Deconfined Quantum Criticality

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The quirky properties of a deconfined quantum critical point are illustrated with a 2+1 dimensional square antiferromagnetic lattice. Deconfined quantum critical points are distinct from critical points described by Landau-Ginzberg theory, possessing emergent fractional excitations and a topological conservation law. The order parameters of the Néel ordered and valence bond solid (VBS) phases of the lattice are found to possess a nonlocal topological relationship. Both phases are described in terms of the emergent degrees of freedom at the critical point.

INTRODUCTION

Landau-Ginzberg theory successfully describes a wide variety of phase transitions, including quantum phase transitions at zero temperature. Deconfined quantum critical points are a distinct class of phase transitions that cannot be described with Landau-Ginzberg theory, distinguished by an emergent conserved topological quantity at the critical point. A 2+1 dimensional antiferromagnetic lattice is considered, which undergoes a quantum phase transition between the Néel and valence bond solid (VBS) ordered phases. The Néel phase and VBS phase break two different symmetries: SU(2) spin symmetry and translation symmetry. Landau theory predicts a first order transition, or an intermediate phase containing a mixture of the Neel and VBS phases [3]. However, a second order transition occurs, crucially dependent on the quantum properties of the system, notably the Berry phase of the spin. The nature of this critical point, as well as the relationship between the order parameters of the Néel and VBS phase, is investigated.

First, the Néel and VBS phases are introduced on a 2+1 dimensional lattice. The topological defects (monopoles) of the Néel phase are related to the magnetic flux of an emergent U(1) gauge field at the critical point. The creation and annihilation of these defects is found to be irrelevant at the critical point due to effects of the Berry phase of spin, yielding a conserved topological quantity. Topological defects (vortices) of the VBS phase are found to be described by a spinor field coupled to the emergent gauge field at the critical point. The phase diagram is then described in terms of the fugacity of monopoles and vortices.

NÉEL PHASE

Consider a 2+1 dimensional lattice with spins $S = \frac{1}{2}$ at each site, with the antiferromagnetic Hamiltonian

$$H = J \sum_{<ij>} \vec{S}_i . \vec{S}_j + \dots \qquad (1)$$

H has $SU(2)$ spin symmetry and the translational symmetry of the square lattice. $J > 0$ is the antiferromagnetic interaction strength. Additional terms not shown are next-nearest neighbor interaction terms that can be tuned to cause a phase transition. In the Néel phase, with $g \to 0$, these terms are small, so that only nearest neighbor interactions are relevant, and the spins align antiparallel to their nearest neighbors . This causes the spins to obtain an expectation value $\langle \vec{S}_i \rangle = (-1)^i \hat{n} \neq 0$, spontaneously breaking the SU(2) spin symmetry [5]. Here, $\hat{n}$ is the Néel order parameter. The Néel phase has gapless spin wave excitations.

The dynamics of the fluctuations of the Néel parameter can be described with an O(3) nonlinear sigma model [4]

$$S_n = \frac{1}{2g} \int d\tau dx dy \left[\frac{1}{c^2} \left(\frac{\partial \hat{n}}{\partial \tau} \right) + (\nabla \hat{n})^2 \right] + S_B[\hat{n}]$$

with Berry phase action

$$S_B[\hat{n}] = iS \sum_j (-1)^{x_j + y_j} \mathcal{A}_j[\hat{n}];$$

The Berry phase contribution from each site j alternates in a checkerboard fashion. $\mathcal{A}_j$ is the area enclosed by the path of $\hat{n}(\tau)$ on the sphere in spin space. We will see that this in fact has a relation to the order parameter of the VBS phase, which breaks translational symmetry.

