Zoom norms: • video on please.
• unmute to ask Q's.

WORK: • weekly psets

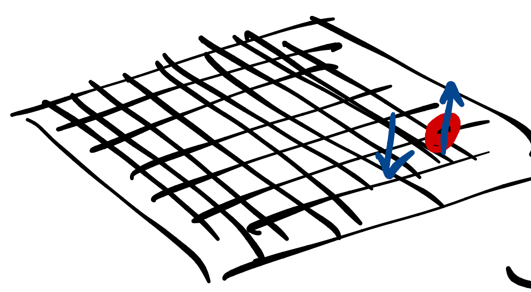
• find typos (send email)

Intro Remarks.

$$\text{QFT} \equiv \text{QM of } \underbrace{\text{extensive dofs}} \equiv \text{FIELD.}$$

at each point of space

something that can wiggle.



UBIQUITOUS:

egs: • atoms in a solid

• e^- in atoms in a solid (metal)

• spins of e^- in atoms in a solid (magnet)

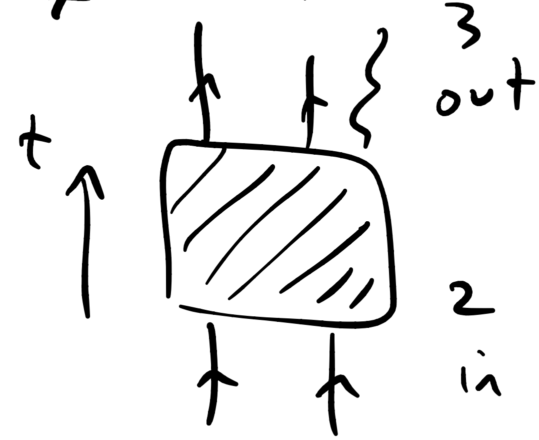
- EM in vacuum.
- electron field in vacuum
- Standard Model

<u>sometimes</u>: Lorentz invariant.

QM + SR $\Rightarrow$ QFT [Weinberg]

QFT $\supsetneq$ Relativistic QFT.

<u>Particle Production</u>:



VS:

<u>Single-particle QM (few)</u>: ① states $|\psi\rangle \in \mathcal{H}_1$

② observables are linear ops on $\mathcal{H}_1$

<u>eg</u>: $\hat{x}, \hat{p}$

③ time evolution: $i\hbar\, \partial_t |\psi\rangle = \hat{H} |\psi\rangle$

Hamiltonian.

④ measurement: measure $A = \sum_a a\,|a\rangle\langle a|$

in state $|\psi\rangle$ we get a w prob $|\langle a|\psi\rangle|^2$.

$$|\psi\rangle = \sum_a \psi(a)|a\rangle \longleftarrow \mathbb{1} = \sum_a |a\rangle\langle a|$$

$$\psi(a) = \langle a|\psi\rangle$$

eg: $\hat{x} = \int dx\; x\,|x\rangle\langle x|$

$|\psi\rangle = \int dx\, \psi(x)\, |x\rangle \qquad \psi(x) = \langle x|\psi\rangle.$

WARNING: NOT A FIELD!!

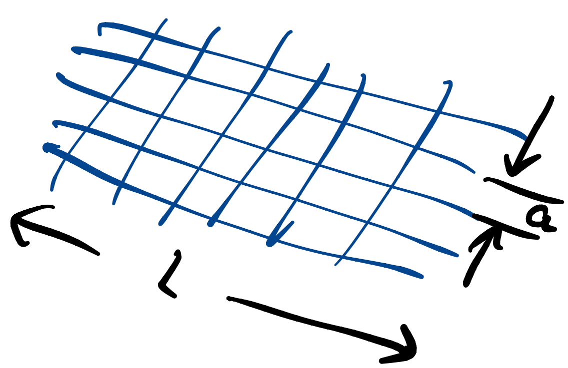
GOAL: how to accomodate particle production?

- "Divergences"

