$$|\psi\rangle = \frac{1}{\sqrt{n^\psi_\alpha!}} b^+(u_1)\cdots b^+(u_N)|0\rangle$$

$$B(x,x',y,y') \equiv \langle\psi| b^+_x b^+_{x'} b_y b_{y'} |\psi\rangle$$

$$= +\underline{G_\psi(x,y)G_\psi(x'y')} + G_\psi\underline{(x,y')G_\psi(x',y)}$$

$$-\sum_\alpha n^\psi_\alpha(n^\psi_\alpha+1)u^*_\alpha(x)u^*_\alpha(x')u_\alpha(y)u_\alpha(y').$$

no wick!

Pair correlator for (spinless) bosons in free space:

$$|\Phi\rangle = |n_{p_0} n_{p_1}\cdots) = \frac{(b^+_{p_0})^{n_{p_0}}}{\sqrt{n_{p_0}!}}\cdots|0\rangle$$

w $\rho(x) = \langle\Phi| b^+_x b_x|\Phi\rangle = \frac{1}{V}\sum_p n_p = n.$

Pair correlator:

$$n^2 g_B(x-y) \equiv \langle\Phi| b^+_x b^+_y b_y b_x |\Phi\rangle$$

$$= \frac{1}{V^2}\sum_{pp'qq'} e^{-i(p-p')\cdot x - i(q-q')y} \langle\Phi| b^+_p b^+_q b_{q'} b_{p'} |\Phi\rangle$$

$$\langle \Phi | b_p^\dagger b_q^\dagger b_{q'} b_{p'} | \Phi \rangle = \delta_{pq} \delta_{pq'} \delta_{qq'} \langle \underbrace{(b_p^\dagger)^2 b_p^2}_{= \hat{n}_p (\hat{n}_p - 1)} \rangle_\Phi$$

$$+ (1-\delta_{pq}) \delta_{pp'} \delta_{qq'} \langle \underbrace{b_p^\dagger b_q^\dagger b_p b_q}_{= \hat{n}_p \hat{n}_q} \rangle_\Phi$$

$$+ (1-\delta_{pq}) \delta_{pq'} \delta_{qp'} \langle \underbrace{b_p^\dagger b_q^\dagger b_q b_p}_{= \hat{n}_p \hat{n}_q} \rangle_\Phi$$

$$= (1-\delta_{pq}) (\delta_{pp'} \delta_{qq'} + \delta_{pq'} \delta_{qp'}) n_p n_q$$

$$+ \delta_{pq} \delta_{pp'} \delta_{qq'} n_p (n_p - 1)$$

$$= \left(\underline{\underline{\delta_{pp'} \delta_{qq'}}} + \delta_{pq'} \delta_{qp'} \right) n_p n_q + \delta_{pq} \delta_{pp'} \delta_{qq'} \underbrace{\left(-2 n_p^2 + n_p (n_p - 1) \right)}_{-n_p (n_p + 1)}$$

$$\rightarrow n^2 g_B (x-y) =$$

$$\frac{1}{V^2} \left(\underbrace{\sum_p n_p}_{N} \sum_q n_q + \sum_p n_p e^{-i p \cdot (x-y)} \sum_q n_q e^{+iq(x-y)} - \sum_p n_p (n_p + 1) \right)$$

$$= n^2 + \left| \frac{1}{V} \sum_p n_p e^{-i p \cdot (x-y)} \right|^2 - \frac{1}{V^2} \sum_p n_p (n_p + 1)$$

eg 1: $|\Phi\rangle = |\Phi_0\rangle = |n_{p_0} = N\rangle$

$$\to \quad n^2 g_B(x-y) = n^2 + n^2 - \frac{N(N+1)}{V^2}$$

$$= \frac{N(N-1)}{V^2} \xrightarrow[V\to\infty]{N\to\infty} n^2 .$$

fixing $n = N/V$

"thermodynamic limit".

