Physics 211C (239):

Phases of Quantum Matter

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OH: after lecture or by email request
or email Q's.

Work: (A) psets < weekly
submit via canvas
typeset preferred. ($\exists$ template)

(B) brief final paper 2 page PRL format

(C) find typos & email me.

Zoom: please video on!

0.1 Goals.

phases of matter $= \left\{ \underset{\sim\sim\sim\sim\sim}{\underline{\text{macroscopic}}\ \text{piles of stuff}} \right\} \Big/ \sim$

$A \sim B$ if <u>adiabatically connected</u>

by deforming H & adding <u>trivial stuff</u>.

"stable equivalence"

(K theory)

what is $\pi_0(\text{stuff})$?

<u>Topological invariant</u> = can't change continuously.

<u>eg</u>: # of groundstates eg of an Ising magnet

unbroken phase ———————— broken phase →

$\# = 1$ $\# = 2$

Goal: understand such topological labels on phases of matter as concretely as possible.

(some of them exist on this planet!)

Topics:

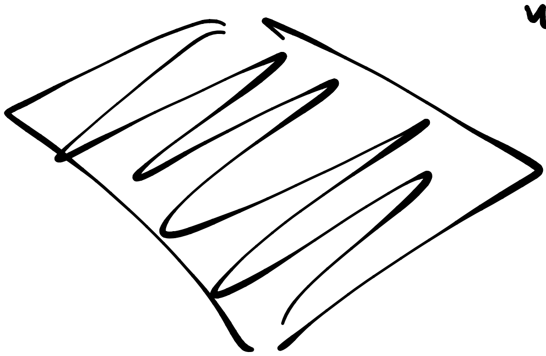
- Defects of ordered media. [Mermin]
- Symmetry-protected topological phases (SPTs)
- Lieb-Schultz-Mattis theorems.

⋮

? FQH ?

1 Defects & textures of ordered media

1.1 Landau-Ginzburg theory of ordering



w/ symmetry G.

eg: $G = U(1)$

(superfluid, particle #
or planar magnet (S_x, S_y))

Quantumly: $[Q, H] = 0$. spectrum of $Q \in \mathbb{Z}$.

SSB: $e^{iQ\phi}|0\rangle = |\phi\rangle$ another g.s. w/ same energy

ϕ = constant $\in [0, 2\pi)$

$$|\phi(x)\rangle \equiv e^{iQ\phi(x)}|0\rangle$$ has $E \propto \partial\phi$

$$E[\phi] = \langle \phi(x) | \hat{H} | \phi(x) \rangle.$$

$$Q = \int d^d x \; j^0(x) \qquad \text{(assumption)}$$

$$\text{"} Q\,\phi(x) \text{"} \equiv \int d^d x \; \underline{\underline{j^0(x)}} \; \underline{\underline{\phi(x)}}$$

<u>Goldstone's theorem:</u>

SSB of continuous sym $\Rightarrow$ gapless mode

Let $\Phi(x)$ transform <u>linearly</u> under G.

$$\Phi(x) \to e^{i\alpha}\,\Phi(x)$$

eg $\Phi(x) = S_x(x) + i S_y(x)$

$$F_{LG}[\Phi] = \int_{\text{space}} d^d x \left[\underbrace{V(|\Phi|)}_{\text{0 derivs}} + \kappa \underbrace{\vec{\nabla}\Phi^* \cdot \vec{\nabla}\Phi}_{\text{2 derivs}} + \text{terms w/ more derivatives} \right]$$

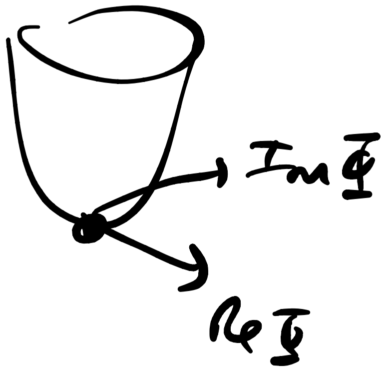
more

- local
- U(1) symmetric
(- rotation sym)

- Φ is slowly varying

$$V(|\Phi|) = r|\Phi|^2 + u|\Phi|^4 + \cdots$$

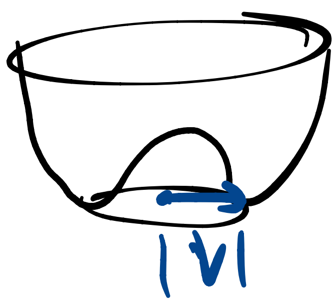
$r > 0$



unique min at $\Phi = 0$

UNBROKEN

★ $r < 0$



ring of minima at $\Phi \neq 0$.

