Last time: an LSMOH theorem

in a crystal w/ conserved atom #

ASSUMING · a gap for non-goldstones

· a unique groundstate

$$\Longrightarrow S[\theta, A] = S_{\text{elastic}}[\theta] + S_\nu[\theta, A] + \dots$$

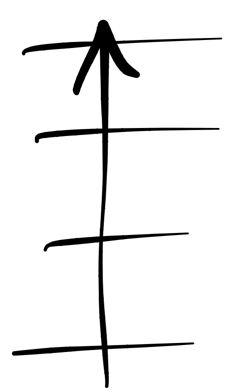
$(\partial\theta)$

$$S_\nu[\theta, A] = \frac{\nu}{(2\pi)^d}\int A \wedge d\theta^1 \wedge \dots \wedge d\theta^d$$

with $\nu \in \mathbb{Z}$.

we saw: $\nu = \dfrac{\#\text{ of atoms}}{\text{unit cell}}$.

Allowed densities:



contrapositive: If $\frac{\# \text{ of particles}}{\text{unit cell}} \notin \mathbb{Z}$

$\Longrightarrow$ g.s. MUST BE INTERESTING

ie either A) gapless

A1) & symmetric.

A2) spontaneously breaks particle #.

($\Rightarrow$ extra goldstone)

OR B) multiple gapped groundstates

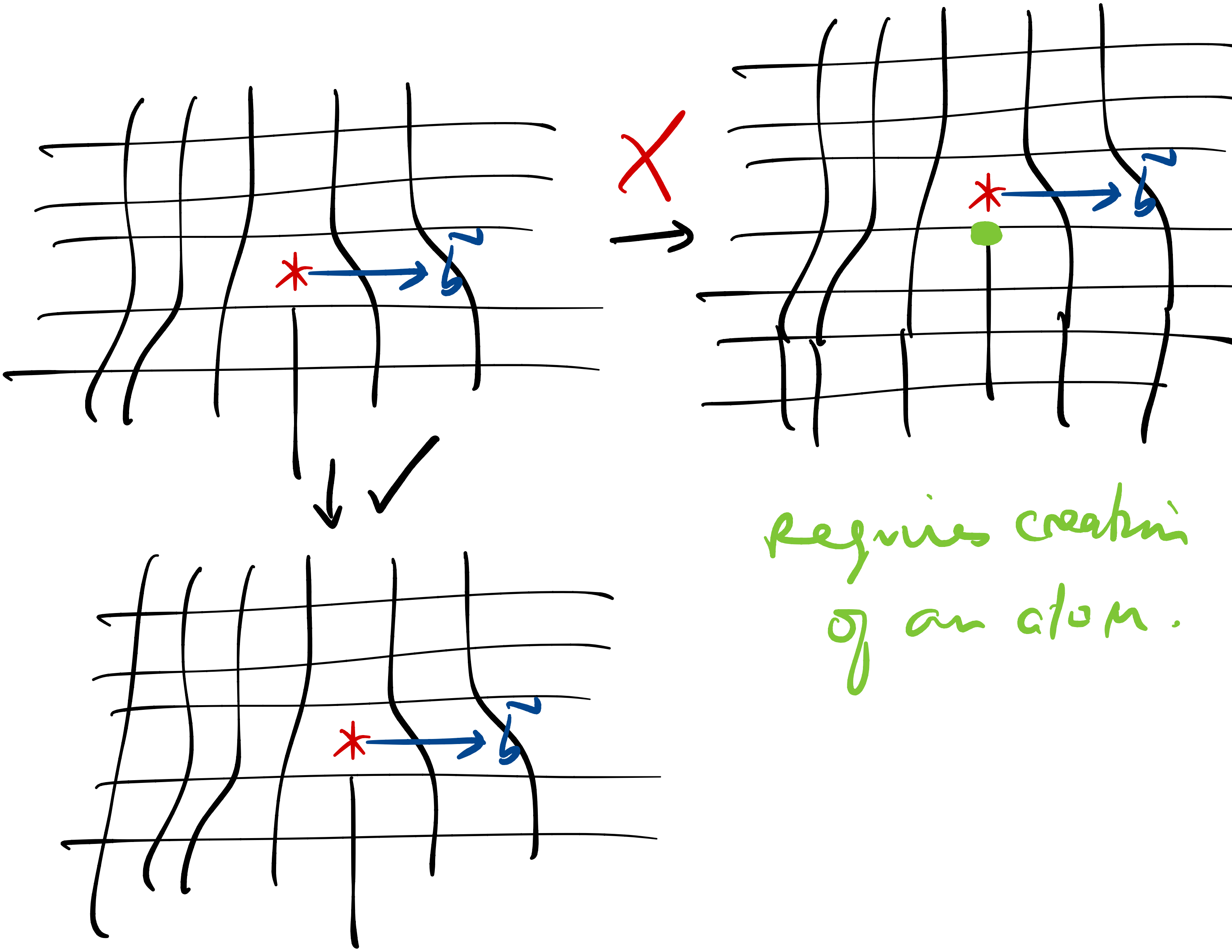
B1) topologically-ordered

B2) charge density wave

ie spontaneously breaks discrete translations.

