Lecture 1: Lightning Overview

McGreevy

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Reading: Polchinski §1.1, 3.1, begin §1.2-1.4, GSW chapter 1 (I find that GSW chapter 1 is worth rereading once a year or so.).

Outline of this lecture:

1. Structure of string perturbation theory.

2. Compactification

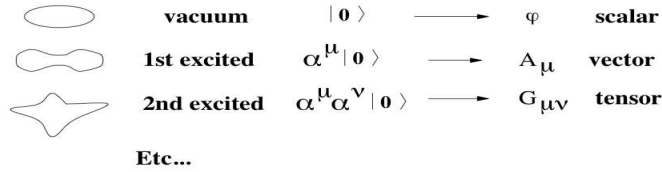3. Superstrings

4. Non-perturbative issues

5. Motivations

1 Big picture

In this first lecture, the goal is to convey the basic structure of the beast under study, and to consider what it might be good for. There will many gaps, which we will start to fill in Lecture 2. At the beginning I will assume you have some reason for wanting to know about string theory, then we'll discuss what such reasons might exist.

We begin with the seemingly silly idea that maybe we've been wrong all along about particles, and maybe we should consider the dynamics of one-dimensional objects instead. They may be closed (*i.e.* without boundary) or open, oriented or unoriented. We'd like to try to generalize for such creatures the 1st quantized Feynman path integral expression for a transition amplitude

$$\int_{x(\tau_1)=x_1}^{x(\tau_2)=x_2} [Dx(\tau)] \; e^{iS[x(\tau)]} = \langle x_2; \tau_2 | x_1; \tau_1 \rangle$$

as a sum over trajectories interpolating between specified inital and final states. Here $x(\tau)$ is a map from the *worldline* of the particle into the *target space*, which has some coordinates x^μ. For

	vacuum	$\lvert 0 \rangle$	$\longrightarrow$	φ	scalar
	1st excited	$\alpha^\mu \lvert 0 \rangle$	$\longrightarrow$	A_μ	vector
	2nd excited	$\alpha^\mu \alpha^\nu \lvert 0 \rangle$	$\longrightarrow$	$G_{\mu\nu}$	tensor
	Etc...				

This figure was stolen from A. Uranga's notes. It shows how different vibrational modes of the string give particles with different spin.

a massive scalar particle, $S[x] = m \int ds = m \int_{\tau_1}^{\tau_2} d\tau \sqrt{\dot{x}^2}$, proportional to the length is a good worldline action. This has the important property that it is invariant under reparametrizations of the worldline $\tau \mapsto \tilde{\tau}(\tau)$. If it weren't there would be some *correct* parametrization, which would be ridiculous.

The object we want will be a sum over paths the string can take

$$\int_{\text{string worldsheet trajectories}} [DX(\tau, \sigma)] \; e^{iS_{ws}[X(\tau,\sigma)]}.$$

Can we define this sum over string embeddings? Notice that we can try to specify our initial and final configurations as disconnected loops.

What might be good about this? Good Thing number 1: many particles from vibrational modes. From far away, meaning when probed at energies $E \ll M_s$, they look like particles. This suggests that there might be some kind of unification, where all the particles (different spin, different quantum numbers) might be made up of one kind of string.

To see other potential Good Things about this replacement of particles with strings, let's consider how interactions are included. In field theory, we can include interactions by specifying initial and final particle states (say, localized wavepackets), and summing over all ways of connecting them, *e.g.* by position-space feynman diagrams. To do this, we need to include all possible interaction vertices, and integrate over the spacetime points where the interactions occur. UV divergences arise when these points collide.

To do the same thing in string theory, we see that we can now draw smooth diagrams with no special points (see figure). For closed strings, this is just the sewing together of pants. A similar procedure can be done for open strings (though we will see that we must also attach handles there).

So, Good Thing Number Two is the fact that one string diagram corresponds to many feynman diagrams. Next let's ask
Q: where is the interaction point?
A: nowhere. It's in different places in different Lorentz frames. This is a first indication that strings will have soft UV behavior (Good Thing Number Three). They are floppy at high energies.