Besides spin waves, the Néel phase permits smoothly varying topological configurations of $\hat{n}$ called skyrmions (see Figure 1). The (integer) number of skyrmions Q is given by

$$Q = \frac{1}{4\pi} \int dx dy \hat{n} \cdot (\partial_x \hat{n} \times \partial_y \hat{n}) \qquad (2)$$

Now, imagine creating or destroying a skyrmion. This requires a singularity in $\hat{n}(x, y, \tau)$ at some point in space and time, creating a monopole in spacetime. The Berry phase associated with a monopole configuration is significant, and oscillates from one site to the next on the lattice. It can be shown [3] that the total berry phase for a collection of monopole events is

$$\prod_n e^{i \frac{\pi}{2} \zeta_k \Delta Q_k}$$

Here k runs over all monopoles, $\Delta Q_k = \pm 1$ is the change in Q associated with each monopole, and $\zeta_k \in \mathbf{Z}_4$ determines the alternating phase on each face of the lattice (see Figure 2). For single monopole events that

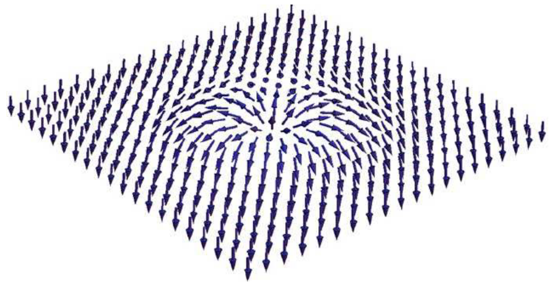

FIG. 1: A skyrmion of the Néel phase

1	*i*	1	*i*
-*i*	-1	-*i*	-1
1	*i*	1	*i*
-*i*	-1	-*i*	-1

FIG. 2: Berry phases of monopoles placed at each plaquette

change Q by 1, the Berry phases destructively interfere and prevent single skyrmion changing events. However, monopole events which change Q by multiples of 4 do not destructively interfere and survive. As a result, quadrupled monopole events are the smallest number allowed. The quadrupled monopole operator has a large scaling dimension, which causes it to be suppressed at the critical point [3]. As we move away from the Néel phase towards the critical point, it becomes convenient to write the Néel order parameter $\hat{n}$ in a spinor representation. This will elucidate the fractionalized fields at the critical point. We map $\hat{n} \in S^2$ to $\binom{z_\uparrow}{z_\downarrow}$ $\in$SU(2) using a Hopf transformation:

$$\hat{n} = z_a^\dagger \vec{\sigma}_{ab} z_b$$

This map has a U(1) gauge redundancy $z \to e^{i\alpha(\vec{r},\tau)} z$. Therefore we can couple z to a U(1) gauge field a_μ. The "magnetic" flux of this gauge field turns out to be $2\pi \times Q$, the skyrmion number. Whenever a skyrmion is created, it changes the gauge flux by 2π. This is only allowed when the gauge group U(1) is compact [4].

At the critical point, the monopoles become irrelevant. This is because the Berry phase suppresses all monopole events that are not quadrupled, and the quadrupled terms have a large scaling dimension that make them irrelevant at the critical point [3]. We can therefore consider a noncompact U(1) gauge group at the critical point. The simplest theory is [4]

$$L_z = \sum_{a=1}^{2} |(\partial_\mu - i a_\mu) z_a|^2 + V(z_a^\dagger, z_a) + \kappa(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa)^2 \quad (3)$$

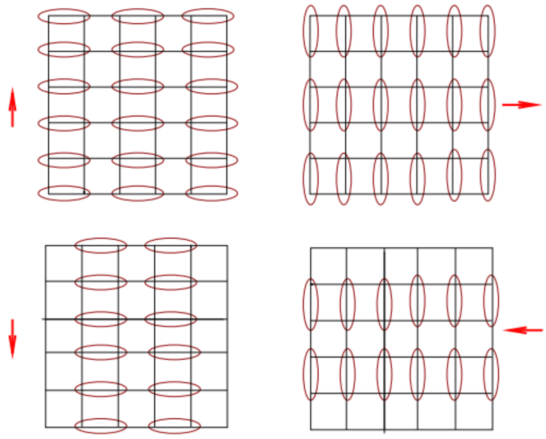

FIG. 3: The four distinct configurations of the VBS state, corresponding to the four values of the order parameter ψ_{VBS}. The ovals represent singlet states. Figure borrowed from [1]