Mobility constraints on dislocations

claim: if $v \neq 0$ dislocation can only move $\parallel$ to burgers' vector.



$$j^{\mu}(x) = \frac{\delta S[\theta, A]}{\delta A_\mu(x,t)} \underset{d=2}{=} \frac{\nu}{8\pi^2} \epsilon^{\mu\nu\rho} \partial_\nu \theta^I \partial_\rho \theta^J \epsilon_{IJ}$$

$S_\nu[\theta, A]$ was gauge-inv't

for smooth θ ie $\underbrace{(\partial_x\partial_y - \partial_y\partial_x)\theta}_{=0}$

$$\left.\begin{array}{l}(\partial_x \partial_y - \partial_y \partial_x)\theta^I = \text{density of dislocations} \\ \qquad\qquad = j_d^I\end{array}\right\}$$

w/ burgers' vector $\propto a^I$.

$$\partial_\mu j^\mu = \frac{\nu}{8\pi^2} (j_d^I)^\rho \partial_\rho \theta^J \epsilon_{IJ}$$

$$\left((j_d^I)^\rho = \epsilon^{\mu\nu\rho} \partial_\mu \partial_\nu \theta^I \right)$$

In eqbm: $\partial_\rho \theta^J = \underline{\underline{K_i}}^J \delta_\rho^i$

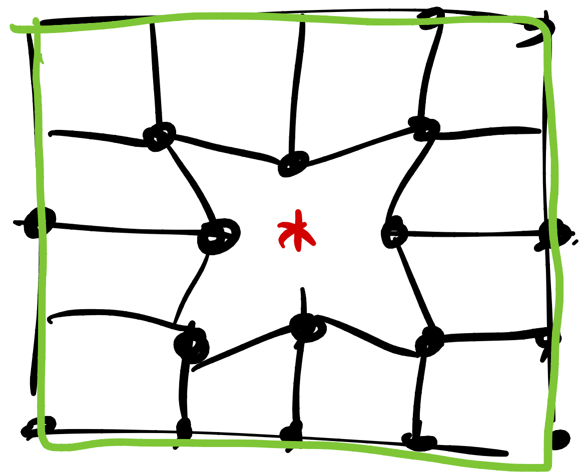
$$\Rightarrow \partial_\mu j^\mu = \frac{\nu}{8\pi^2} (j_d^I)^i K_i^J \epsilon_{IJ} \neq 0$$

if $\vec{j}^I \perp$ burgers vector, a^I.

a dislocation is a "lineon"

"vacancy"

- mobile
- stable.

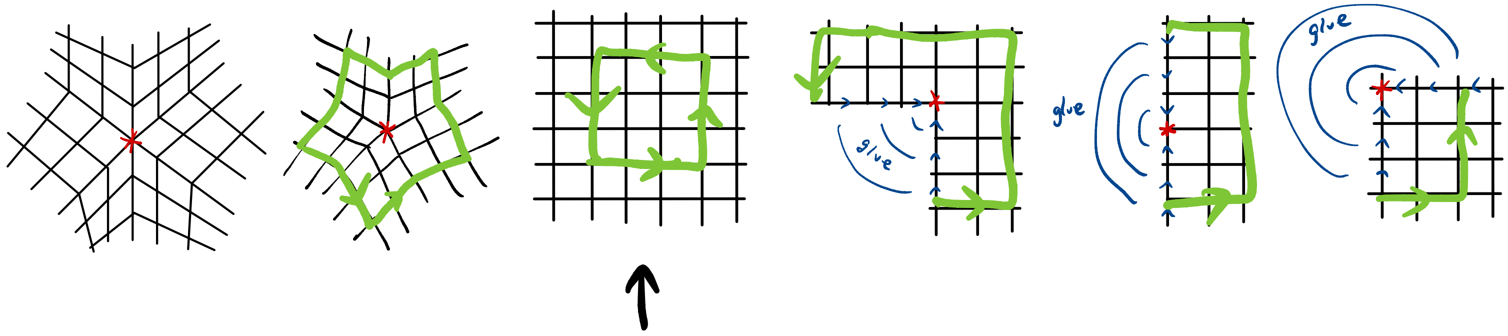


ex: describe in terms of $S[\theta, A]$.

