MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
String Theory (8.821) – Prof. J. McGreevy – Fall 2007

Problem Set 7

D-branes and orientifolds

Reading: Polchinski, Chapter 13 and v. I pp. 189-192, 226-229, §8.8, v. II pp. 29-31, §10.8, §13.2; Uranga, chapter 10.

Due: Thursday, November 29, 2007 at 11:00 AM in lecture or in the box.

1. **BPSness and multiple branes.**

   (a) Which of the 32 supercharges of type II are conserved in the presence of a Dp-brane and a D(p+2k)-brane which lie along the 01...p and 01..(p+2k) coordinate axes, respectively?

   How much supersymmetry does the D0-D4 system preserve?

   (b) Consider on the worldvolume of $N > 1$ D4-branes a configuration of the gauge field F which carries a topological charge

   $$\frac{1}{16\pi^2}\int_{\text{space}} \text{tr}_{\ N\times N} F\wedge F = 1$$

   (here 'space' is meant to denote an integral over the four spatial dimensions of the D4-brane at fixed time); this quantity is called the 'instanton number' because in 4d gauge theory there exist finite-action solutions of the euclidean EOM which carry such charge[1].

   Show that any such configuration carries the RR charge of one D0-brane. (Such configurations which preserve supersymmetry describe the condensation of the massless scalars arising from the 0-4 strings – on this *Higgs branch* of the moduli space, the D0-brane can be said to dissolve into the D4-brane.)

   (c) [More optional than usual] Show that if we take a pair of coincident Dp-branes and rotate one relative to the other, the amount of preserved supersymmetry is determined by whether the rotation matrix can be written as an element of a subgroup of $SO(n)$ which preserves a spinor.

[1] There are of course many configurations which carry the charge but do not solve the equations of motion. The gauge field configurations which actually describe instantons are (anti-)self-dual: $F = \pm\star F$. The instanton number is also called the 'second Chern class' or 'first Pontryagin class'.

Consider the case when the branes share 3+1 dimensions, so that the relative rotation is in general an element of $SO(6)$. Show that the spectrum contains chiral matter under the *relative* $U(1)$ which couples to the strings stretched between the two branes.

If you get stuck, see BDL `hep-th/9606139`.

2. **Dp-D(p+2) system.**

   (a) In type II string theory in flat space, consider a Dp-brane and a D(p+2)-brane which are parallel in $p+1$ flat dimensions. Show that the spectrum of strings stretching between them includes a tachyon if they are close enough together.

   (b) Make a conjecture for the endpoint of the condensation of this tachyon.

   Hint:

$$S_{Dp} \supset \int_{Dp} \sum_q C_{RR}^{(q)} \wedge \mathrm{tr}\ e^{aF} = \int_{Dp} \left(C_{RR}^{(p+1)} + a\mathrm{tr}\ F \wedge C_{RR}^{(p-1)} + ... \right)$$

   where a is some numerical stuff with factors of α'.

3. **O-plane charge.**

   How many Dp-branes cancel the RR charge of an Op^+ plane (the SO kind)?

   Hint: consider what happens when you T-dualize type I, and notice that the number of connected components of the fixed locus of $x^i \to -x^i, \quad i = 1..q$, with x^i periodic, is 2^q.

   [OK, this is an uninspired question, but I didn't get to mention it in lecture.]

4. **A string duality.**

   Consider the worldvolume theory of a D-string in the type I theory in ten flat dimensions. (Recall that unlike the type I F-string, the D-string carries a charge because the RR 2-form potential survives the orientifold projection.) Stretch the D1-brane along the 01 directions. What is the spectrum of 11-strings after performing the orientifold projection? Recall that the type I vacuum contains 32 D9-branes; what is the spectrum of 1-9 strings?

   Show that the orientifold projection leaves a $\mathbb{Z}_2$ subgroup of the worldvolume gauge group which acts on the 1-9 strings.

   Where have you seen this theory before?[2]

[2] This wonderful observation is from `hep-th/9510169`.