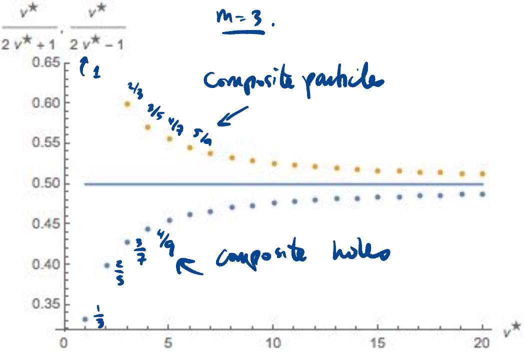


$\frac{\nu^\star}{2\nu^\star+1}, \frac{\nu^\star}{2\nu^\star-1}$
m = 3.
Composite particles
composite holes
2/3
3/5
4/7
5/9
1/3
2/5
3/7
4/9
$\nu^\star$

$$\tilde{\Psi}_\nu(z) = P_{LLL}\left[\prod_{i<j} z_{ij}^{m-1}\, \tilde{\Psi}_{cf}(z,\bar{z})\right]$$

N-electron wavefn

$\{z_i\}_{i=1}^{N}$ $\quad z_i = x_i + i y_i$

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_e^*} + \sum_{i<j} V(r_i - r_j)$$

Composite fermions as partons: $\quad c = f\, b$

U(1) gauge redundancy $\begin{cases} f \to e^{-i\alpha} f \\ b \to e^{i\alpha} b \\ a_1 \to a_1 - d\alpha \end{cases}$

Choice: put b in a $\nu = \frac{1}{m-1} = \frac{1}{2}$ Laughlin state.

(gap)

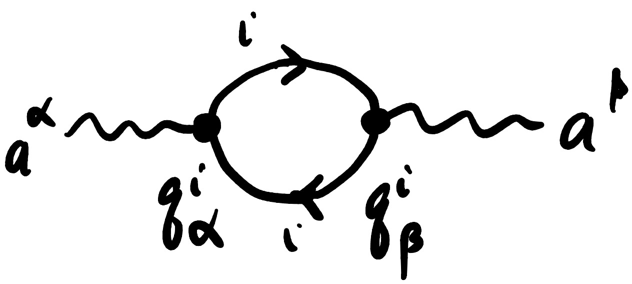
To do that: $b = d_1 d_2$

let $d_{1,2}$ fill a Chern band (LL)

(IQH)

	A	a_1	a_2
d_1	q_1	0	1
d_2	q_2	1	-1
f	q_f	-1	0

$(q_1 + q_2 + q_f = 1.)$



$$L_i = \frac{c_i}{4\pi} \sum_{\alpha\beta} \left(q_i^\alpha a_\alpha \right) d \left(q_i^\beta d a_\beta \right) \quad *$$

contribution from parton i: $\quad a_{\alpha=0} \equiv A$.

$$L_{eff} = \frac{c_1}{4\pi} \left(a_2 + q_1 A \right) d \left(a_2 + q_1 A \right)$$

$$+ \frac{c_2}{4\pi} \left(a_1 - a_2 + q_2 A \right) d \left(a_1 - a_2 + q_2 A \right) + L\left(f, a^\alpha \right)$$

f is the composite fermion.

If we let f fill ν^* LLs.

(integrate out f, integrate out a_i)

using $*$

$$\longrightarrow S_{eff}[A) = \frac{\nu}{4\pi} AdA \qquad c_1 = c_2 = c.$$

$$\nu = \frac{c\nu^*}{c+2\nu^*}\Bigg|_{c=1} = \frac{\nu^*}{1+2\nu^*} \quad \lambda$$

$$(q_1 + q_2 + q_f)$$
$=1$

Hierarchy & K-matrices : $\partial_\mu j^\mu_{qp} = 0$

$$\Longrightarrow j^\mu_{qp} = \frac{1}{2\pi}\epsilon^{\mu\nu\rho}\partial_\nu \tilde{a}_\rho$$

$$4\pi L = m\,a\,da + 2A\,da + 2a\,d\tilde{a} + \tilde{k}\,\tilde{a}\,d\tilde{a}$$

