University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215C QFT Spring 2017
Assignment 1

Due 12:30pm Wednesday, April 12, 2017

1. **Brain-warmer.**

   Show that, for two variables K, X,

   $$\text{if} \quad e^{-2K} = \tanh X \quad \text{then} \quad e^{-2X} = \tanh K \tag{1}$$

   (*i.e.* this relation between K and X is 'self-dual'). Show that the relation (1) can be made more manifestly symmetric under interchange of X and K by writing it as

   $$1 = \sinh 2X \sinh 2K \ .$$

   (This is the combination that appears in the Kramers-Wannier self-duality condition on the square lattice Ising model coupling.)

2. **Trotterization practice.**

   Here's an exercise in understanding the quantum-to-classical correspondence.

   Suppose we add a term

   $$\Delta \mathbf{H}_1 = -v_x \sum_j \mathbf{X}_j \mathbf{X}_{j+1}$$

   to the hamiltonian of the transverse-field Ising model.

   (a) Does this term preserve the $\mathbb{Z}_2$ symmetry generated by $\mathbf{S} = \prod_j \mathbf{X}_j$?

   (b) Construct a corresponding statistical mechanics model. That is, find C and S so that

   $$\sum_C e^{-S} = \text{tr}_{\mathcal{H}} e^{-\frac{1}{T}(\mathbf{H}_{\text{TFIM}} + \Delta \mathbf{H}_1)}$$

   Are the resulting Boltzmann weights e^{-S} positive?

   (c) Answer the previous two questions for

   $$\Delta \mathbf{H}_2 = -v_y \sum_j \mathbf{Y}_j \mathbf{Y}_{j+1} \ .$$

   (d) Answer the previous two questions for

   $$\Delta \mathbf{H}_3 = -g_y \sum_j \mathbf{Y}_j \ .$$

3. **Ising gauge theory.**

Show that the statistical mechanics model associated (by the quantum-classical correspondence) with the toric code Hamiltonian (on the last homework of 215B) is Wilson lattice gauge theory with gauge group $\mathbb{Z}_2$.

4. **Warmup for the next problem.**

Parametrize the general pure state of a qbit in terms of two real angles. (For example: find the eigenstates of

$$\boldsymbol{\sigma}^n \equiv \check{n} \cdot \vec{\boldsymbol{\sigma}} \equiv n_x \mathbf{X} + n_y \mathbf{Y} + n_z \mathbf{Z}$$

where $\check{n}$ is a unit vector.)

Compute the expectation values of $\mathbf{X}$ and $\mathbf{Z}$ in this state.

5. **Mean field theory is product states.**

Consider a spin system on a lattice. More specifically, consider the transverse field Ising model:

$$\mathbf{H} = -J \left(\sum_{\langle x,y \rangle} Z_x Z_y + g \sum_x X_x \right) .$$

Consider the mean field state:

$$|\psi_{\text{MF}}\rangle = \otimes_x \left(\sum_{s_x \pm} \psi_{s_x} |s_x\rangle \right) . \tag{2}$$

Restrict to the case where the state of each spin is the same.

Write the variational energy for the mean field state.

Assuming s_x is independent of x, minimize it for each value of the dimensionless parameter g. Find the groundstate magnetization $\langle \psi | Z_x | \psi \rangle$ in this approximation, as a function of g.