BROKEN.

you are here.

$\left(v = |r|/2u \right)$

One perspective on $F_{LG}[\Phi]$: mean field theory.

ansatz $\quad |\Phi\rangle = \bigotimes_x \underbrace{|\Phi(x)\rangle_x}$

$$F_{LG}[\Phi] \sim \langle \Phi | H | \Phi \rangle$$

If H is local, $F_{LG} = \int d^dx \left(\ldots \right)$

$$\left(H = \int d^dx \, H_x \right)$$

$$f_{LG}\left[\Phi = v e^{i\phi(x)}\right]$$

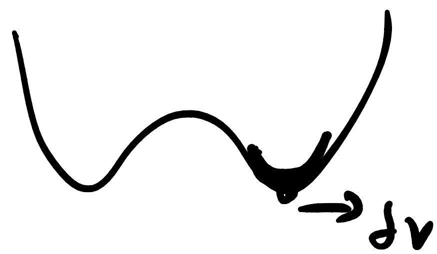
$$= \int_{\text{space}} d^d x \left(\underline{\underline{\underline{\rho}}} \, (\nabla \phi)^2 + \text{terms w/ more derivs.} \right)$$

No potential for the goldstone mode $\phi(x)$.

(if instead $\Phi = (v + \delta v(x))\, e^{i\phi}$

$F = \ldots \quad (\delta v(x))^2$)

gradients of ϕ cost energy

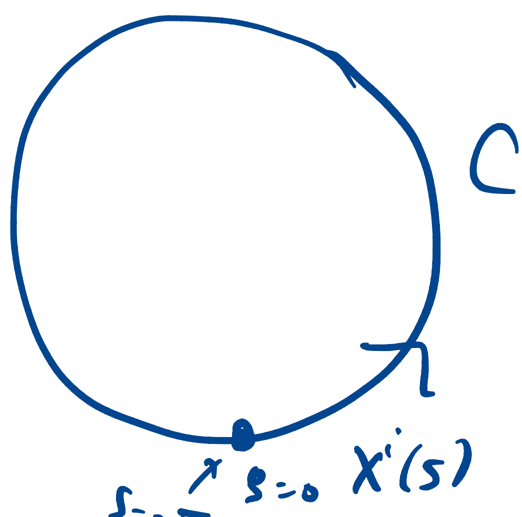


$\equiv$ "generalized rigidity"

(absent in the unbroken phase $\langle \Phi \rangle = 0$.)

Vortices

$s \in [0, 2\pi)$



given: $\phi(x)$ phase of order parameter $\Phi = e^{i\phi}$

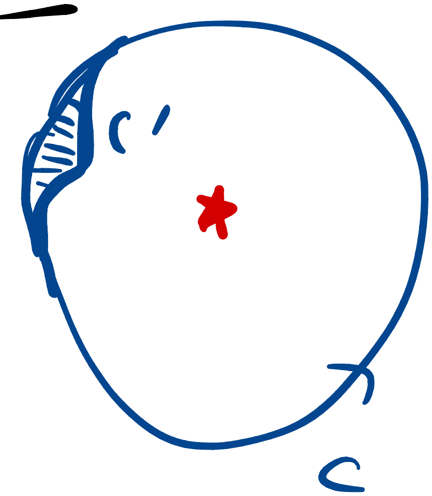
$$2\pi\, w[C] \equiv \oint_C d\phi = \oint \frac{\partial \phi}{\partial x^i}\, dx^i = \int_0^{2\pi} \frac{\partial \phi}{\partial x^i} \frac{dx^i}{ds}\, ds$$

$$\stackrel{\text{FTC}}{=} \phi(x(s))\Big|_{s=0}^{s=2\pi} \in 2\pi\mathbb{Z}.$$

$$\phi \simeq \phi + 2\pi.$$

Suppose $w[C] \equiv w \neq 0$.

Fix $\phi(x)$, vary C.



{pts across which $w[C]$ jumps}

= vortices.

