University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215A QFT Fall 2021
Assignment 9

Due 11:59pm Wednesday, November 23, 2021 (but a day or two late is fine)

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1. **Brain warmers on $\mathsf{SO}(3)$.**

   (a) Consider the statement that the rotation generators transform as a vector under rotations:

   $$\left(D_{(j=1)}(\theta)\right)^k_j \mathbf{J}^j = D_R(\theta)\mathbf{J}^k D_R(\theta)^\dagger, \tag{1}$$

   where $D_R(\theta) = e^{-\mathbf{i}\theta^i \mathbf{J}^i}$ and $D_{(j=1)}(\theta) = e^{-\mathbf{i}\theta^i J^i_{(j=1)}}$, with $(J^i_{(j=1)})^j_k = -\mathbf{i}\epsilon^{ijk}$. Show that to leading nontrivial order in θ (about $\theta = 0$) this is equivalent to the $\mathsf{so}(3)$ Lie algebra,

   $$[\mathbf{J}^i, \mathbf{J}^j] = \mathbf{i}\epsilon^{ijk}\mathbf{J}^k. \tag{2}$$

   (b) Starting from the form of the generators in the vector (spin 1) representation,

   $$(\mathbf{J}^i)^j_k = -\mathbf{i}\epsilon^{ijk} \tag{3}$$

   (with $\epsilon^{123} = 1$) construct the matrix realizing a rotation by angle θ about the z axis on a vector.

2. **Decay of a scalar particle.**

   Consider the following Lagrangian, involving two *real* scalar fields Φ and ϕ:

   $$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2 + \partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right) - \mu\Phi\phi^2.$$

   The last term is an interaction that allows a Φ particle to decay into two ϕs, if the kinematics allow it. Calculate the lifetime of the Φ particle to lowest order in μ. In this problem you can set $d = 3$. What is the condition on the masses for a finite lifetime?

3. **Scalar particle scattering cross-sections.**

   What is the leading-order differential cross-section $\frac{d\sigma}{d\Omega}$ for $2 \to 2$ snucleon-snucleon scattering in $d = 3$ space dimensions in the center-of-mass frame?

   What is the total cross section in the limit that the snucleons (the particles being scattered) are massless?