

String-net condensation: A mechanism for topological phases, and a possible origin of light and electrons

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Gauge theories can be equivalently written as string-net theories, and by this string-net picture, we can understand the topological phases which can't be explained by the usual symmetry breaking picture. Also, string-net condensation can naturally emerge both gauge bosons and fermions, which offers a possibility of origin of light and electron.

INTRODUCTION

Landau theory, which says that phase transition is because of symmetry breaking, has been succeeded to explain all the phase transition for a long time. However, recent discoveries, especially the discovery of fractional quantum hall states, showed us some new phases beyond the Landau theory. These new phases are defined by ground states degeneracy, and they are called topological phases. The appearance of the word topological will be explained later in this paper. Topological phases can be explained by string-net condensation theory.

String-net condensation can naturally give birth to both fermions and gauge bosons, so it implies that, our real vacuum might be a string-net material.

WHAT IS STRING-NET

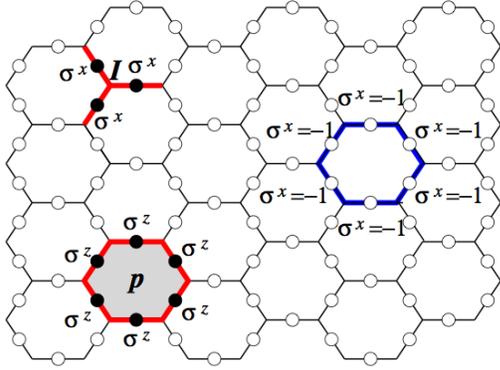


FIG. 1: The lattice configuration for Z_2 lattice gauge theory. Source: [1]

In general, each gauge theory is equivalent to a string-net condensation theory. In order to introduce the concept, we start from the simplest example, Z_2 lattice gauge theory. The lattices are assumed to be honeycomb for simplicity, see FIG. 1. The Hamiltonian can be written as the form of:

$$H_{Z_2} = -U \sum_i \sigma_i^x + t \sum_p \prod_{\text{Edges of } p} \sigma_j^z \quad (1)$$

where $\sigma^{x,y,z}$ are Pauli matrices, $I, i(j)$ and p labels the sites, links, and plaquettes respectively. For every site I , the Hilbert space is formed by the states satisfying

$$\prod_{\text{Legs of } I} \sigma_i^x |\Phi\rangle = |\Phi\rangle \quad (2)$$

This constraint is extremely important to understand string-net theory. If we define the state $\sigma_i^x = -1$ as being occupied by a string, and $\sigma_i^x = +1$ as not being occupied, then (2) tells us that, an end of a string (one -1 state) can not exist, a cross point (three -1 states) also can not exist. So only closed strings are allowed in the theory. FIG. 2 (a) shows one example of possible string-net configurations for Z_2 lattice gauge theory.

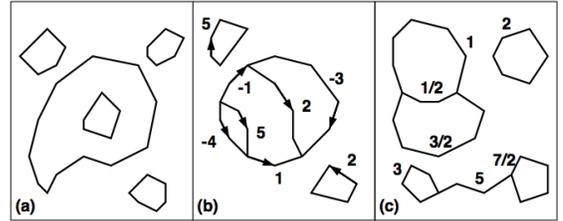


FIG. 2: Some examples of string-net configuration of different gauge theories. Source: [1].

Now we have an equivalent closed string picture of Z_2 gauge theory. Let's look at the Hamiltonian again, the meaning will be more clear. The "electric energy" U term is a string tension or potential energy, and "magnetic energy" t term is a string kinetic energy.

In general, for every gauge theory, we can write a string-net Hamiltonian $H = UH_U + tH_t$. For each gauge theory we also have a constraint like (2), here are some examples. $U(1)$ gauge symmetry: see FIG. 2 (b), the strings are orientated, and can be labeled only by integers, and they must satisfy branching rules $E_1 + E_2 + E_3 = 0$; $SU(2)$ gauge symmetry: the strings are unorientated, and can be labeled only by half-integers, and the branching rule is the triangle inequality.

FIXED POINT WAVE FUNCTIONS

We expect the universal features of string-net condensed phases that belong to the same quantum phase look the same in the long distances, and the only differences happen in small scales. According to standard renormalization group analysis, this requires the wave functions to be fixed point wave functions in the RG flow picture. A fixed point ground state wave function, is equivalent to a wave function satisfying some specific local constraints. The local constraints equations are shown in FIG. 3. In these equations, a grey block represents any string-net configuration.