The crucial point: near *any* point in the worldsheet (contributing to some amplitude), it is just free propagation. This implies a certain uniqueness (Good Thing Number Four): once the free theory is specified, the interactions are determined. The structure of string perturbation theory is very
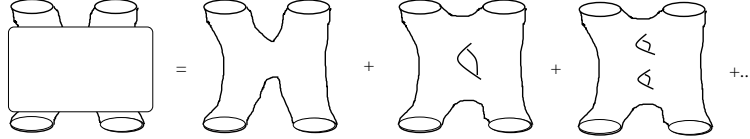
Crudely rendered perturbative interactions of closed strings. Time goes upwards. Slicing this picture by horizontal planes shows that these are diagrams which contribute to the path integral for 2 to 2 scattering. Tilting the horizontal plane corresponds to boosting your frame of reference and moves the point where you think the splitting and joining happened.

rigid; it is very difficult to mess with it (*e.g.* by including nonlocal interactions) without destroying it. [1]

We'll talk much more about what the worldsheet action S_{ws} is. The simplest guess is that it should just be the *area* of the embedded surface, times some factor to make up the dimensions:

$$S_{ws} = T \times \text{Area}.$$

This is called the *Nambu-Goto* action. The factor T is called the *string tension*, because it suppresses worldsheets that are floppy. It has units of mass^2 and sets the scale of the whole discussion. It appears often and has many names:

$$T \equiv M_s^2 \equiv l_s^{-2} \equiv \frac{1}{4\pi\alpha'}.$$

An important point about this action is that it shouldn't depend on how we parametrize it. What coordinates are special? None. This amounts to a principle (a useful one) of general coordinate invariance on the worldsheet. This is familiar from general relativity, and for the same reasons, one way to implement this principle is to introduce a dynamical metric γ_{ab} on the worldsheet. We will find a relation of the form:

$$\int [DX(\tau,\sigma)] \ e^{iS_{HORRIBLE}[X(\tau,\sigma)]} \widetilde{\equiv} \int [DX(\tau,\sigma) \ D\gamma(\tau,\sigma)] \ e^{iS_{NICE}[X(\tau,\sigma),\gamma]}.$$

In this equation, $S_{HORRIBLE}$ here is the Nambu-Goto action proportional to the area of the embedding, S_{NICE} is the 'Polyakov' action

$$S_{NICE} = T \int d\sigma d\tau \sqrt{\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu.$$

($\mu,\nu = 0..D-1$ are target space indices, $a,b = 1,2$ are worldsheet indices.) [2]

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[1] Why stop at one-dimensional objects? Why not move up to membranes? It's still true that the interaction points get smoothed out. But the divergences on the worldvolume are worse. $d = 2$ is a sort of compromise. In fact, some sense can be made of the case of 3-dimensional base space, using the BFSS matrix theory. They turn out to be already 2d quantized! The spectrum is a continuum, corresponding to the degree of freedom of bubbling off parts of the worldvolume. It's not entirely clear to me what to make of this.

[2] The strange symbol in the above relation should be read 'is defined to be approximately equal to,' or something like that. The basic issue is that the left hand side is hard to define because of the sqrt. What we will actually show is that the two actions are classically the same.

This Polyakov action is more symmetric, more redundant. There's an extra redundancy, Weyl invariance, under which

$$\gamma_{ab} \mapsto \Lambda\gamma_{ab}$$

which didn't act on the Nambu-Goto description at all, so is a gauge symmetry. The Polyakov action is just the kinetic term for a free boson coupled to (2d) gravity.

What's 2d gravity? Locally, it's nothing: $\gamma_{ab} = \gamma_{ba}$ has $n(n+1)/2|_{n=2} = 3$ independent components. There are two independent coordinate redefinitions we can do, leaving one degree of freedom left over. In fact, any two-dimensional metric is (locally) conformally flat:

$$ds^2 = e^{2w(\sigma,\tau)}(d\sigma^2 + d\tau^2),$$

i.e. flat up to some (position-dependent) scale factor. Choosing these coordinates on the worldsheet is called 'conformal gauge'.

We can then try to use the extra Weyl redundancy to eliminate the remaining metric degree of freedom, w. But...

Important Plot Twist: **The effect of some reparametrizations on the form of the metric can be undone by a Weyl transformation.** Let's go to light cone coordinates $\sigma^\pm = \frac{1}{\sqrt{2}}(\sigma \pm \tau)$, where the conformal gauge metric looks like

$$ds^2 = e^{2w(\sigma,\tau)}d\sigma^+d\sigma^-.$$

Consider a coordinate change of the form

$$(\sigma^+, \sigma^-) \mapsto (\tilde{\sigma}^+(\sigma^+), \tilde{\sigma}^-(\sigma^-))$$

(the important thing here is that $\tilde{\sigma}^\pm$ depends only on $\sigma^\pm$). This is called a conformal transformation. It leads to

$$ds^2 = e^{2w(\sigma,\tau)}\left(\frac{\partial\sigma^+}{\partial\tilde{\sigma}^+}\frac{\partial\sigma^-}{\partial\tilde{\sigma}^-}\right)d\tilde{\sigma}^+d\tilde{\sigma}^-,$$

it just changes the overall factor in the metric, and hence preserves coordinate angles (though not coordinate lengths). Hence it can be absorbed by a Weyl transformation.