where $V(z_a^\dagger, z_a) = s|z|^2 + u|z|^4$ is a potential with constants s and u determined by the coupling strengths of the original theory(1). Since monopoles are irrelevant at the critical point, the topological number Q is conserved. The critical point also has spin 1/2 excitations with mass s that do not appear in the Néel phase. We will see that the same critical theory is obtained by attacking the critical point from the VBS phase.

VBS PHASE

The valence bond solid (VBS) phase is a paramagnetic phase in which pairs of spins form singlets (see Figure 3). This phase preserves the spin SU(2) symmetry but breaks translational symmetry. The valence bond phase is characterized by a $\mathbf{Z}_4$ order parameter $\psi_{VBS} = e^{i\chi}$, $\chi = 0, \pi/4, \pi/2, 3\pi/4$, corresponding to the four possible orientations and positions of the singlet pairs. Unlike the Néel phase, VBS phase has gapped spin 1 "triplon" quasiparticle excitations [2].

Topological defects in this phase are domain walls, separating regions with different values of the VBS angle χ [1]. By intersecting domain walls at a site on the lattice, a $\mathbf{Z}_4$ "vortex" can be constructed. Shifting the vortex by a single site reverses the direction of the vortex. An unpaired spin occupies the center of the vortex, where the domain walls meet (see Figure 4). We will see that this lone spin corresponds to the spinor z introduced in the last section. As we move away from the VBS ordered phase towards the critical point, the domain walls of the vortices thicken, with thickness ξ_{VBS}. Within the domain walls, on the scale of ξ, the correlation length of the spins, the $\mathbf{Z}_4$ anisotropy breaks down and χ varies smoothly. At the critical point, χ describes the phase of an XY model, which has local U(1) symmetry. It can

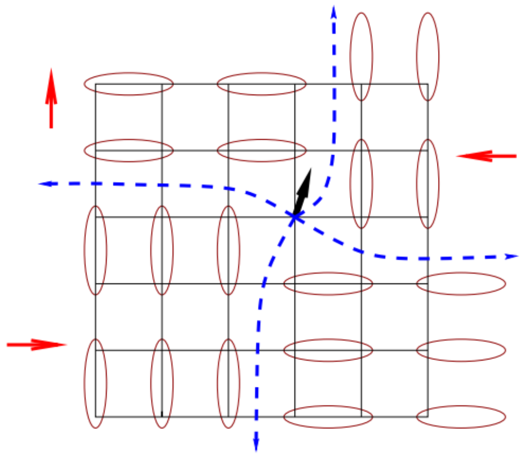

FIG. 4: $\mathbf{Z}_4$ vortex formed by domain walls (blue), with an unpaired spin in the center. Figure borrowed from [1]

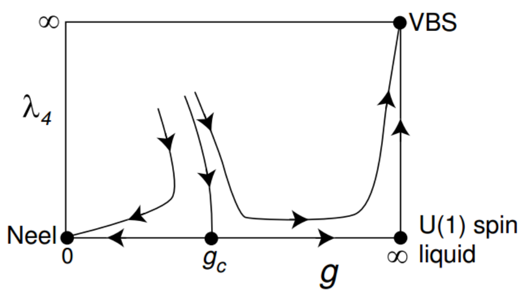

FIG. 5: RG flow of the system, where g is the strength of interactions in (1) and λ_4 is the monopole fugacity. Figure borrowed from [5]

be shown that the vortices with unpaired spins z couple to the gauge field of this U(1) symmetry, yielding exactly the same theory near the critical point that was obtained starting from the Néel phase (3). The conserved quantity $J_0 = K\partial_0\chi = \frac{1}{2\pi}\epsilon^{ij}\partial_i a_j$ is the magnetic flux density of the emergent gauge field. This means that the operator $\psi_{VBS} = e^{i\chi}$ increases the magnetic flux by 2π. Thus the VBS order parameter describes the creation of monopoles in the Néel phase [1].