$$\Phi \left(\begin{array}{c} \text{---} i \text{---} \\ \text{---} \end{array} \right) = \Phi \left(\begin{array}{c} \text{---} \curvearrowright \text{---} \\ \text{---} \end{array} \right) \quad (4)$$

$$\Phi \left(\begin{array}{c} \text{---} \square^i \\ \text{---} \end{array} \right) = d_i \Phi \left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \quad (5)$$

$$\Phi \left(\begin{array}{c} \text{---} k \text{---} \\ \uparrow \quad \downarrow \\ \text{---} i \quad \text{---} j \end{array} \right) = \delta_{ij} \Phi \left(\begin{array}{c} \text{---} k \text{---} \\ \uparrow \quad \downarrow \\ \text{---} i \quad \text{---} i \end{array} \right) \quad (6)$$

$$\Phi \left(\begin{array}{c} \text{---} i \text{---} \\ \diagup \quad \diagdown \\ j \quad k \end{array} \right) = \sum_n F_{kln}^{ijm} \Phi \left(\begin{array}{c} \text{---} i \text{---} \\ \diagup \quad \diagdown \\ j \quad n \quad k \end{array} \right) \quad (7)$$

FIG. 3: The local constraints equations. Source: [1]

Now let's understand the meaning of these constraint equations. (4) basically says that, a straight line is equivalent to any curved line in string-net theories. This rule tells us that if two string-net configurations are topologically equivalent, then the corresponding wave function Φ are the same. This topological feature is closely related to the terminology "topological phase". (5) is motivated by renormalization group point of view, the wave functions should look the same at all scales, up to a constant. Since a separate closed loop will disappear at larger scales, so the wave function of a string-net configuration with a closed loop is proportional to the wave function without. (6) can also be understood by renormalization group, at large scales the loop in the middle will disappear, so i must be equal to j otherwise the wave function is 0. (7) is more subtle, the reason why we need it is that (4)-(6) isn't complete to uniquely specify a ground state wave function, and (7) is the simplest local constraint to finish the confinement.

Now the universal features of string-net condensed state can be captured by (d_i, F_{kln}^{ijm}) . Note that not every choice of (d_i, F_{kln}^{ijm}) is self-consistent, there are some equations that they must satisfy in order to be self-consistent. A mathematical object that satisfy some certain algebra equations like F_{kln}^{ijm} is called tensor category.

MECHANISM FOR TOPOLOGICAL PHASES

Topological phases mean ground states degeneracy, and it can be explained by string-net condensation.

In the Hamiltonian (1), when $U \gg t$, the string tension term dominates, so there should be only a few strings, there's no ground state degeneracy; however, when $U \ll t$, the string kinetic energy is more important, in this case the ground state will consist many large strings full of space, and thus it's called the "string-net condensation" state. In string-net condensed states, the ground states might degenerate or say might have topologically different string-net configurations, they are corresponding to different topological phases.

TABLE I shows the comparison between usual phases and topological phases. Just as usual phases can be classified by different symmetries; topological phases can be classified by different string-net configurations, or say different (d_i, F_{kln}^{ijm}) pair. So the usual phase transition is caused by symmetry breaking (according to Landau theory); topological phase transition is caused by changing of string-net configuration. The mathematical framework for symmetry is group theory, so each phase can be labeled by a group; the math language for string-net theory (or say knot theory) is tensor category theory, and each topological phase can be labeled by a tensor category (d_i, F_{kln}^{ijm}) .

TABLE I: Comparison between usual and topological phases

	Usual phases	Topological phases
Arise via	Particle condensation	String-net condensation
Classified by	Different symmetries	Different string-net configurations
Phase transition	Symmetry breaking	String-net configuration changing
Math theory	Group theory	Tensor category theory

POSSIBLE ORIGIN OF LIGHT AND ELECTRONS

Before this section we only discussed the ground states, if we consider excitations, open strings are possible. Open string itself is not an observable, only the two ends are, so the two ends of an open string can be seen as two quasi-particles that can move freely. If we analyze the statistics that are satisfied by these quasi-particles, we can find that different types of string-net theory can either give rise to boson statistics or fermion statistics (in some dimension string-net theory can also give birth to anyons.). So string-net theory gives us a unified view of bosons and fermions: they can both be emerged from string-net condensation state. Xiaogang Wen even claims that our true vacuum might be a string-net condensed material, and light and electrons are emerged from it.

However, this point of view offers only a possibility, it's not a complete theory yet: the biggest problem is that, this concept is quite similar to ether, and from ether we can't derive Lorentz invariance. So there are still a lot to be done about this point of view.

CONCLUSION

String-net condensation is a novel and interesting idea, it can explain topological phases in an elegant way. Also both bosons and fermions can be emerged from string-net condensation, which makes us think about an interesting

possibility that our real vacuum is a string-net condensed material.

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- [1] Michael A Levin and Xiao-Gang Wen. String-net condensation: A physical mechanism for topological phases. *Physical Review B*, 71(4):045110, 2005.
 - [2] X.G. Wen. *Quantum Field Theory of Many-Body Systems: From the Origin of Sound to an Origin of Light and Electrons*. Oxford Graduate Texts. OUP Oxford, 2004.