$\Rightarrow$ ϕ is not well-defined there

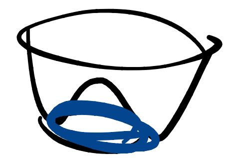
If $\Phi(x_0) = 0$ $\Rightarrow$ ϕ is not well-def'd at x_0.

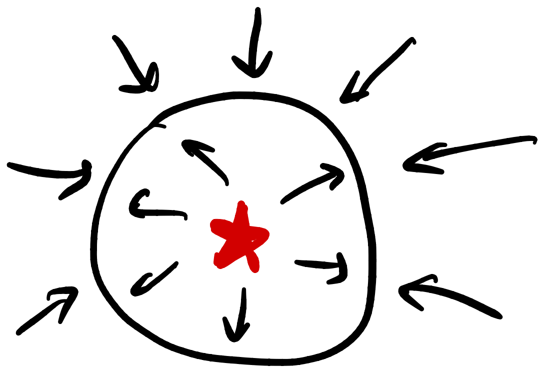
Vortices occur at codim 2 $\equiv$ must specify 2 coords to specify locatn.

In $d=2$ vortices are particles

In $d=3$ " " strings.

$$\phi : \mathbb{R}^2 \setminus \text{vortices} \longrightarrow S^1$$





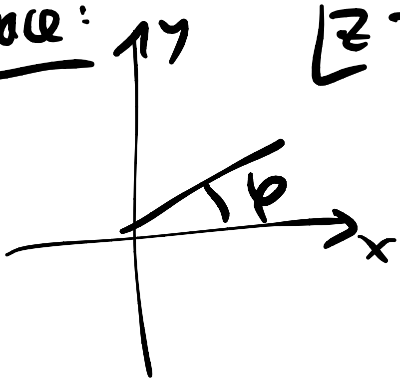
if all vortices in a clump at 0

$$\mathbb{R}^2 \setminus \{0\} \simeq S^1$$

$$\phi : S^1 \longrightarrow S^1$$

$$[\phi] \in \pi_1(S^1) = \mathbb{Z}. \Rightarrow w\left[\text{unit circle}\right]$$

space:



$z = x + iy = re^{i\varphi}$

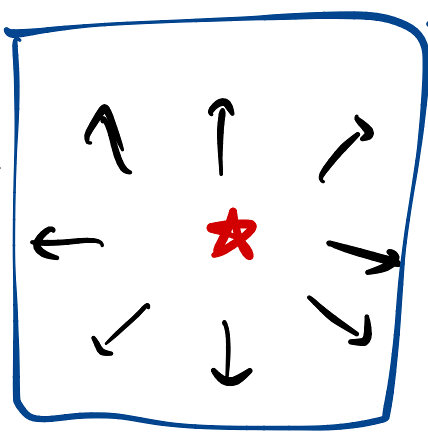
- If $\phi = \phi_0$

  $w[C] = 0$.