Disclinations   Include rotations $\subset G$.

$$\text{defects} \overset{?}{\longleftrightarrow} \pi_{q-1}(G/H)$$

Square lattice.   codim 2. $\pi_1(G/H)$



$\cdots \quad q=-2 \quad q=-1 \quad q=0. \quad q=1 \quad q=2 \quad q=3$

$q=4$ is no lattice!

discliation charge & group.

$$\pi_1(V) = \pi_1\left(\frac{\text{translations} \ltimes \text{rotations}}{\Gamma \ltimes \text{point group}, K}\right)$$

$$= \Gamma \ltimes \hat{K} \qquad \text{nonabelian}$$

$\Rightarrow$ $\underbrace{\text{disclination} + \text{anti-disclination}}_{\text{dipole of discl. charge}} = \text{dislocation}.$

Square lattice:

$$\pi_1(V) = \mathbb{Z}^2 \rtimes \langle R \equiv \tfrac{\pi}{2} \text{ rotations} \rangle$$

$$= \{ (\vec{a}, R^k) \}$$

$$(\vec{a}, R^k)(\vec{a}', R^{k'}) = (\vec{a} + R^k \vec{a}', R^{k+k'})$$

$$(\vec{a}, R^k)^{-1} = (-R^{-k}\vec{a}, R^{-k})$$

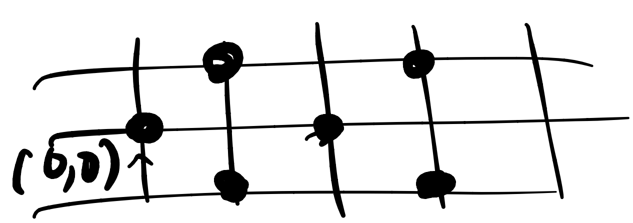
conj. class of $\pi/2$ dislocation $(\vec{0}, R)$

