University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 239/139 Spring 2018
Assignment 1

Due 12:30pm Monday, April 9, 2018

1. **Too many numbers.**

   Find the number of qbits the dimension of whose Hilbert space is the number of atoms in the Earth. (It's not very many.) Now imagining diagonalizing a Hamiltonian acting on this space.

2. **Warmup for the next problem.**

   Parametrize the general pure state of a qbit in terms of two real angles. A good way to do this is to find the eigenstates of

   $$\boldsymbol{\sigma}^n \equiv \check{n} \cdot \vec{\boldsymbol{\sigma}} \equiv n_x \mathbf{X} + n_y \mathbf{Y} + n_z \mathbf{Z}$$

   where $\check{n}$ is a unit vector.

   Compute the expectation values of $\mathbf{X}$ and $\mathbf{Z}$ in this state, as a function of the angles θ, φ.

3. **Mean field theory is product states.**

   Consider a spin system on a lattice. More specifically, consider the transverse field Ising model:

   $$\mathbf{H} = -J \left(\sum_{\langle x,y \rangle} Z_x Z_y + g \sum_x X_x \right).$$

   Consider the mean field state:

   $$|\psi_{\text{MF}}\rangle = \otimes_x \left(\sum_{s_x \pm} \psi_{s_x} |s_x\rangle \right). \tag{1}$$

   Restrict to the case where the state of each spin is the same.

   Write the variational energy for the mean field state, *i.e.* compute the expectation value of $\mathbf{H}$ in the state $|\psi_{\text{MF}}\rangle$, $E(\theta, \varphi) \equiv \langle \psi_{\text{MF}} | \mathbf{H} | \psi_{\text{MF}} \rangle$.

   Assuming s_x is independent of x, minimize $E(\theta, \varphi)$ for each value of the dimensionless parameter g. Find the groundstate magnetization $\langle \psi | Z_x | \psi \rangle$ in this approximation, as a function of g.