$\nu = \frac{1}{m}$ AJ $a_\mu j^\mu_{qp}$ FQHE of qps.

$$= K_{IJ}\, a^I da^J + 2 t_I\, a^I dA$$

$$K = \begin{pmatrix} m & 1 \\ 1 & \tilde{k} \end{pmatrix} \qquad t = (1, 0)$$

$$\sigma^{xy} = t K^{-1} t = \frac{1}{m - \frac{1}{\tilde{k}}} \overset{\tilde{k}=2}{=} \frac{2}{2m-1}$$

$$m = \nu_* .$$

$$GSD = |\det K|^g$$

$$g \text{ at } \nu = \frac{2}{5} \quad \begin{pmatrix} m = 3 \\ \tilde{k} = 2 \end{pmatrix}$$

$$\hat{\tilde{a}}\, j_{\tilde{q}\tilde{p}} = \frac{2\tilde{a}\, d\tilde{\tilde{a}}}{4\pi} \qquad \Delta L = \frac{\tilde{\tilde{k}}\, \hat{\tilde{a}} \wedge d\hat{\tilde{a}}}{4\pi}$$

$$K = \begin{pmatrix} k & 1 & 0 \\ 1 & \tilde{h} & 1 \\ 0 & 1 & \tilde{\tilde{h}} \end{pmatrix} \qquad t = (1\ 0\ 0)$$

$$\nu = \frac{1}{k - \frac{1}{\tilde{k} - \frac{1}{\tilde{\tilde{k}}}}}$$

...

Boson $\nu = \frac{1}{2}$ Laughlin:

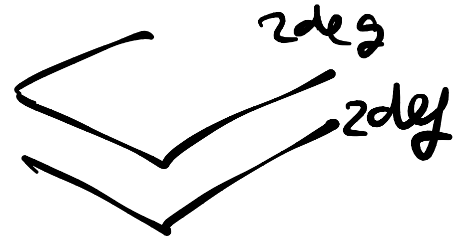
$$\tilde{\Psi}(z) = \prod_{i<j} z_{ij}^2$$

$$L_{eff}[a] = 2\frac{a\,da}{4\pi} + \frac{A\,da}{2\pi}$$

$$\longrightarrow \sigma^{xy} = \frac{1}{2}.$$

["Kalmeyer-Laughlin chiral spin liquid" $\longleftarrow$

egs. $K = \begin{pmatrix} m_1 & n \\ n & m_2 \end{pmatrix}$, $t = (1,1)$

- Layered (F)QH states $\Big\{$ 

[see Wen]

. equivalence rel'n: $a^I \to W^I{}_J a^J$ $\quad W \in GL(n,\mathbb{Z})$

($\det W = \pm 1$.)

(flux quantization)

$$\begin{cases} K \to W^T K W \\ t \to W t \end{cases}$$

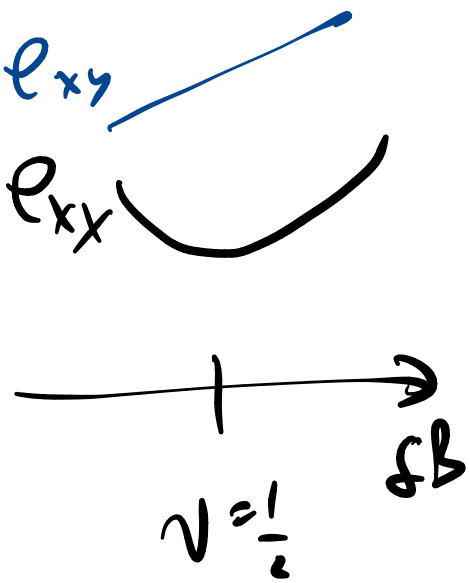
- $L = a\,db/4\pi$ is trivial.

QH Metal : as $\nu^* \to \infty$

$$\nu = \frac{\nu^*}{(m-1)\nu^* \pm 1} \longrightarrow \frac{1}{m-1}.$$

$$B^* = B\big(1-(m-1)\nu\big) \overset{\nu=\frac{1}{m-1}}{=\!=} 0 \quad !$$

$c = fb$



<u>ansatz</u>: $\left\{ \begin{array}{l} b \text{ is } \nu = 1/(m-1) = 1/2 \ \underline{\text{Laughlin}} \\ f \text{ can fill a Fermi sea!} \end{array} \right.$

Compressible!