$$(\vec{a}, R^k)(\vec{0}, R)(\vec{a}, R^k)^{-1}$$

$$= (\vec{a} - \tilde{R}\vec{a}, R)$$

$$(\vec{0}, \underline{\underline{R}}) \simeq ((1,1), R) \simeq ((1,-1), R)$$

$$\simeq ((n,m), \underline{\underline{R}}) \text{ w/ } n+m \text{ even}$$

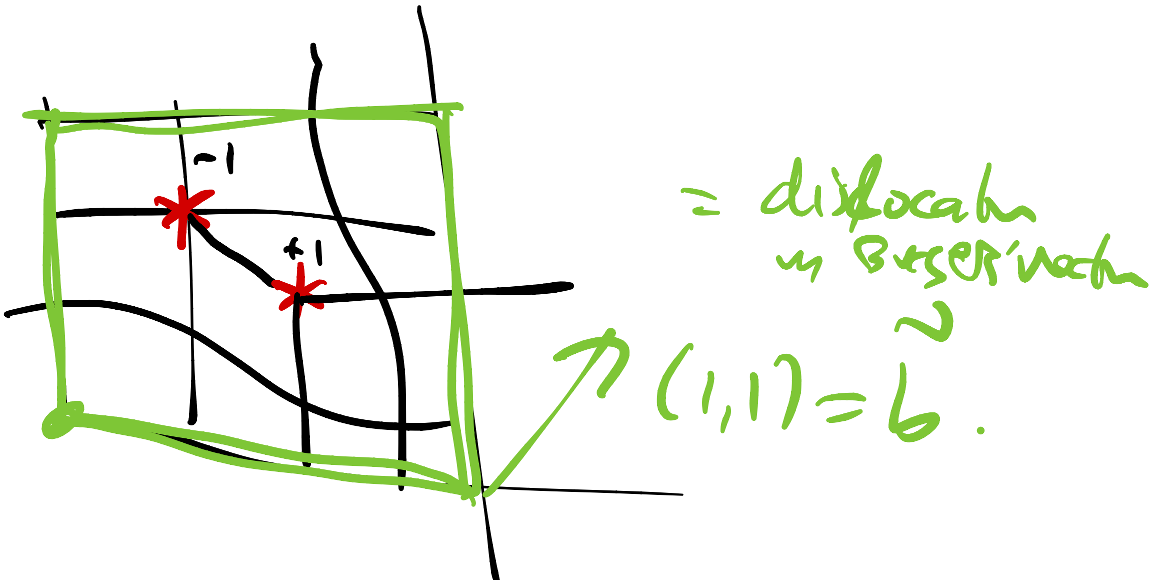


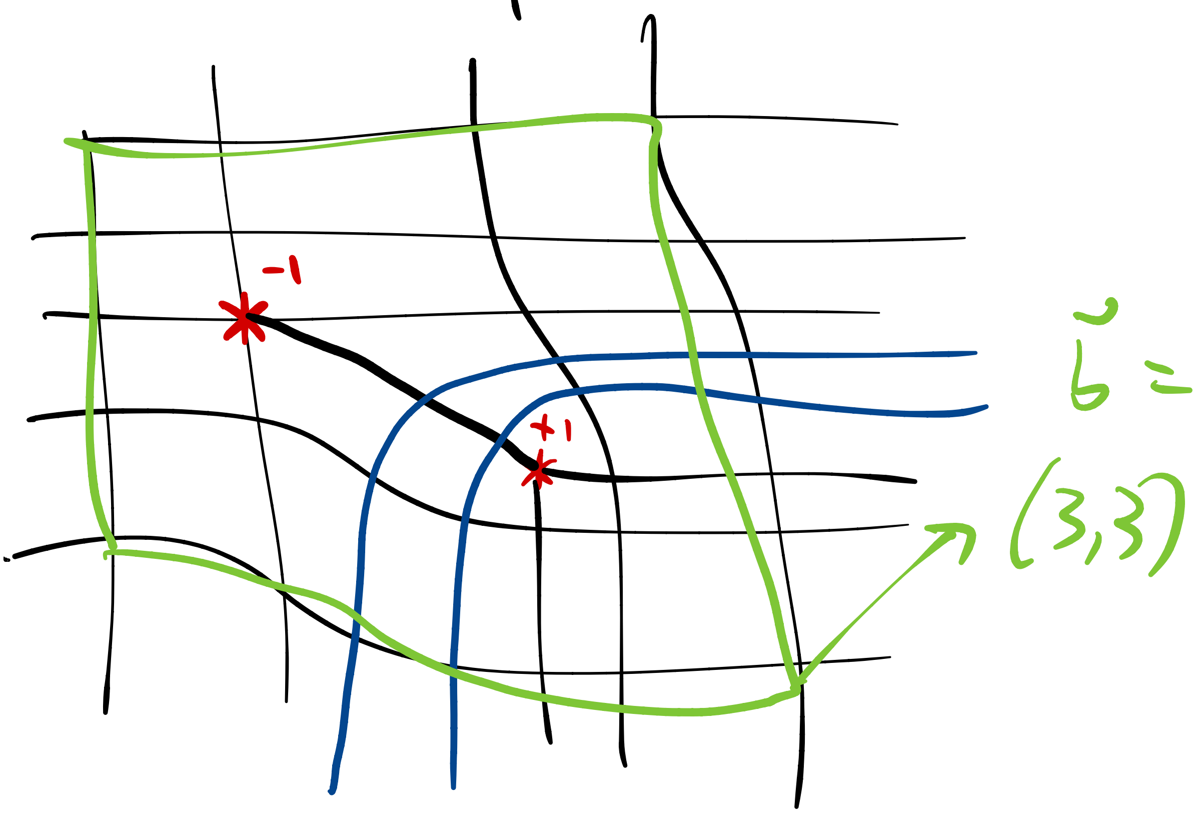
$\Rightarrow$ $\{ (0, R)$ and $(0, -R) \}$

have the same charge as

$(n+m, R^0)$ $n+m$ even

ie any dislocation is even Burgers vector.





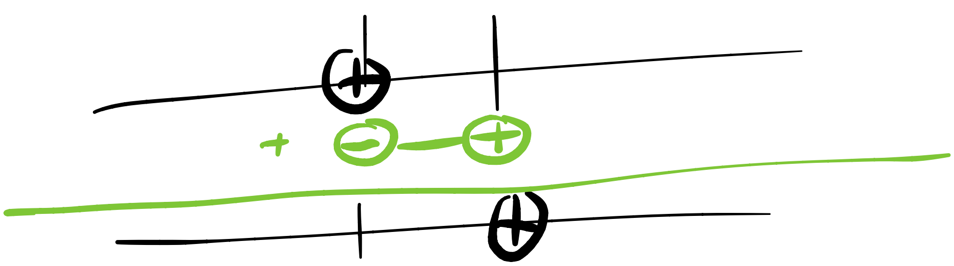
- An isolated disclination is immobile

  (even w/o $\partial^\mu j_\mu = 0$)

- A $+/-$ pair, a dipole

  can move $\perp$ dir. of separation.