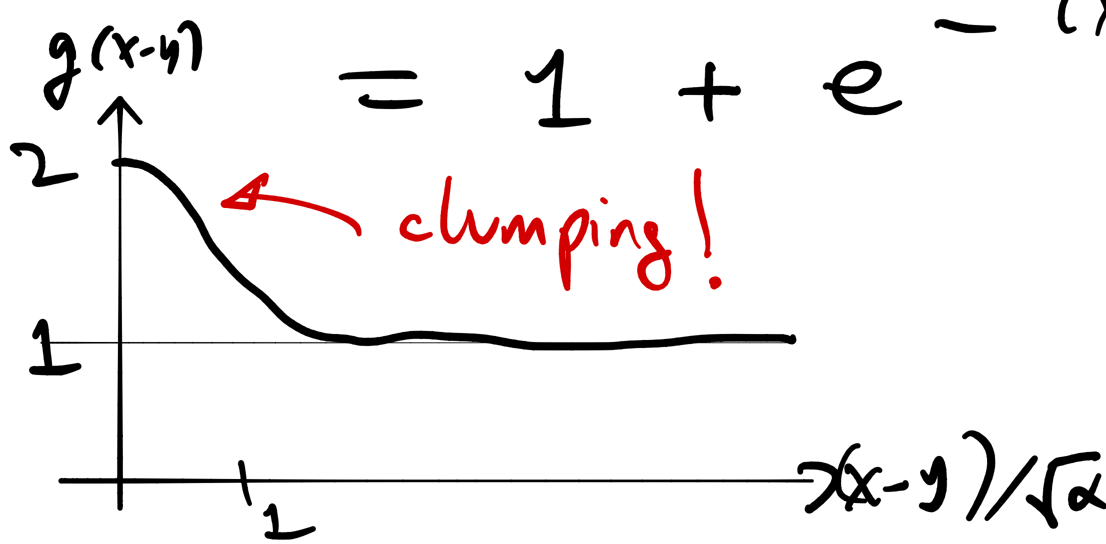
eg 2: A beam of particles

$$n_p = c\, e^{-\alpha(p-p_0)^2/2}$$

$$n = \int d^3p\, n_p \implies c = c(n,\alpha)$$

$$g(x-y) \simeq \frac{1}{n^2}\left(n^2 + \left|\int d^3p\, n_p\, e^{-ip\cdot(x-y)}\right|^2\right)$$

$$= 1 + e^{-(x-y)^2/\alpha} .$$



Intensity Interferometry

Ingredients:

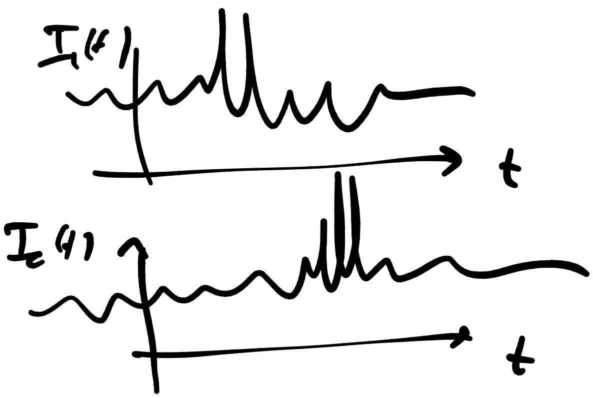
- A beam of incoherent light.