Thus, conformal invariance is a gauge symmetry left unfixed by our choice of conformal gauge.

1.1 spectrum of closed bosonic string

Now we are ready to quantize the closed bosonic string (!). The equation of motion for the X fields (just free massless bosons!) is

$$0 = (\partial_\sigma^2 + \partial_\tau^2)X^\mu = \partial_{\sigma^+}\partial_{\sigma^-}X^\mu$$

whose solution is a superposition of *leftmovers and rightmovers*:

$$X^\mu(\sigma,\tau) = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-).$$

When we put the theory on a circle these have mode expansions

$$X^\mu(\sigma, \tau) = \sum_n e^{in\sigma^+} \tilde{\alpha}_n^\mu + \sum_n e^{in\sigma^-} \alpha_n^\mu.$$

Canonical quantization ($[x, p] = i$) leads to the commutation relations

$$[\alpha_n^\mu, \alpha_m^\nu] = m\delta_{n+m}\eta^{\mu\nu}.$$

(Notice that if the target space has Minkowski signature, there is a scary sign on the RHS for $\mu = \nu = 0$. More about this next week.) So this is just a bunch of SHOs.

All the complications (and the resolution of the scary sign) are in the constraints arising from the coupling to 2d gravity: although we gauged away the metric, we must still impose its equation of motion (it's like a lagrange multiplier) on physical states. Its EOM

$$0 = \frac{\delta S}{\delta \gamma^{ab}} = T_{ab}$$

says that the worldsheet stress tensor should vanish. This includes, in particular, the hamiltonian

$$H = p^2 + \hat{N} + \text{const},$$

where $p^\mu \equiv \alpha_0^\mu$ is the COM momentum of the string, and $\hat{N}$ is an operator that counts the number of excited oscillator modes. The condition that this vanishes is of the form

$$0 = p^2 + m^2$$

which is reasonably called the mass-shell condition and tells us the target-space mass of a string state in terms of its worldsheet data.[3]

Now we can build physical states. [4] The result looks like

$\|0\rangle$	$\mathcal{T}$	$m^2 = -M_s^2$
$\alpha_{-1}^{\{\mu}\tilde{\alpha}_{-1}^{\nu\}}\|0\rangle$	$G_{\{\mu\nu\}}$	$m^2 = 0$
$\alpha_{-1}^{[\mu}\tilde{\alpha}_{-1}^{\nu]}\|0\rangle$	$B_{[\mu\nu]}$	$m^2 = 0$
$\alpha_{-1}^\mu \tilde{\alpha}_{-1\ \mu]}\|0\rangle$	Φ	$m^2 = 0$
$\alpha_{-1}^{\mu_1}\alpha_{-1}^{\mu_2}...\alpha_{-1}^{\mu_{n-1}}\tilde{\alpha}_{-n}^{\mu_n}\|0\rangle$	$T_{\mu_1...\mu_n}$	$m_n^2 = nM_s^2$

$$...$$

(where [..] is meant to indicate antisymmetrization of indices, {..} is meant to indicate symmetrization of indices). A massless symmetric tensor like $G_{\mu\nu}$ which interacts must couple to stress energy, and hence is a graviton. Using the machinery we'll build from the above picture, this mode can be seen to participate in nontrivial scattering amplitudes, which indeed reduce to those computed from GR at low energies.

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[3] once we figure out the additive constant.

[4] There's one more important constraint I didn't mention, called *level-matching* which says that the number of right-moving excitations $\hat{N}$ and the number of left-moving excitations $\hat{\tilde{N}}$ should be equal. This follows from demanding that there be no special point on the worldsheet.

1.2 effective action for small fluctuations

The action which reproduces these scattering amplitudes for the small fluctuations is (at tree level):

$$\int d^D x \sqrt{G}\, e^{-2\Phi} \left(R - c_1 (\nabla \Phi)^2 + c_2 H_{MNP} H^{MNP} + \cdots \right) + \text{tachyonterms}$$

A few comments about this.

1. The modes enumerated in the spectrum above are the *fluctuations* of these fields. There should be little deltas in front of them.

2. The massless scalar Φ is called the *dilaton*. This is because of how it sits in front of the above action – its vev determines the coupling constant:

$$\langle e^{-2\Phi} \rangle \equiv \frac{1}{g_s^2}.$$

   This is exciting, since it means that the coupling can be determined dynamically.