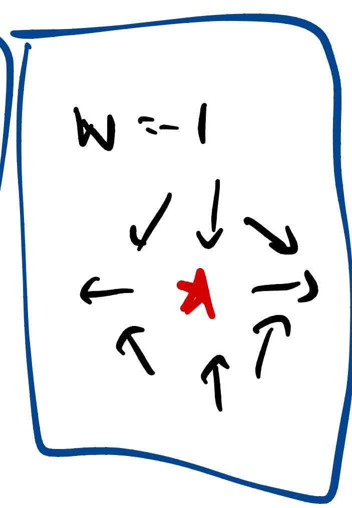
- If $\phi = \phi_0 + w\varphi$

  $w\left[\text{unit circle}\right] = w.$

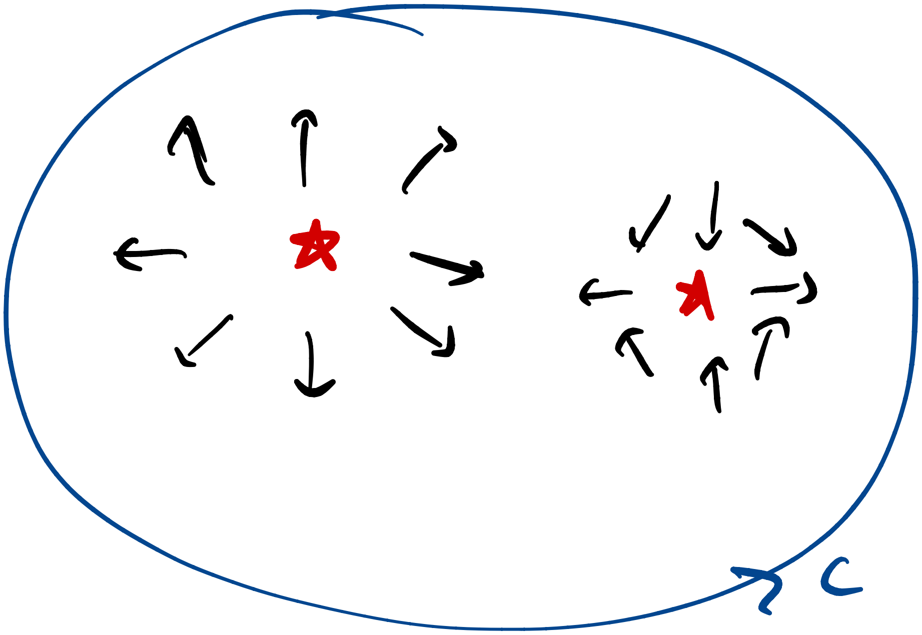
$w = +1$:



vortex



antivortex.



$W[c] = 0.$

<u>Energetics:</u> $F^{\circ}_{LG}[\phi = \varphi] = \int_L \rho(\partial\phi)^2 \Big|_{\phi=\varphi}$

$= \log(L) \longrightarrow \infty$ in Thermodyn limit.

But:

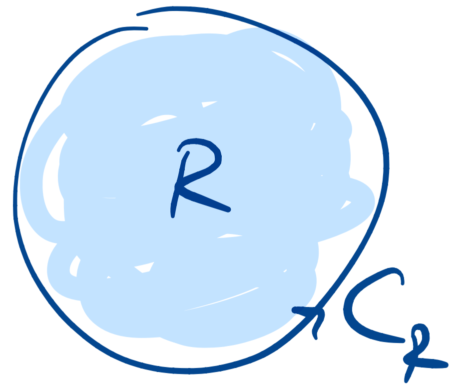
- V-$\bar{V}$ pair has finite energy
- vortex in the <u>abelian Higgs model</u> has finite energy.

(superconductor)

$$2\pi W[C] = \oint_{C_R} d\phi \qquad \Phi = v e^{i\phi}$$

$$d\phi = -\tfrac{1}{2} i\, \Phi^* d\Phi + \tfrac{1}{2} i\, d\Phi^* \Phi$$

$$\overset{\text{Stokes}}{=} \underbrace{\int_R d\left(\quad \downarrow \quad \right)}_{=\ \text{density of vortices}}$$



$$\partial R = C_R$$

$W[C] = \#$ of vortices in R

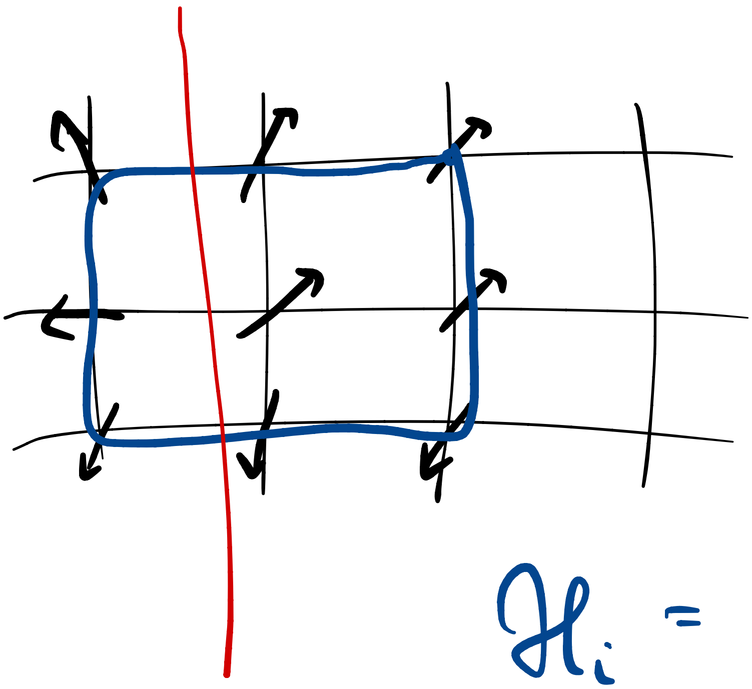
$$j_0^v(x) = \frac{1}{2\pi} \epsilon_{ij} \partial_i \left(-\tfrac{1}{2} i\, \Phi^* \partial_j \Phi + h.c. \right)$$

density of vortices at x.