# of points in space $= ?\ \infty$.

"$a \to 0$" UV divergence

"$L \to \infty$" IR divergence



Weak vs Strong Coupling

Some QFTs are well-described as weakly coupled springs

$$L = \sum_n \dot{q}_n^2 - \sum_n q_n^2 + \lambda q^3$$

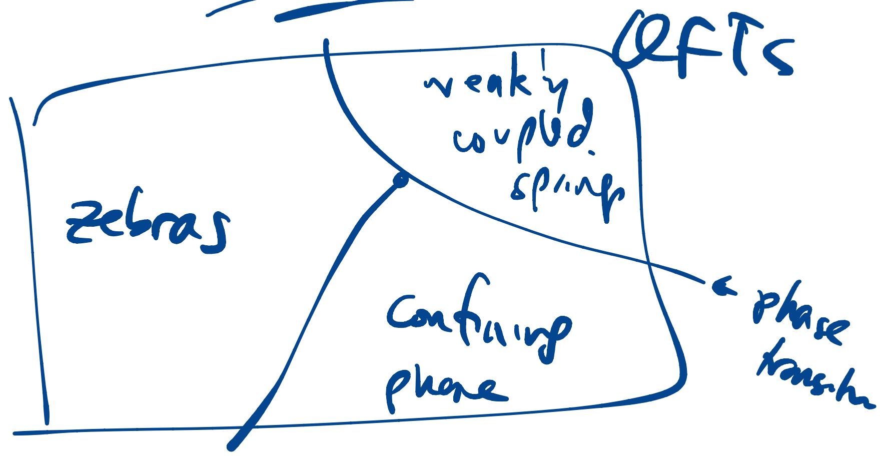
requires

$|0\rangle_\lambda$ adiabatically connected to $|0\rangle_{\lambda=0}$.

Not always true. (eg: QCD)

ie no phase transition in between.

multiplicity of phases of matter is reflected in phases of QFT



Summary: QFT $\supset$ pert. theory.

(this quarter: mostly pert. theory...)

Books:
- Peskin
- Zee
- Schwartz (SM)
- Tong
- Fradkin

Conventions: • $ds^2 = +dt^2 - d\tilde{x} \cdot d\tilde{x}$

$(+ - - -)$

• $\hbar = \frac{h}{2\pi} = 1$ units, $c = 1$

$\Rightarrow ([\cdot] \equiv \text{units of})$

$[\text{mass}] = [\frac{1}{\text{length}}] = [\text{energy}]$

- $đk \equiv \frac{dk}{2\pi}$ & $f(x) = \int đ^D k \, e^{ikx} \hat{f}(k)$.

$$đ^D k \equiv \frac{d^D k}{(2\pi)^D}$$

- $D = d + 1$ = # dims of spacetime.

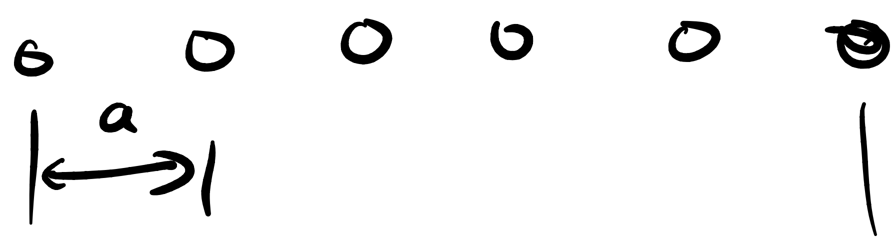
---

(1) Particles → fields → particles

[particles are quanta of excitation of fields]

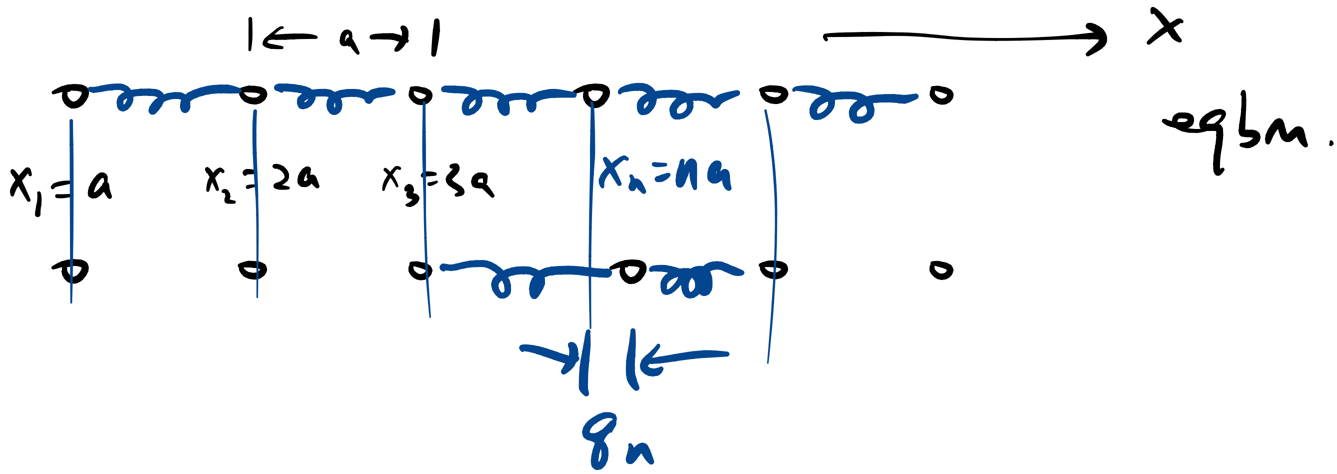
1.1 Quantum sound (phonon)

crystalline solid



atoms fixed in eqbm by $V(\{q_n\}) = V(q_n - q_{n-1})$

interaction w/ neighbors.