<u>metal!</u>

$$\tilde{\Psi}(z) = P_{LL}\left[\prod_{i<j} z_{ij}^{m-1} \det_{ij} e^{i k_i \cdot r_j} \right]$$

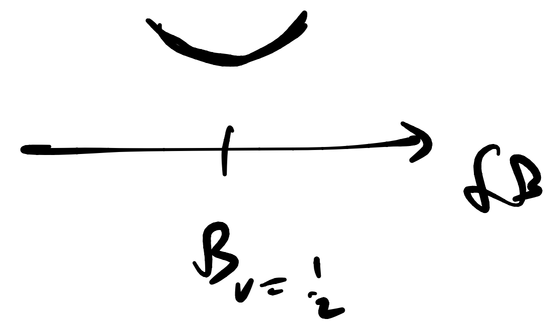
"composite FL"

$\left[\{k_i\} = \text{lowest } \epsilon \text{ single particle states} \right)$

"HLR"

cfs are coupled to a CS gauge field $\longrightarrow$ "Non-Fermi Liquid".

$$B = B_{\nu=\frac{1}{2}} + \delta B$$



FL in δB

$\rightarrow$ quantum oscillations

observables are periodic in $\frac{1}{\delta B}$.

$$\frac{1}{\delta B} = \pm \frac{1}{e\Phi_0} \nu^* \qquad \nu^* \in \mathbb{Z}$$

$$\Longrightarrow \nu(\delta B) = \frac{\nu^*}{(m-1)\nu^* \pm 1}.$$

INCompressible states at even denominators:

$(\nu = \frac{1}{2}, \nu = \frac{1}{4} \ldots)$

spinless cfs $\rightarrow$ p-wave.

If $p+ip$ superconductor $\xrightarrow[\text{wavef'n}]{\text{BCS}}$ Moore-Read Pfaffian state.

$\nu = \frac{1}{2}$ $\qquad$ $\nu = \frac{3}{2}$ $\qquad$ $\nu = \frac{5}{2}$ $\qquad$ $\nu = \frac{7}{2}, \frac{9}{2}$

HLR $\qquad$ incompressible.