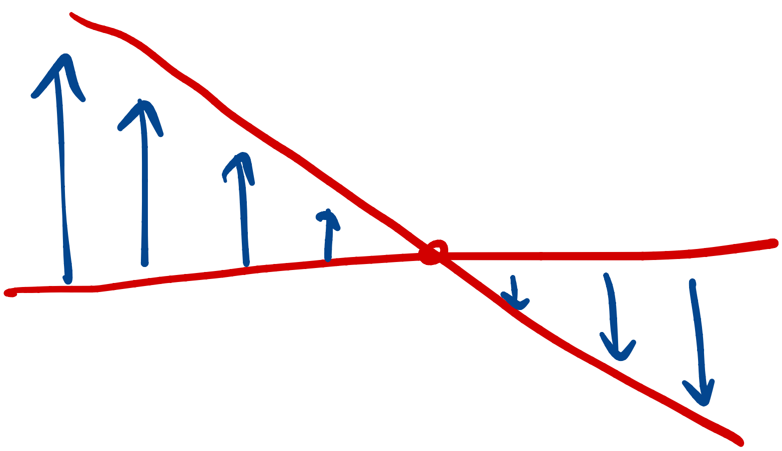
$$= \int_R d^2\phi \qquad \boxed{?}$$

vs $d^2 = 0$.

$\phi_i \in S^1$ at each lattice site i.



$$\mathcal{H}_i = \text{span}\left\{ |\phi\rangle \right\}_{\phi \in [0, 2\pi)}$$



$$S[\Phi] = \int \partial_\mu \Phi^* \partial^\mu \Phi \qquad \underline{z=1}$$

$$\rightarrow H = \int \Pi^2 + |\nabla \Phi|^2$$

or

$$\left[\int \Phi^* i \partial_t \Phi - |\nabla \Phi|^2 \right. \qquad \underline{z=2}.$$

$$G(T) \sim e^{-1/TL} \qquad L = \text{radius of curvature}$$

$$\underline{S[\phi] = \int d^d x \sqrt{g}\, \partial_\mu \phi\, \partial_\nu \phi\, g^{\mu\nu}}$$

$$ds^2 = \underline{e^{+2y}}\, dx^2 + dy^2$$

$$\sqrt{g} \equiv \sqrt{\det g} = e^{(d-1)y}$$

$$S = \int d^{d-1}x\, dy\; e^{(d-1)y} \left[e^{-2y}\left(-p^2\phi^2\right) + (\partial_y \phi)^2 \right]$$

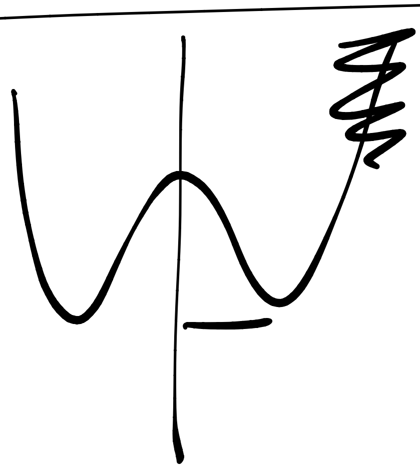
$$0 = \frac{\delta S}{\delta \phi_{p,y}} = \left[-\partial_y \left(e^{(d-1)y}\right) \partial_y \phi - p^2 \phi_p\, e^{(d-3)y} \right]$$

