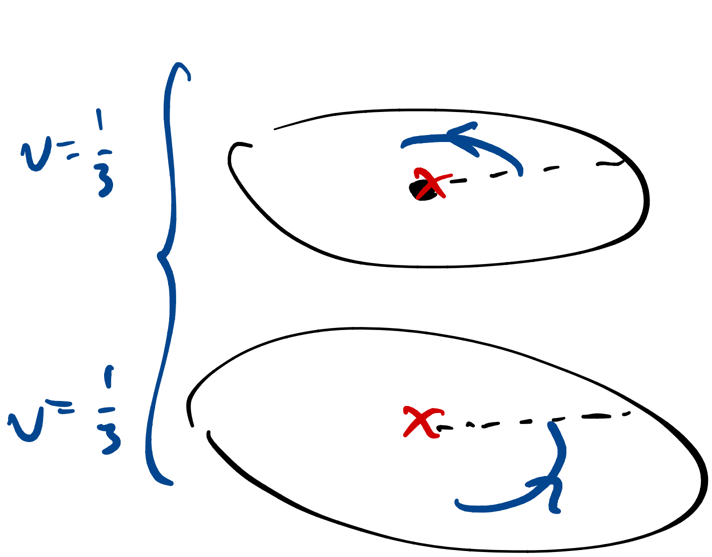
"Bulk-edge correspondence"

SPTS (in D dims) (phases of matter) .... anomalous systems in D-1 dims. on the edge.

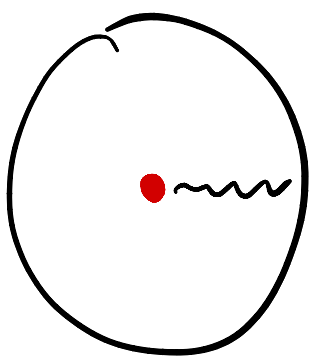
possible edges in $D \geq 3+1$:

- can break G. $\longrightarrow$ (GSD or goldstones)
- can be gapless.
- can be (anomalous) TO ($\rightarrow$ GSD)
- can attach any non-anomalous system!

[Qi, Barkeshli, Jian]



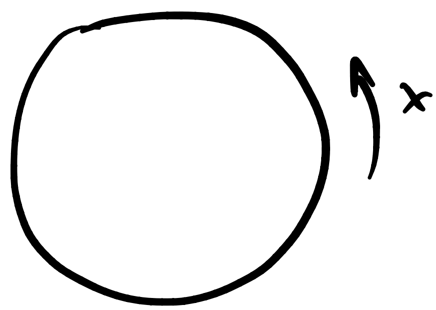
σ anyon:



$$\gamma(x+2\pi) = +\gamma(x)$$

R

no anyon

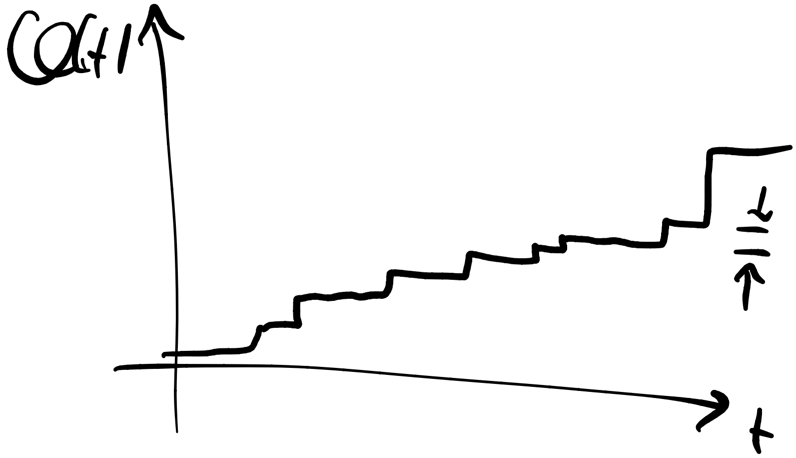


$$\gamma(x+2\pi) = -\gamma(x)$$

NS

$$\Rightarrow \gamma(x) = \sum_{n \in \mathbb{Z}} e^{inx} \gamma_n$$

$$[H, \gamma_n] = i n \gamma_n$$



$$\int_{}^{T} dt\, \frac{I^2(t)}{T}$$