- discl. charge is conserved
- so is disclination dipole moment



1.9 The boojum & relative homotopy

<u>^{3}He A</u> : $\quad A_{\alpha i} = \hat{d}_\alpha (e^{(1)} + i e^{(2)})_i$

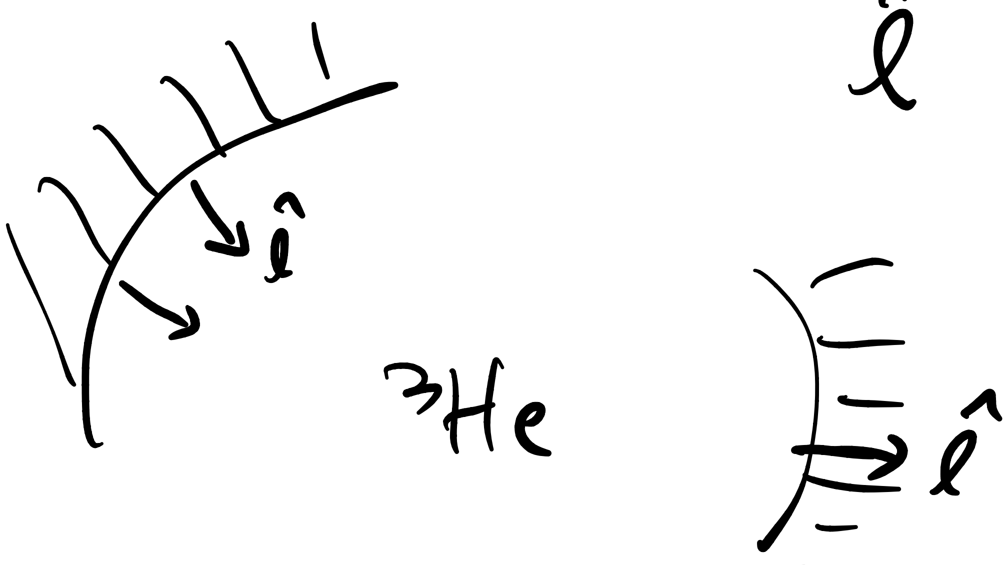
$\{ e^{(1)}, e^{(2)}, \hat{\ell} = e^{(1)} \times e^{(2)} \}$ form a ON frame

$$V_A = (S^2 \times SO(3))/\mathbb{Z}_2$$

("dipole-free phase")

<u>Put in a container</u> : Boundary condition :

$\hat{\ell} \parallel \hat{n}$ to bdy



on the bdy

A is restricted to

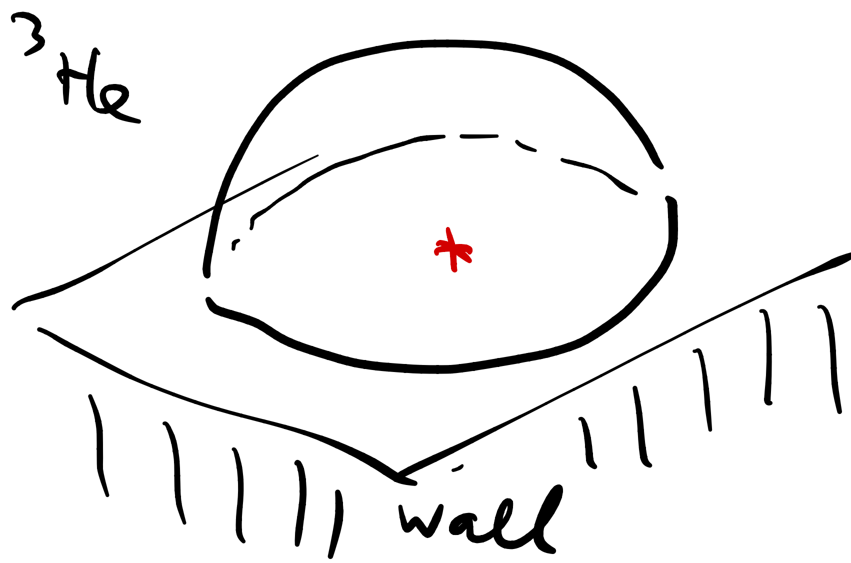
in or out

$$V_A^{wall} = \left(S^2 \times U(1)_\theta \times \mathbb{Z}_2\right)/\mathbb{Z}_2$$