RENORMALIZATION GROUP FLOW

The renormalization group flows for the antiferromagnetic lattice model are shown above. Here, g is the strength of long range interactions in (1) and λ_4 is the monopole fugacity. As $g \to 0$, the monopoles vanish ($\langle\psi_{VBS}\rangle = 0$) and the vortices z of the VBS phase condense ($\langle\hat{n}\rangle \neq 0$). As $g \to \infty$, the monopoles condense ($\langle\psi_{VBS}\rangle \neq 0$) and the vortices become confined ($\langle\hat{n}\rangle = 0$). The U(1) spin liquid phase appears at large g, but is unstable to monopole creation events. The deconfined critical point at $g = g_c$ possesses properties not seen in either the Néel or VBS phase: The conservation of an integer topological number Q (2), given by the magnetic flux of the emergent U(1) gauge field, and fractional excitations z. Interestingly, these two degrees of freedom connect the two phases in a nonlocal, topological way: Q counts the number of skyrmions in the Néel phase, and z corresponds to vortices in the VBS phase.

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APPENDIX

To obtain the theory at the critical point in a dual vortex description, we note that as we approach the critical point, the domain walls thicken and the $\mathbf{Z}_4$ symmetry approaches a $U(1)$ symmetry. Since the vortices are massless in this limit, the XY Lagrangian in the continuum limit is

$$\mathcal{L}_{XY} = \frac{1}{2}\kappa(\partial\chi)^2 \tag{4}$$

where ϕ is the vortex phase angle (VBS orientation), which changes by 2π going around a vortex, and κ is a constant.Clearly $J_\mu = K(\partial_\mu\chi)$ is the conserved current, with $\partial_\mu J^\mu = 0$. In 2+1 dimensions, this means we can write J_μ in terms of a U(1) gauge field a_μ:

$$J_\mu = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda \tag{5}$$

Indeed, J_μ is invariant under the gauge transformation $a_\lambda \to a_\lambda + \partial_\lambda\Lambda$. Plugging in (5) into $\mathcal{L}_{XY}$ and integrating by parts [6],

$$\mathcal{L}_{XY} = \frac{1}{4\pi}a_\lambda\epsilon^{\lambda\mu\nu}\partial_\mu\partial_\nu\chi \tag{6}$$

Since the phase field χ is not defined globally, $\epsilon^{\lambda\mu\nu}\partial_\mu\partial_\nu\chi$ does not vanish. Integrating the component coupled to a_0 in a region of space containing the vortex,

$$\int d^2x\epsilon^{ij}\partial_i\partial_j\chi = \oint d\vec{x}\cdot\vec{\nabla}\chi = 2\pi \tag{7}$$

This shows that $\epsilon^{\lambda ij}\partial_i\partial_j\chi$ is the density of vortices, and the vortex current is

$$j_\mu = \epsilon^{\lambda\mu\nu}\partial_\mu\partial_\nu\chi \tag{8}$$

Plugging in (5) for $\kappa\partial_n u\chi$,

$$j_\mu = \frac{1}{4\pi^2\kappa}\epsilon^{\mu\nu\lambda}\partial_\nu b_\lambda \tag{9}$$

where $b_\lambda = \epsilon^{\lambda\alpha\beta}\partial_\alpha a_\beta$ is the field strength. Including terms $z_a, z_a^\dagger$ which create and annihilate vortices, the critical theory is [1]

$$L_z = \sum_{a=1}^{2} |(\partial_\mu - ia_\mu)z_a|^2 + V(z_a^\dagger, z_a) + \kappa(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa)^2$$

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