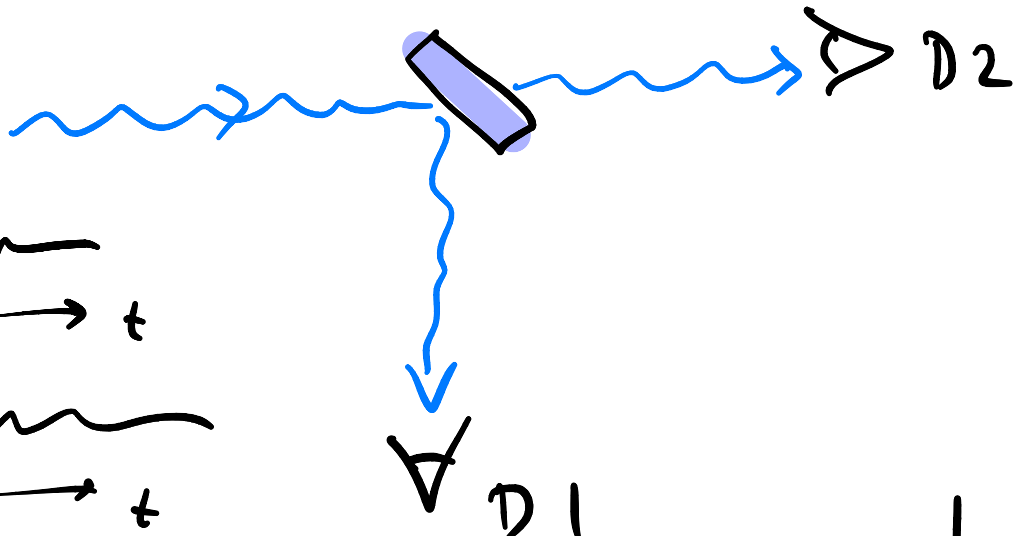
  eg: a distant star.

- A beam-splitter. (half-silvered mirror)

- Two detectors.

Recipe:



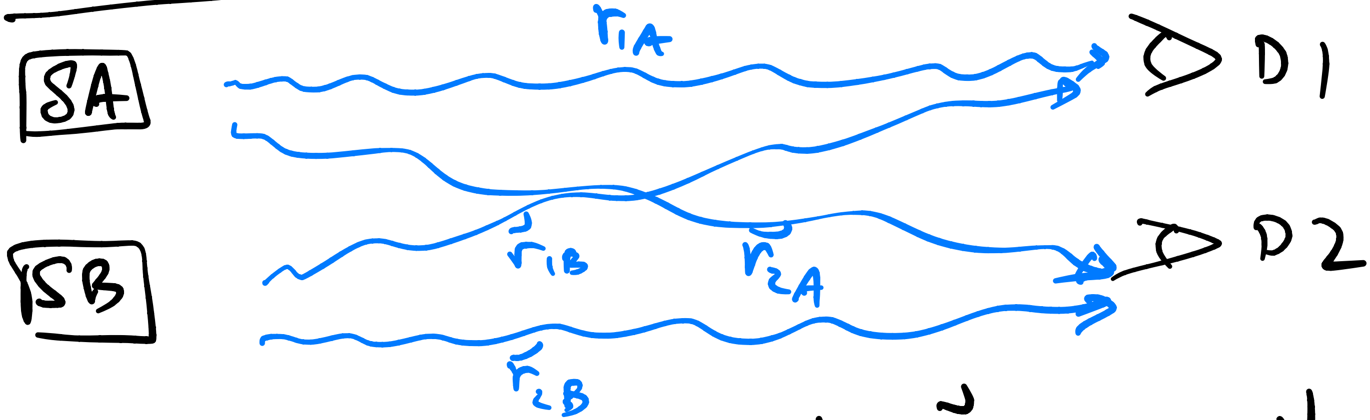


$$I_1(t)\, I_2(t+\tau) \propto \text{Prob}\left(\begin{array}{c}\text{observe } \gamma \text{ at} \\ \text{D2 at time} \\ t+\tau\end{array} \,\middle|\, \begin{array}{c}\text{observed } \gamma \\ \text{at D1 at} \\ \text{time } t\end{array}\right).$$

$$\overline{I_1 I_2} = \int dt\, I_1(t)\, I_2(t+\tau)$$

$$= \bar{I}_1\, \bar{I}_2\, g_B(\tau/_c) = 1 + e^{-(\tau/_c)^2/\alpha}$$

why: light is a beam of bosons.