$\phi\big|_{hemisphere}$:

hemisphere $\to V_A$

∂ hemisph $\to V_A^{wall}$

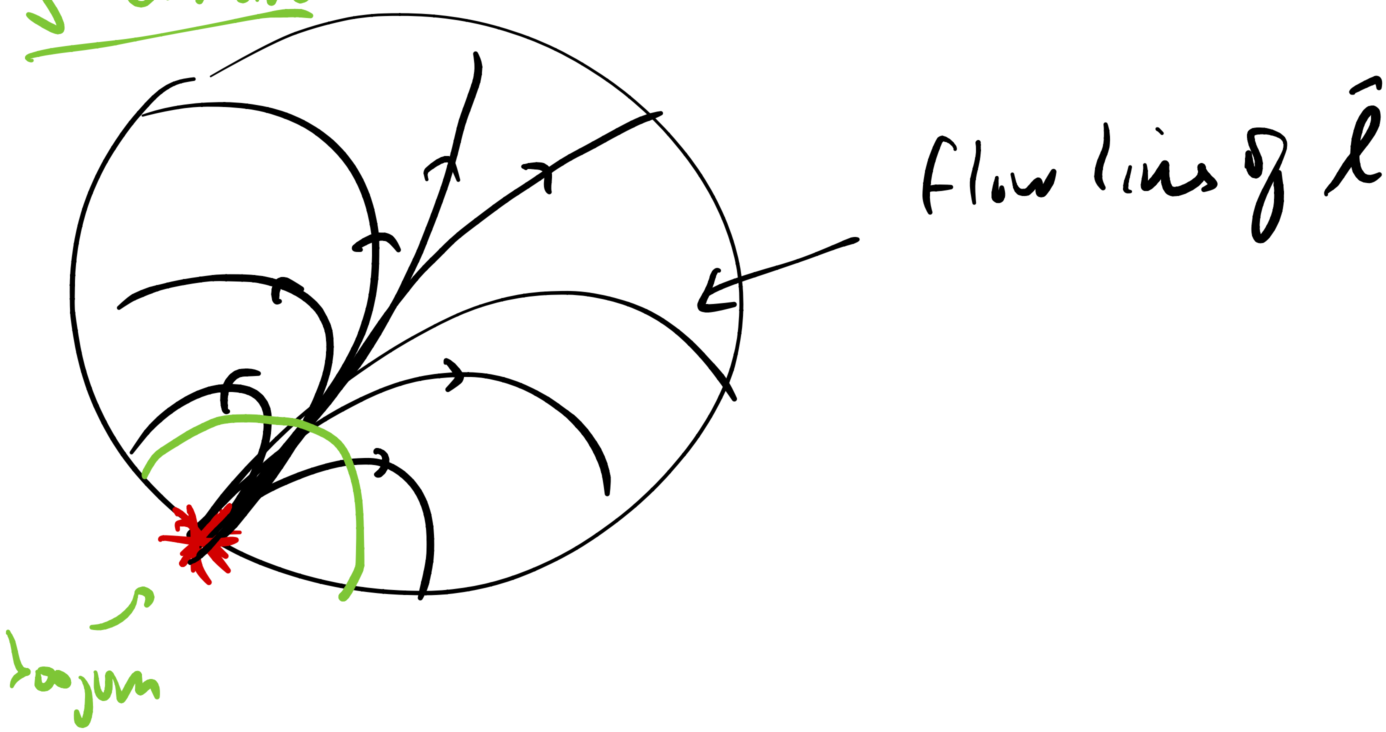


$$[\phi\big|_{hemisphere}] \in \pi_2(\underset{\sim}{V_A}, \underset{\sim}{V_A^{wall}})$$

long exact seq $\Longrightarrow \pi_2(V_A, V_A^{wall}) = \mathbb{Z}$.

$= \langle$ boojum $\rangle$

S^2 container:



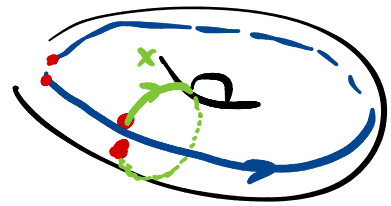
2. Some quantum Hall physics

"topological phase of quantum matter"

can mean
- protected edge modes
- topological order (TO)

TO:
1) fractionalization of quantum #s.
2) Robust g.s. degeneracy depending on topology of space.
3) Long-range entanglement

1) $\Rightarrow$ 2) : eg space = T^2

$W_x =$  $= W_y$

$W_x |gs\rangle = |gs'\rangle$

assume anyons $\Rightarrow$ $[W_x, W_y] \neq 0$.

2) $\Rightarrow$ 1) If gsd is geometry-indep

$W_x |gs\rangle = |gs'\rangle$

can associate $\{W\}$ with cycles in space.

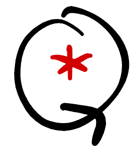
& interpret as ferrying anyons.

(loophole: in type II fractons W is supported on a fractal.)