stripes

$SU(2)_2$ CS. $\qquad c_L - \cancel{c_R}^{0} = \frac{3}{2}$

κ^{xy} thermal Hall conductivity

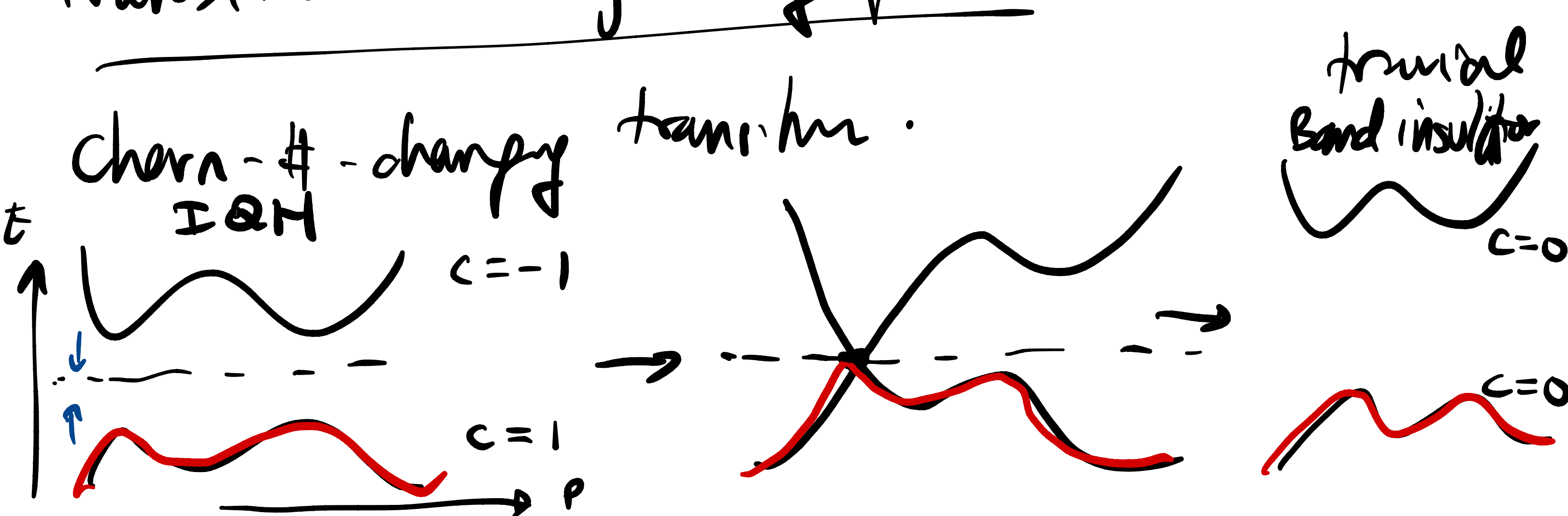
measures $c_L - c_R$

$=$ # rightmovers $-$ # leftmovers

PH Pfaffian has $\kappa_{xy} = 5/2$.

Transitions to neighboring phases.

Chern-#-changing transition.



$$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$$\delta S = \int \left(e^{i(\phi_L + \phi_R)(x)} + h.c. \right)$$

is a local operator

vs: $e^{i\phi_L(x)}$ is not.

$$L = \partial_t \phi_L \partial_x \phi_L + (\partial_x \phi_L)^2$$

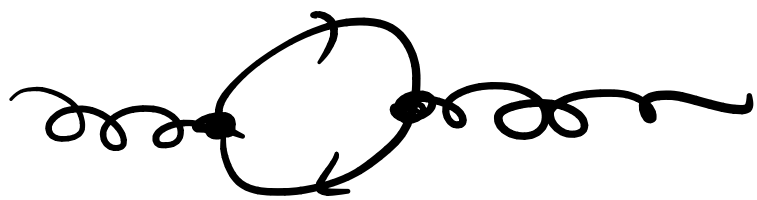
$$\Pi_L = \frac{\partial L}{\partial \dot{\phi}_L} = \partial_x \phi_L$$

$$[\phi_L(x), \partial_x \phi_L(y)] = i\,\delta(x-y)$$

$$\Rightarrow [\phi_L(x), \phi_L(y)] = i\,\Theta(x-y)$$

U(1) anomaly

$$\sum_i {q_i}_L^2 - \sum_i {q_i}_R^2$$



$O(2)_{2,L}$

$\uparrow \quad \uparrow$

$SO(2) \quad \mathbb{Z}_2$

$$L_1 = |\partial\Phi|^2 - \underline{V(|\Phi|^2)}$$

$\underline{\Phi = v e^{i\theta}}$

gauge the U(1)

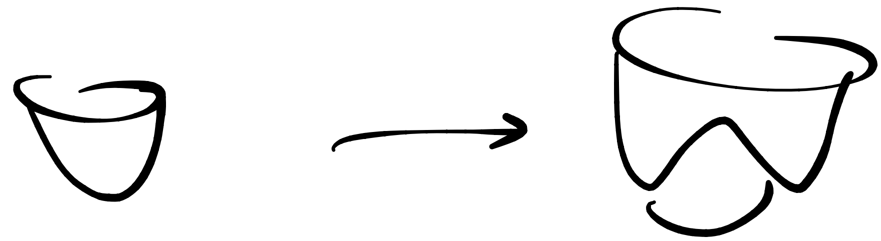
$$L_2 = |(\partial + A)\Phi|^2 - \underline{V(|\Phi|^2)} - F^2$$

$$= v^2 (A + \partial\theta)^2 - F^2$$

choose $\underline{\theta = 0}$

$$L_2 = A^2 - F^2$$

<u>In L_1</u>:



SSB

<u>In L_2</u>:

Higgsing

$$\int \mathcal{D}a \; e^{\int \frac{a\,da}{4\pi} + \frac{a\,dA}{2\pi}}$$

$$\sim e^{\int \sigma^{xy} \frac{A\,dA}{4\pi}}$$