MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
String Theory (8.821) – Prof. J. McGreevy – Fall 2007

Problem Set 6
Superstrings

Reading: Polchinski, Chapters 10, 11, 12, except for the bits about unoriented strings.

Due: Tuesday, November 20, 2007 at 11:00 AM in lecture or in the box.

1. **Bosonization for interacting theories.**

   There are many ways to deform the free boson = free complex fermion theory, besides the linear dilaton deformation discussed in class.

   (a) Change the radius of the boson. What operator accomplishes this in the boson theory? Describe this operator in terms of the fermions. Adding this operator to the free fermion lagrangian makes it no longer free (though still solvable); it's called the *Thirring model*.

   (b) Add a mass for the fermions. Describe this operator in terms of the bosons. The result is called the *sine-Gordon* model.

   If you get stuck here, or want to learn more, see page 246 of Coleman, *Aspects of Symmetry*, or *Phys. Rev.* **D11** (1975) 2088.

2. **Operator algebra of spin fields.**

   (a) Using bosonization and constraints from Lorentz invariance, convince yourself that

   $$\psi^\mu(z)\Theta_\alpha(0) \sim \frac{1}{\sqrt{2}}\frac{1}{\sqrt{z}}\Gamma^\mu_{\alpha\beta}\Theta_\beta(0)$$

   and

   $$\Theta_\alpha(z)\Theta_\beta(0) \sim C_{\alpha\beta}\frac{1}{z^{5/4}} + \frac{1}{\sqrt{2}}\frac{1}{z^{3/4}}(C\Gamma^\mu)_{\alpha\beta}\psi_\mu(0)$$

   where C is the charge conjugation matrix $C\Gamma^\mu C^{-1} = -(\Gamma^\mu)^T$. It may help to note that these are closely related to eqns (12.4.7) and (12.4.18) of Polchinski, and that the power of z is determined by dimensional analysis. Note that the first term in the second equation vanishes when the Θs have the same chiralities, such as in part (b) of this problem.

If you want to worry about signs and factors, it might be helpful to come to terms with the equations of p. 77 of Peskin's notes, where he gives an explicit prescription for the cocyles for the spin fields.

(b) Verify the algebra of spacetime supersymmetry generators

$$Q_\alpha = \oint \frac{dz}{2\pi i} \mathcal{V}_\alpha(z) = \oint \frac{dz}{2\pi i} e^{-\phi/2} \Theta_\alpha(z).$$

3. **A little more superstring scattering.**

Polchinski, problem 12.8.

4. **The bosonic tachyon and the superstring.**

The bosonic string tachyon, whose vertex operator is $e^{ik \cdot X}$, is not a physical state of the superstring because it isn't supersymmetric, *i.e.* it is not killed by the T_F part of the BRST operator. However, a skeptic might worry that it would somehow try to appear in interactions of allowed states (even though the BRST construction guarantees that it does not). In fact, it comes dangerously close to appearing, as follows. [1]

Consider the OPE of two (-2)-picture NS tachyons (ignore the contribution of the superconformal ghosts here, which gives an extra factor of $e^{-2\phi}$):

$$V_k^{(-2)} \equiv ik \cdot \psi e^{ik \cdot X}.$$

Show that it contains pole terms that look very much like the bosonic tachyon vertex $e^{ik \cdot X}$. Become afraid. Then show that the coefficient of this term vanishes when the on-shell condition for the superstring tachyon is imposed.

The following problems don't actually require a written response:

5. **Anomalies.** Read Polchinski section 12.2 about anomalies in type I supergravity, and their cancellation using the Green-Schwarz mechanism. Look at section 12.6 where JP shows that for the heterotic string the required $B \wedge \mathrm{tr}\ F^4$ term is generated at one loop.

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[1]This problem follows some comments in section 9.13 of Polyakov's book.

6. **The oracle speaks on GSO.**

   Try to understand the following statements in favor of the GSO projection, from Polyakov's book (p. 251). "*...only under the above prescription* [*i.e.* only when making the GSO projection and summing over R and NS sectors] *is it possible to treat the system in terms of spin operators* [*i.e.* is it equivalent to an Ising model]. *For that matter, take an Ising model on a surface with high genus. We know that usually* [*i.e.* on the plane] *this model can be replaced by free fermions. Is this still true? In fermionization of Ising spins a crucial role is played by Kramers-Wannier duality... Fermionic lines are essentially the boundaries of drops containing reversed spins. However, if the surface is homologically nontrivial* [*i.e.* genus > 0], *there are closed paths which do not form boundaries of anything. We must ensure that fermionic trajectories corresponding to these paths do not contribute. The way to achieve this is just to sum over spin structures, since then each homologically nontrivial path will be cancelled by one of the opposite spin structure.*"

7. **Geometry.**

   Soon we are going to start using some fancy geometry. Start looking at section 2 of hep-th/9702155.