University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215C QFT Spring 2017
Assignment 4

Due 12:30pm Wednesday, May 3, 2017

1. **Brain-warmer.** Find the coefficient $\mathcal{N}_s$ in the coherent state representation of the spin operator for general spin s
$$\mathbf{S}^a = \mathcal{N}_s \int dn \, |\check{n}\rangle \langle \check{n}| \, .$$

2. **Topological charge.** How does the theta term appear in the $\mathbb{CP}^1$ representation of the NLSM on S^2? Show that
$$\epsilon_{abc} n^a dn^b \wedge dn^c = \alpha dA$$
for some constant α, and find α.

3. **Large n.** Consider the NLSM on S^{n+1} in terms of the $\check{n}$ variables, in D spacetime dimensions. Impose the constraint $\check{n} \cdot \check{n} = 1$ by Lagrange multiplier, $\int [d\sigma] e^{\mathbf{i} \int \sigma \left(n^2 - 1\right)}$. so that the integral over n is Gaussian. Do the gaussian integral and find an effective action for σ. Find the saddle point equation for σ. Find a translation-invariant saddle point. Compare and contrast the saddle point condition for $D = 2$ and $D > 2$. For $D > 2$ you should find a critical value of the coupling.

   Compare the behavior near the critical point with the large-n limit of the Wilson-Fisher fixed point in the ϵ expansion.

   Evaluate the two point function $\langle n^a(x) n^a(0) \rangle$ at the saddle point.

4. **Reminder.** If you didn't do the problem on the Haldane phase on the previous problem set, try it now.

5. **Fermionic coherent state exercise.**

   Consider a collection of fermionic modes c_i with quadratic hamiltonian $H = \sum_{ij} h_{ij} c_i^\dagger c_j$, with $h = h^\dagger$.

   (a) Compute $\text{tr}e^{-\beta H}$ by changing basis to the eigenstates of h_{ij} (the single-particle hamiltonian) and performing the trace in that basis: $\text{tr}... = \prod_\epsilon \sum_{n_\epsilon = c_\epsilon^\dagger c_\epsilon = 0,1} \cdots$.

   (b) Compute $\text{tr}e^{-\beta H}$ by coherent state path integral. Compare!

   (c) [super bonus problem] Consider the case where h_{ij} is a random matrix. What can you say about the thermodynamics?