

Quantum Field Theory C (215C) Spring 2015 Assignment 5

Posted May 11, 2015

Due 11am, Tuesday, May 19, 2015

1. All possible terms.

Perturb the gaussian fixed point of the XY model in $D > 3$ by a term

$$\delta S_6 = \int d^D x g_6(x) (\phi^* \phi)^3$$

where g_6 is a short-ranged coupling function. Only if you find *new* relevant perturbations (this does not include terms that may be absorbed in changes in the bare values of perturbations that we have already studied) must you keep track of the numerical coefficients.

What would you find if you included also δS_{2n} with $n > 3$?

2. (Upper) critical dimension. [from Kaplan, nucl-ph/0510023]

Define the ‘critical dimension’ d_c for an operator to be the spacetime dimension for which that operator is marginal. How will that operator behave in dimensions $d > d_c$ and $d < d_c$? In a theory of interacting relativistic scalars, Dirac fermions, and gauge bosons, determine the critical dimension for the following operators:

- A gauge coupling to either a boson or a fermion through the covariant derivative in the kinetic term (‘minimal coupling’).
- A Yukawa interaction, $\phi \bar{\psi} \psi$.
- An anomalous magnetic moment coupling for a fermion $g \bar{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi$ (where $\sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu]$ is the generator of spin rotations).
- A four-fermion interaction $(\bar{\psi} \psi)^2$

3. Non-relativistic QFT and the Schrödinger equation. [from Kaplan, nucl-ph/0510023]

We’ve discussed in lecture a Lagrangian for a non-relativistic field theory in d spatial dimensions:

$$S = \int d^d \vec{x} dt \left(\psi^*(\vec{x}, t) \left(\mathbf{i} \partial_t - \frac{\nabla^2}{2m} \right) \psi(\vec{x}, t) - \frac{g}{8m^2} (\psi^*(\vec{x}, t) \psi(\vec{x}, t))^2 \right) .$$

Translate this into ‘first-quantized’, single-particle language: that is, write down the Hamiltonian \mathbf{H} acting on the sector of the Hilbert space where there are two particles. This sector is closed because the particle number is conserved.

Spoilers: show that this interaction is equivalent to a $\delta^d(\vec{x}_1 - \vec{x}_2)$ interaction potential between two particles in d spatial dimensions. Relate the strength of the potential to g by matching the Born approximation (=tree level) amplitudes.

What does the RG scaling of the interaction (it can be relevant, marginal or irrelevant, depending on d) mean for the solution of the Schrödinger problem? What is the critical dimension for the $\delta^d(\vec{r})$ potential?

4. **1, 2, many.**

By integrating momentum shells, derive the RG equations for the quartic and quadratic operators of the relativistic $O(N)$ vector model in D dimensions,

$$S_E[\phi^a] = \int d^D x \left(\frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a + \frac{1}{2} r \phi^a \phi^a + u (\phi^a \phi^a)^2 \right) .$$

Here $a = 1..N$.