$$\partial_y^2 \phi + \# \partial_y \phi + p^2 \phi = 0$$

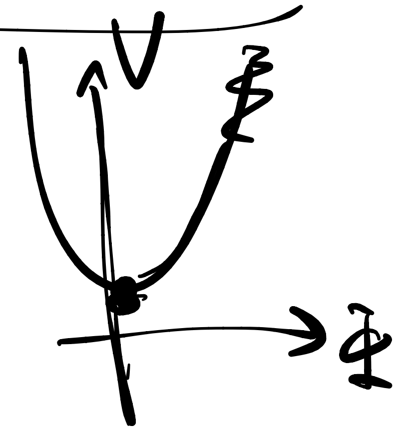
$$\phi(x) = e^{ikx}$$

$$\partial \phi \sim k$$

$r<0$



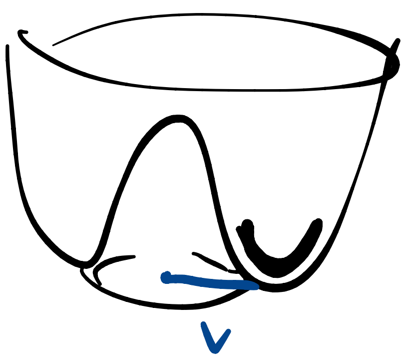
$r>0$



eg: $|\Phi(i)\rangle_i = \Phi_x(i)|\sigma^x=+\rangle_i + \Phi_y(i)|\sigma^y=+\rangle_i$

$$\sigma^x|\sigma^x=+\rangle = |\sigma^x=+\rangle$$

$$|\Phi\rangle = \bigotimes_{i\in \text{space}} |\Phi(i)\rangle_i$$

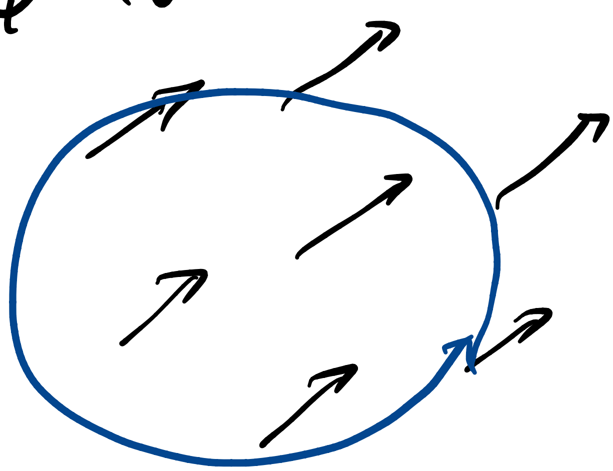


$V(v)=0$.

ex:

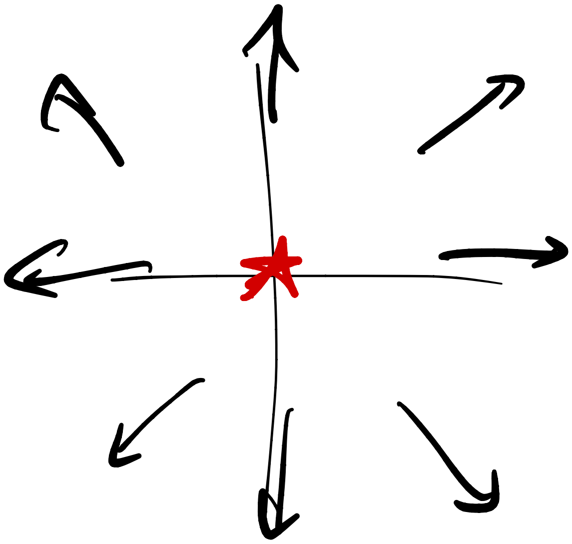
$F_{LG}[\Phi(x)=\rho(x)e^{i\phi(x)}]$

$\phi=\phi_0$

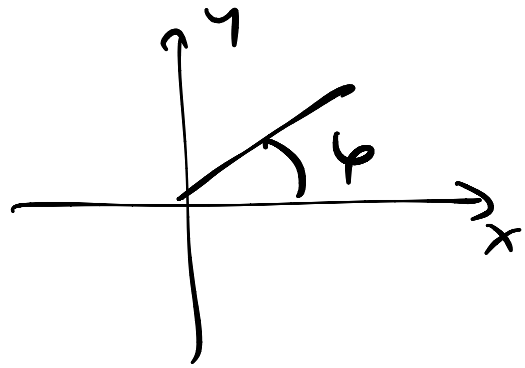


$d\phi=0 \to \oint d\phi=0$

$$\phi = \underline{\underline{w}}\,\varphi$$



$$x + iy = re^{i\varphi}$$



$$d\phi = d\varphi$$

$$\oint_{\substack{\text{unit}\\ \text{circle}}} d\phi = \int_0^{2\pi} d\varphi$$

$$= 2\pi$$

single-valued
under $\varphi \to \varphi + 2\pi$

$$\Longrightarrow \underline{w \in \mathbb{Z}}\,.$$