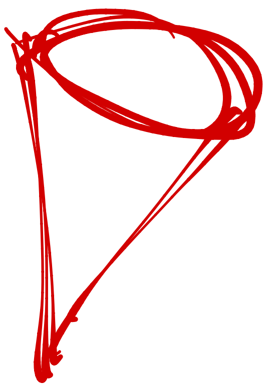
[Else 2103 . . .]

$\theta^I = 1 \ldots D > d$

$$b^I = \frac{1}{2\pi} \oint_C \frac{\partial \theta^I}{\partial x^i} dx^i \in \mathbb{Z}^D$$



$q > 0$:



$q < 0$

$R \propto q f^2(x)$

$$2 - 2g = \int R/2\pi$$

In $\mathbb{H}^d$,

$G/H = ?$

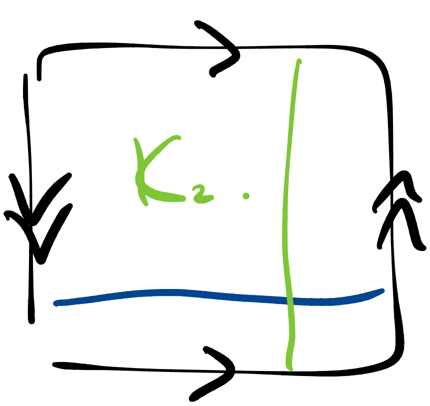
$G/H = \mathbb{R}^d/\Gamma$

$$e^{iS_{eff}[\theta, A]} = \int D(\text{stuff})\, e^{iS_{micro}[\text{stuff}, \theta, A]}$$

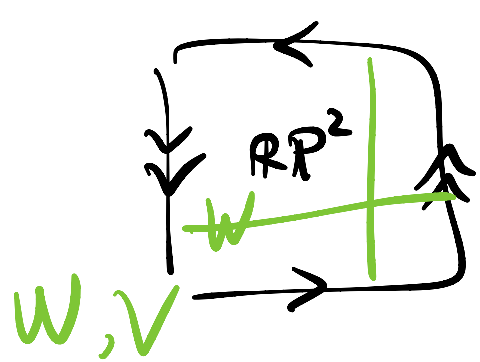
ψ, ϕ

a) $\psi(x^i + u^i)$

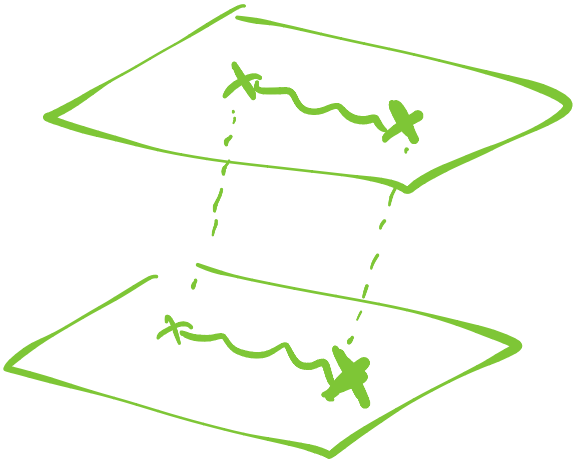
b) derivative coupling of ψ ...