$$A_{at\,1} = \alpha_1 e^{ik_A\cdot\vec{r}_{1A}} + \beta_1 e^{ik_B\cdot\vec{r}_{1B}}$$

$$A_{at\,2} = \alpha_2 e^{ik_A\cdot\vec{r}_{2B}} + \beta_2 e^{i\vec{k}_B\cdot\vec{r}_{2B}}.$$

α_i, β_i have $\boxed{\text{random}}$ phases

$$\Longrightarrow \overline{\alpha_i} = \overline{\beta_i} = 0.$$

$$\left(\text{But } \overline{\alpha^*\alpha} \neq 0.\right)$$

$$I_{i=1\sim 2} = |A_{at\,i}|^2 = |\alpha|^2 + |\beta|^2 + 2\,\mathrm{Re}\,\alpha^*\beta\, e^{i(k_B\cdot r_{iB} - k_A\cdot r_{iA})}$$

$$\overline{I_i} = |\alpha|^2 + |\beta|^2.$$

$$I_1 I_2 = |A_1 A_2|^2$$

$$= \Big| \alpha^2 \underbrace{e^{ik_A\cdot(r_{1A}+r_{2A})}}_{\text{Both particles come from A}} + \beta^2 \underbrace{e^{ik_B(r_{1B}+r_{2B})}}_{\text{Both come from B}}$$

$$+ \alpha\beta\Big(e^{\underbrace{ik_A\cdot r_{1A}+ik_B r_{2B}}_{\substack{A\to 1\\ B\to 2}}} + e^{\underbrace{ik_B r_{1B}+ik_A r_{2A}}_{\substack{A\to 2\\ B\to 1}}}\Big)\Big|^2$$

$$\overline{I_1 I_2} = |\alpha|^4 + |\beta|^4 + |\alpha\beta|^2 \left| e^{ik_A r_{1A}+ik_B r_{2B}} + e^{ik_B r_{1B}+ik_A r_{2A}} \right|^2$$

$$= \bar{I}_1 \bar{I}_2 + 2|\alpha|^2|\beta|^2 \cos\big(k_B(r_{1B}-r_{2B}) - k_A(r_{1A}-r_{2A})\big)$$

$r_{1A}-r_{2A} \simeq r_{1B}-r_{2B} \equiv r$

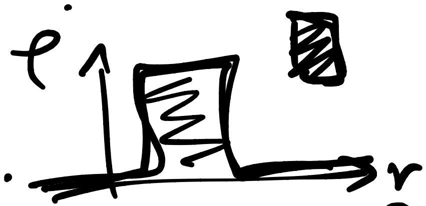
$$\simeq \bar{I}_1 \bar{I}_2 + 2|\alpha|^2|\beta|^2 \cos\big((k_B-k_A)\cdot r\big)$$

<u>Suppose:</u> $|\alpha_k| = c\, e^{-\frac{\alpha(k-k_0)^2}{2}} = |\beta_k|$

$$\frac{\overline{I_1 I_2}}{\overline{I_1}\,\overline{I_2}} = \frac{c^2}{\overline{I_1}\,\overline{I_2}} \int d^3k_A \int d^3k_B\, e^{-\frac{\alpha(k_A-k_0)^2}{2} - \frac{\alpha(k_B-k_0)^2}{2}} \left(2 + \mathrm{Re}\, e^{i(k_B-k_A)\cdot r}\right)$$

$$= 1 + e^{-r^2/\alpha}$$

∘ <u>Improvement</u>: SA, SB $\longrightarrow$ $\underline{\rho(r)}$.



