University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215C QFT Spring 2017
Assignment 5

Due 12:30pm Wednesday, May 10, 2017

1. **Grassmann brain-warmers.**

   (a) A useful device is the integral representation of the grassmann delta function. Show that
   $$-\int d\bar\psi_1 e^{-\bar\psi_1(\psi_1-\psi_2)} = \delta(\psi_1-\psi_2)$$
   in the sense that $\int d\psi_1 f(\psi_1)\delta(\psi_1-\psi_2) = f(\psi_2)$ for any grassmann function f.

   (b) Recall the resolution of the identity on a single qbit in terms of fermion coherent states
   $$\mathbb{1} = \int d\bar\psi d\psi \; e^{-\bar\psi\psi} \left|\psi\right\rangle \left\langle\bar\psi\right| . \tag{1}$$
   Show that $\mathbb{1}^2 = \mathbb{1}$. (The previous part may be useful.)

   (c) In lecture I claimed that a representation of the trace of a bosonic operator was
   $$\text{tr}\mathbf{A} = \int d\bar\psi d\psi \; e^{-\bar\psi\psi} \left\langle -\bar\psi\right| \mathbf{A} \left|\psi\right\rangle \; ,$$
   and the minus sign in the bra had important consequences.
   (Here $\left\langle -\bar\psi\right| \mathbf{c}^\dagger = \left\langle -\bar\psi\right| (-\bar\psi)$).
   Check that using this expression you get the correct answer for
   $$\text{tr}(a + b\mathbf{c}^\dagger\mathbf{c})$$
   where a, b are ordinary numbers.

   (d) Prove the identity (1) by expanding the coherent states in the number basis.

2. **Fermionic coherent state exercise.**

   Consider a collection of fermionic modes c_i with quadratic hamiltonian $H = \sum_{ij} h_{ij} c_i^\dagger c_j$, with $h = h^\dagger$.

   (a) Compute $\text{tr}e^{-\beta H}$ by changing basis to the eigenstates of h_{ij} (the single-particle hamiltonian) and performing the trace in that basis: $\text{tr}\ldots = \prod_\epsilon \sum_{n_\epsilon = c_\epsilon^\dagger c_\epsilon = 0,1} \cdots$.

(b) Compute $\mathrm{tr} e^{-\beta H}$ by coherent state path integral. Compare!

(c) [super bonus problem] Consider the case where h_{ij} is a random matrix. What can you say about the thermodynamics?

3. **Gross-Neveu model.**

Now that we've learned about fermionic path integrals, consider the partition function for an N-vector of fermionic spinor fields in D dimensions:

$$Z = \int [d\psi d\bar{\psi}] e^{\mathbf{i} S[\psi]}, \quad S[\vec{\psi}] = \int \mathrm{d}^D x \left(\bar{\psi}^a \mathbf{i} \partial\!\!\!/ \psi^a - \frac{g}{N} \left(\bar{\psi}^a \psi^a \right)^2 \right).$$

(a) At the free fixed point, what is the dimension of the coupling g as a function of the number of spacetime dimensions D? Show that it is classically marginal in $D = 2$, so that this action is (classically) scale invariant.

(b) Follow the steps from the $\mathsf{O}(N)$ saddle problem on the previous homework to show that this model in $D = 2$ exhibits dimensional transmutation in the form of a dynamically generated mass gap.