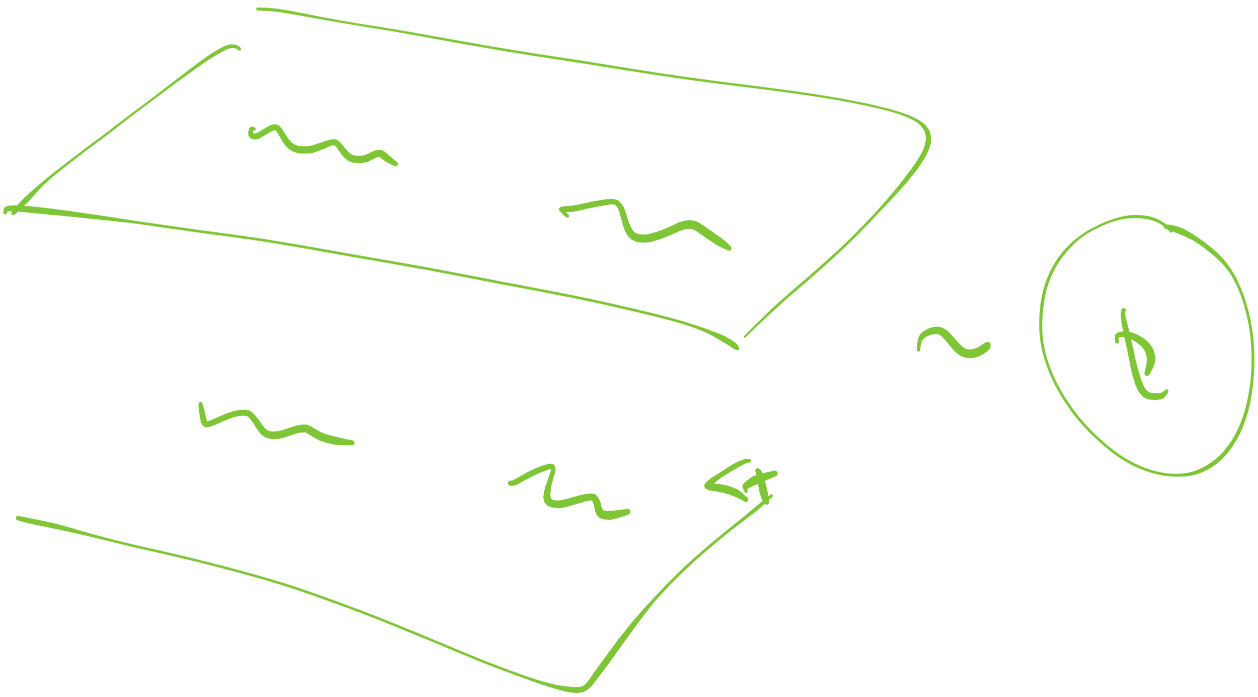


$V_x W_y = -W_y V_x$



Qi, Jian,
Barkeshli





$$y^2 = W(x)$$

$$V = \left\{ \sum_{a=1}^{2} \left(\mathrm{Re}\,\phi_a^2 + \mathrm{Im}\,\phi_a^2 \right) = v^2 \right\}$$

$$\cong S^3$$

$$\Rightarrow \pi_1(V) = \{e\}$$

vs $\quad V = SU(2) \times U(1)_Y / U(1)_Q$

$$\mathcal{L} \ni \left| \left(\partial_\mu + \underbrace{\cos\theta_W B_\mu + \sin\theta_W Y_\mu}_{\equiv Q_\mu^\perp} \right) \phi \right|^2$$

$$- \lambda \left(\sum |\phi|^2 - v^2 \right)^2$$

$$\phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \xrightarrow{U(1)_{T^3}} \begin{pmatrix} 0 \\ e^{i\cos\theta_W \alpha} v \end{pmatrix} \xrightarrow{U(1)_Y} e^{i\sin\theta_W \beta} \begin{pmatrix} v \\ \cdots \end{pmatrix}$$

$$H \equiv H_{\phi_0} = U(1)_Q \qquad Q = \sin\theta_W T_3 - \cos\theta_W Y$$

$$\left\{\begin{array}{l} \phi \xrightarrow{SU(2)} \underline{e^{i\cos\theta_W \frac{\vec{\sigma}}{2}\cdot\vec{\alpha}}\phi} \quad ? \\ B^A_\mu \to B^A_\mu - \partial_\mu \alpha^A \end{array}\right.$$

$$(\partial_\mu + B_\mu + \tan\theta_W Y_\mu)\phi$$

$$\left[\frac{SU(2)\times U(1)_Y}{U(1)_Q} \cong S^3 \quad ? \right.$$

G acts transitively on V $\Rightarrow$ $\underline{V = G/H}$

$$SU(2)\times U(1)_Y \ni \left(e^{i\cos\theta_W \alpha\cdot\frac{\vec{\sigma}}{2}}, e^{i\sin\theta_W\beta}\right)$$

$$\gamma \in U(1)_Q \longrightarrow \left(e^{i\cos\theta_W \alpha\cdot\frac{\vec{\sigma}}{2} + i\sin\theta_W\gamma\frac{T_3}{2}}, e^{i\sin\theta_W\beta - i\cos\theta_W\gamma}\right)$$

$$2\pi\mathbb{Z} \ni \sin\theta_W\beta - \cos\theta_W\gamma \Rightarrow \boxed{\gamma = \cancel{\tan\theta_W\beta} + \dots}$$

$$2\pi n = \sin\theta_W \beta - \cos\theta_W \gamma$$

$$\Rightarrow \boxed{\gamma = \tan\theta_W \beta - \frac{2\pi n}{\cos\theta_W}}$$

$$U(1)_Q \ni e^{i\gamma(\sin\theta_W T_3 - \cos\theta_W Y)}$$

$$= e^{i\tan\theta_W \beta (\;\;)} \underbrace{e^{-i2\pi n \tan\theta_W T_3}}$$

$$V \stackrel{?}{=} \underline{SU(2)/\mathbb{Z}} .$$