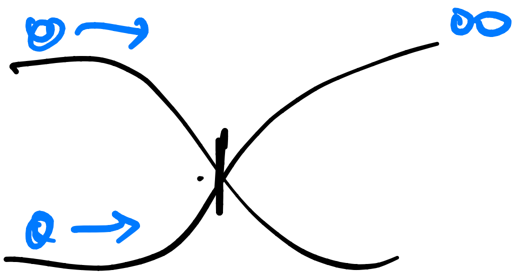
$$\frac{\overline{I_1 I_2}}{\overline{I_1}\,\overline{I_2}} = 1 + \left|\int d^3r\, \rho(r)\, e^{ik\cdot r}\right|^2$$

$\longrightarrow$ How BIG is the source.

eg: Hanbury-Brown & Twiss

<u>Baym review article.</u>

$$|1,1\rangle \xrightarrow{\text{interferometer}} |2,0\rangle + |0,2\rangle$$



"Hong-Ou-Mandel effect"

$$|2,0\rangle \longrightarrow |2,0\rangle + 2|1,1\rangle + |0,2\rangle$$

$$|0,2\rangle \longrightarrow |2,0\rangle - 2|1,1\rangle + |0,2\rangle$$

(check my signs!)

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2. Hand Problems

2.1 A simple magnet

$$\mathcal{H} = \bigotimes_{x=1..V} \mathcal{H}_{1/2} \qquad \mathcal{H}_{1/2} = \mathrm{span}\{|\uparrow\rangle, |\downarrow\rangle\}$$

on $\mathcal{H}_{1/2}$:

$$\begin{cases} Z|\uparrow\rangle = |\uparrow\rangle \\ Z|\downarrow\rangle = -|\downarrow\rangle \end{cases} \qquad \begin{cases} X|\uparrow\rangle = |\downarrow\rangle \\ X|\downarrow\rangle = |\uparrow\rangle. \end{cases}$$

$$\mathbb{X}_x = \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes X \otimes \mathbb{1} \cdots\cdots \otimes \mathbb{1}$$

the xth site

$$\mathcal{H} = \text{span}\{ |\sigma_1\rangle \otimes \cdots \otimes |\sigma_V\rangle \quad \sigma_{1\ldots V} = \uparrow \text{ or } \downarrow \}$$

$$\equiv |\sigma_1, \sigma_2 \ldots \sigma_V\rangle$$

$$X_x |\sigma_1 \ldots \sigma_V\rangle = |\sigma_1 \ldots \bar{\sigma}_x \ldots \sigma_V\rangle$$

$$(\bar{\uparrow} = \downarrow, \; \bar{\downarrow} = \uparrow.)$$

$\boxed{\text{Hardcore bosons}}$:

$$|\downarrow\rangle = |0\rangle = |\sqcup\rangle$$

$$|\uparrow\rangle = |1\rangle = |\overset{\bullet}{\sqcup}\rangle$$

$$\hat{n}_x | \overset{0 \text{ or } 1}{n_1 \ldots n_V} \rangle \equiv n_x |n_1 \ldots n_V\rangle.$$

$$\sigma_x^{\pm} \equiv \frac{1}{2}(X_x \pm i Y_x)$$

$$\boxed{\frac{\sigma^x + i\sigma^y}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}$$

creates/annihilates a particle at x.

$$(\sigma_x^{\pm})^2 = 0. \qquad \text{"fermionic".}$$

hardcore

$$\sigma^+ |0\rangle = |1\rangle. \qquad \sigma^+ |1\rangle = 0.$$

But $[\sigma_x^\alpha, \sigma_y^\beta] = 0$ for $x \neq y$.