3. Actually this is all true only if $D = 26$! Otherwise the Weyl symmetry, which we need to eliminate the scale factor of the metric, is anomalous.

4. The fact that the mass^2 of the field $\mathcal{T}$ ($\mathcal{T}$ is for Tachyon) is negative means that this vacuum around which we're doing perturbation theory is unstable. To what is still a bit of a mystery. The best guess is Nothing, meaning a much more serious kind of Nothing than just empty space. Nevertheless, we'll spend a lot of time thinking about the bosonic string in perturbation theory.

Given that we don't have a tachyon in our spectrum, or 26 dimensions, how can we change this story to get a more favorable outcome? What could we have done differently to avoid these somewhat disappointing conclusions?

1.3 deforming the string background

The first idea is the answer to the question
Q: What's special about D free bosons?
A: Nothing. We were just studying them because of some strange idea from particle physics that space is flat. Space can be curved. So just like we could coupling a particle to a background metric and gauge field by the replacement

$$\int \sqrt{\dot{X}^2} \mapsto \int \sqrt{\dot{X}^\mu \dot{X}^\nu G_{\mu\nu}} + \int A_\mu \dot{X}^\mu$$

we can couple the Polyakov action to a background by the replacement

$$S_{NICE} \mapsto M_s^2 \int d^2\sigma \sqrt{\gamma} \partial_a X^\mu \partial_a X^\nu \left(G_{\mu\nu}(X)\gamma^{ab} + B_{\mu\nu}(X)\epsilon^{ab}\right) + \dots.$$

Here the Xs should be thought of as *local* coordinates on some manifold whose metric is G. This is the action for a *nonlinear sigma model* (NLSM). It can be thought of as related to the previous flat-space theory by making the fluctuations big – add in so many gravitons that they actually change the background that the string propagates through.

While we're at it, why can't we just take any 2d QFT and couple it to worldsheet gravity and say ha! there's a string theory? In some ways, we can. There is one important dividing line, though. If our 2d QFT is *conformally invariant* (a CFT), the modes of the 2d metric decouple, and the story above goes through. If the worldsheet theory violates conformal invariance, sometimes sense can be made of it by including the modes of the metric; this endeavor goes under the general name *Liouville*.

Back to the NLSM, what is the condition that it is a CFT? The free theory, with flat-space target, is clearly scale invariant (assign the scalars mass dimension zero). We can think of a big smooth curved space as a perturbation around this fixed point. The metric and antisymmetric (AS) tensor B are the couplings. The beta function for the metric is

$$\beta_{\mu\nu} \propto R_{\mu\nu} + ...$$

where $R_{\mu\nu}$ is the ricci curvature of the metric $G_{\mu\nu}$, and ... are terms that are small when the space is large compared to the string length l_s. A *Calabi-Yau manifold* (CY) admits a metric which is Ricci flat, and hence Calabi-Yau nonlinear sigma models are an interesting class to consider for string compactification.

What do I mean by compactification? Consider the target space to be of the form $R^{3,1} \times X_{D-4}$ where X is some compact manifold, whose size we'll characterize by some length scale L. If L is large compared to l_s, we can describe this using the effective field theory and expand the D-dimensional fields in modes of the laplacian on X (this is called KK reduction).

In the regime where $L \sim l_s$, we need information from the worldsheet CFT. More generally, the worldsheet CFT needn't have any geometric interpretation at all; it could be some bunch of critical potts models tensored together. In this sense the target space geometry, even in perturbative string theory, is emergent. The fact that geometry is a derived quantity makes it possible for string theory to do things like resolve singularities of GR and allow interesting topology-changing processes.

What picks out the value of L? This is the important question of 'moduli stabilization'. The answer lies in thinking of L (really, all the parameters of the geometry) as a 4d field, deriving an effective potential, and finding its minima. This is a big, ongoing project.

1.4 superstrings

Compactification can fix the number of dimensions, but there's still a tachyon, and there aren't any fermions in the target space spectrum of the bosonic string.

The second way to change the conclusion is to modify the worldsheet gauge group. If we replace 2d gravity with 2d supergravity, the residual gauge symmetry (previously conformal symmetry) be-

comes *super*conformal symmetry. We'll need to include *worldsheet* fermions in a 2d supersymmetric way. This is the RNS description of superstrings.

