University of California at San Diego – Department of Physics – Prof. John McGreevy

Quantum Field Theory C (215C) Spring 2013
Assignment 6

Posted May 30, 2013 **Due 11am, Thursday, June 6, 2013**

Problem Set 6

1. **Matching with massive electrons.** [from Iain Stewart]

   Consider QED in the regime where the photon momenta q^μ are much smaller than the electron mass m_e. In this regime, we can integrate out the electron and write an effective field theory involving only the photon.

   (a) Calculate the QED one-loop vacuum polarization diagram using dimensional regularization, in the $\overline{\text{MS}}$ scheme. Expand $\Pi(q^2)$ through first order in $\frac{q^2}{m_e^2}$.

   (b) Write down a Lagrangian involving only the photon field operator that reproduces the first two terms in the expansion. (Hint: it should be gauge invariant and Lorentz invariant. There is essentially (up to integration by parts) only one addition to the Maxwell term). Use the calculation above to match between the photon-only EFT and QED at $\mu = m_e$, at this order in the fine structure constant α.

   (c) What symmetry of QED forbids dimension-6 operators involving three field strengths?

   (d) At dimension 8, we can write down operators in the photons-only EFT which describe light-by-light scattering. Write them down (there are two). Draw the QED Feynman diagram which matches to these terms, and determine the number of factors of α in their coefficients. (Don't do the integrals.)

   (e) [bonus] Use dimensional analysis in the low-energy EFT to estimate the size of the $\gamma\gamma \to \gamma\gamma$ cross section.

2. **Right-handed neutrinos.** [from Iain Stewart, and hep-ph/0210271]

   Consider adding a right-handed singlet (under all gauge groups) neutrino N_R to the Standard Model. It may have a majorana mass M; and it may have a coupling g_ν to leptons, so that all the dimension ≤ 4 operators are

$$\mathcal{L}_N = \bar{N}_R \mathbf{i}\partial\!\!\!/ N_R - \frac{M}{2}\bar{N}_R^c N_R - \frac{M}{2}\bar{N}_R N_R^c + \left(g_\nu \bar{N}_R H_i^T L_j \epsilon^{ij} + h.c.\right)$$

   where $N_R^c = C\left(\bar{N}_R\right)^T$ is the the charge conjugate field, $C = \mathbf{i}\gamma_2\gamma_0$ (in the Dirac representation), H is the Higgs doublet, L is the left-handed lepton doublet, containing

ν_L and e_L. Take the mass M to be large compared to the electroweak scale. Integrate out the right-handed neutrinos at tree level. [Hint: you may find it useful to work in terms of the Majorana field

$$N \equiv N_R + N_R^c$$

which satisfies $N = N^c$.]

Show that the leading term in the expansion in $1/M$ is a dimension-5 operator made of Standard Model fields. Explain the consequences of this operator for neutrino physics, assuming a vacuum expectation value for the Higgs field.

Place a bound on M assuming that the observed neutrinos have masses $m_\nu < 0.5$ eV.