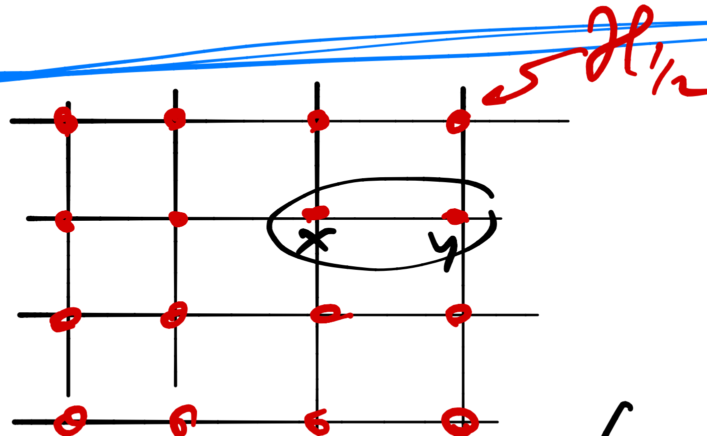
Bosons

different sites

$$[X_x, Z_y] = 0.$$

Warnings: (1) even $H_{\text{quadratic in } \sigma^\pm}$ is not solvable.

(2) H doesn't conserve $\sum_x n_x$ particle #.



(A model of an insulating magnet)

$\langle xy \rangle$ = a link of the lattice.

$$H_{TFIM} = -J\left(\underbrace{\sum_x g X_x} + \underbrace{\sum_{\langle xy \rangle} Z_x Z_y}\right)$$

like $\vec{h}\cdot\vec{\sigma}$ $\leadsto$ $\vec{h} = \hat{x} g J$ "transverse" to $\hat{z}$.

ferromagnetic coupling in $\hat{z}$.

Competition: $X_j |\rightarrow\rangle_j = |\rightarrow\rangle_j$.

$$|\rightarrow\rangle_j = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)_j.$$

vs
$$\begin{cases} -Z_j Z_{j+1} |\uparrow_j \uparrow_{j+1}\rangle = -|\uparrow_j \uparrow_{j+1}\rangle \\ -Z_j Z_{j+1} |\downarrow_j \downarrow_{j+1}\rangle = -|\downarrow_j \downarrow_{j+1}\rangle. \end{cases}$$

$a|\uparrow_j\uparrow_{j+1}\rangle + b|\downarrow_j\downarrow_{j+1}\rangle$ has eval. -1 for $-Z_jZ_{j+1}$.

<u>Compromise</u>: $H_{boring} = -\left(\sin\theta\sum_j X_j + \cos\theta\sum_j Z_j\right)$

$\theta \in [0, \pi/2]$

$H_1 = -(\sin\theta X + \cos\theta Z)$

has g.s. $|\theta\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}|\downarrow\rangle$.

$|\uparrow_z\rangle \longrightarrow |\uparrow_x\rangle = |\rightarrow\rangle$

$\theta = 0$ $\qquad\qquad$ $\theta = \pi/2$

<u>Symmetry</u>: $\boxed{\text{Ising}}$ model has a $\boxed{\mathbb{Z}_2}$ symmetry

$$S = \prod_{x=1}^{V} X_x$$

$$\left\{ \begin{aligned} S|\sigma_1 \ldots \sigma_V\rangle \\ = |\bar\sigma_1 \ldots \bar\sigma_V\rangle \end{aligned} \right.$$

$$\left\{ \begin{aligned} Z_j &\to S Z_j S^\dagger = -Z_j \\ X_j &\to S X_j S^\dagger = X_j \end{aligned} \right.$$

$\underline{X_j Z_i = (-1)^{ij} Z_i X_j}$

$\mathbb{Z}_2$: $\underline{S^2 = \mathbb{1}}$.

$(\underline{X_x^2 = \mathbb{1}.})$

$$[H_{TFIM}, S] = 0.$$

$n_{gs} = 2$!!! 1

g

$|\uparrow\cdots\uparrow\rangle$ S

$|\downarrow\cdots\downarrow\rangle$

$|\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rangle$

$$S|\uparrow\cdots\uparrow\rangle = |\downarrow\cdots\downarrow\rangle$$

$$S|\rightarrow\cdots\rightarrow\rangle = |\rightarrow\cdots\rightarrow\rangle.$$

2 groundstates S

one groundstate

$\Longrightarrow$ no compromise.