The connection between worldsheet fermions and spacetime ones is somewhat indirect (this translation between worldsheet things and spacetime things is a big part of our business here). Part of the sum over 2d geometries includes a sum over *spin structures*, which goes under the name 'GSO projection'. There are a few consistent choices for how to do this but in general, it does a bunch of good things:
it removes the tachyon from the spectrum
it adds in spacetime fermions
it changes the critical dimension to 10
sometimes it implies target space supersymmetry.

There are 5 apparently different 10d supersymmetric superstrings called IIB, IIA, type I, heterotic $E_8 \times E_8$ and heterotic $SO(32)$, which differ mainly in the ways of choosing the worldsheet gauge group.

Even at the level of perturbation theory, some of these are related by 2d field theory dualities upon compactification on a circle (T-duality).

Compactification of superstrings is ridiculously successful at reproducing or predicting ideas for physics of the standard model and beyond: gauge theory, chiral matter, multiple generations, supersymmetry, supersymmetry-breaking...

1.5 perspectives of other probes

D-branes exist. They are very important. They are objects with masses proportional to $\frac{1}{g_s}$, which means you don't make them in perturbation theory around a vacuum without them (though they are present in the 10d type I vacuum). However, their fluctuations have a beautiful desciption in perturbation theory: just add in a new sector of open strings that end only on the place where you want to put the D-brane. The modes of these strings allow the brane to wiggle. In general, they comprise a Yang-Mills gauge theory – the fact that matrices are involved is simply because strings have two ends.

1.6 Non-perturbative questions

So far, all of this was visible in perturbation theory and effective field theory. At this point I have to confess that we don't really know what the theory is at finite coupling. One thing we do know, using supersymmetry, is that the 10d superstrings aren't independent of each other, but are all limits of some mysterious thing that we don't understand. The relations between the string theories are like $g \to 1/g$ dualities (S-duality) of field theory.

1.7 the many roles of field theory in string theory.

Part of why string theory is not the fringe exercise that the newspaper portrays it to be is that in studying string theory the basic tool is field theory, which is what most theoretical physicists have been working on for almost a century. Field theory plays many roles in string theory, and it's important to keep them clearly in mind. These include:

1. The 2d field theory on the worldsheet is a 2d conformal field theory (CFT). Such theories sometimes also manage to describe systems that exist in the real world.

2. The effective field theory of the actual full-blown with-gravity string theory is some (super)gravity theory coupled to matter fields, which is an expansion in derivatives.

3. There are *decoupling limits* of string theory, where we consider energies much less than the string scale, and the effects of oscillator modes go away. Notable examples are the gauge theories living on piles of D-branes, which can be decoupled from gravity if we consider the low-energy limit at weak coupling.

4. Something very interesting happens if we instead take the same limit at larger coupling, where the gravitational back-reaction of the branes is important. In this regime they curve the space into a *black brane* (like a black hole but extended in dimensions other than time). Taking the same limit as we took at weak coupling now focuses in on the near-horizon region of this object, and (in the case of D3-branes) the space there is $AdS_5 \times S^5$. The conclusion we come to is that these two systems (the gauge theory on the D3-branes and the string theory in AdS) are the same up to some variation of the coupling. The more general claim is that string theories in some backgrounds (notably $AdS_5 \times S^5$) are *equivalent* to field theories on some lower dimensional space. This is called *holographic duality*. It's not yet known how to systematically construct such dual pairs, but a lot can be learned in both directions from the ones we know.

2   What might it be good for?

Claim: String theory is the most mysterious obviously-consistent construct in theoretical physics. It's an alien artifact from the 20th century. What is it?? We don't know.

What is it for? Here are some possible answers:

- GR isn't UV complete. (not even N=8 supergravity, even if it's finite.)
  Something happens at high energy (perturbation theory is an expansion in E/M_{planck})
  Something happens at singularities. (GR is provably wrong there!)

  String theory is a quantum theory of gravity, with GR in low-energy limits. The fact that it also includes particle physics means that it might be very useful for learning more about the early universe, such as the period of inflation.

  At short distances, string theory changes our notions of geometry (T-duality, topology change..)

- Many questions aren't askable in field theory.

    Parameters and content of SM of particle physics demand explanation: why three generations? why gauge theory? why this one?

    Beyond particle physics, we can try to ask questions like:

    are there extra dimensions? of what shape?

    why are there 3+1 big dimensions?

    can the topology of space change? (yes.)

    was there something 'before' the big bang?

    do black holes eat probability? (no.)

    why are there light vectorlike particles?

    ...

    This fact that string theory allows more questions to be asked makes it scientifically valuable.

- Via dualities like AdS/CFT, string theory can be used to study strongly-coupled field theories, like QCD in the IR, condensed matter systems, who knows what else?