$$V(q_n - q_{n-1}) = \underline{\underline{0}}\,\cancel{(q_n - q_{n-1})} + \underline{\underline{g_2}}\left(\frac{q_n - q_{n-1}}{a}\right)^2 + \underline{\underline{g_3}}\left(\frac{q_n - q_{n-1}}{a}\right)^3 + \dots$$

$g_2 \sim g_3 \sim g_4 \dots$

suppose $q_n - q_{n-1} \ll a$.

$$\Longrightarrow \quad H = \sum_{n=1}^{N} \Big(\frac{p_n^2}{2M} + \underbrace{\tfrac{1}{2} K (q_n - q_{n-1})^2}_{\equiv V(q)} \Big) + \cancel{\dots}$$

$$V(q) = K_{nm} q_n q_m \qquad K_{nm} = (K^T)_{nm}$$

evecs of K $\equiv$ normal modes.

Assume P.B.C. $x_{N+1} = x_1$. $q_{N+1} = q_1$.

observation: $[\underline{\underline{K}}, T] = 0$

auxiliary 1-particle QM problem : diagonalize

$$\underline{K} \equiv \sum_{n,m} K_{nm} |n)(m|$$

translation op

$$T \equiv \sum_n |n+1)(n|$$

$$\mathcal{H}_1 \equiv \text{span}\{|n\rangle\}_{n=1..N}$$

$$T_{nm} = \begin{pmatrix} 0 & 1 & & & \\ 0 & 0 & 1 & & \\ & 0 & 0 & 1 & \\ & & & \ddots & \\ 1 & & & & \end{pmatrix}$$

$$\underline{|n+N) \equiv |n\rangle.}$$

$$T \sum_n e^{in\theta} |n\rangle = \sum_n e^{in\theta} |n+1\rangle$$

$$= \left(\sum_{\tilde{n}=n+1} e^{i\tilde{n}\theta} |\tilde{n}\rangle \right) \underline{\underline{e^{-i\theta}}}$$

SLOGAN: linear + transl. inv't $\Rightarrow$ Fourier transform.

Regulators:

lattice spacing $\Longrightarrow$ $\underline{\underline{n\in\mathbb{Z}}} \Rightarrow e^{in\theta} = e^{in(\theta+2\pi)}$

$\Longrightarrow$ Range of wavenumbers is finite

$\boxed{\text{UV regulator}}$

$\theta \stackrel{?}{=} \theta + 2a$

$\Rightarrow$ biggest k smallest $\lambda = 2\pi/k \geq a$

N is finite $\Longrightarrow$ $e^{in\theta} \stackrel{!}{=} e^{i(n+N)\theta}$

$n = 1 \dots N$

$\boxed{\text{IR regulator}}$

$\Longrightarrow \{\theta\}$ is discrete.

$(x_n \equiv na)$

$$\begin{cases} q_k = \dfrac{1}{\sqrt{N}} \displaystyle\sum_{n=1}^{N} e^{ikx_n} q_n = \sum_n \underline{\underline{U_{kn}}}\, q_n \\[2ex] p_k = \dfrac{1}{\sqrt{N}} \displaystyle\sum_{n=1}^{N} e^{ikx_n} p_n \end{cases}$$

$U^\dagger U = \mathbb{1} = UU^\dagger$

unitary.

$$k \in \left\{ \frac{2\pi j}{Na} , j = 0 \dots N-1 \right\}$$

$$\sum_n p_n^2 = \frac{1}{N} \sum_{k,k'} p_k p_{k'} \underbrace{\sum_n e^{-i(k+k')na}}_{N\,\delta_{k+k'}}$$

$$\left(p_n = \frac{1}{\sqrt{N}} \sum_k e^{-ikna} p_k \right)$$

$$= \sum_k p_k p_{-k}$$

is Hermitian

$$p_n = p_n^* \rightarrow p_k^* = p_{-k}.$$

$$\sum_n (q_n - q_{n-1})^2 = \frac{1}{N} \sum_{kk'} q_k q_{k'} \left[(1 - e^{-ika})(1 - e^{-ik'a}) \right]$$

$$1 + 1 - e^{-ika} - e^{+ika}$$

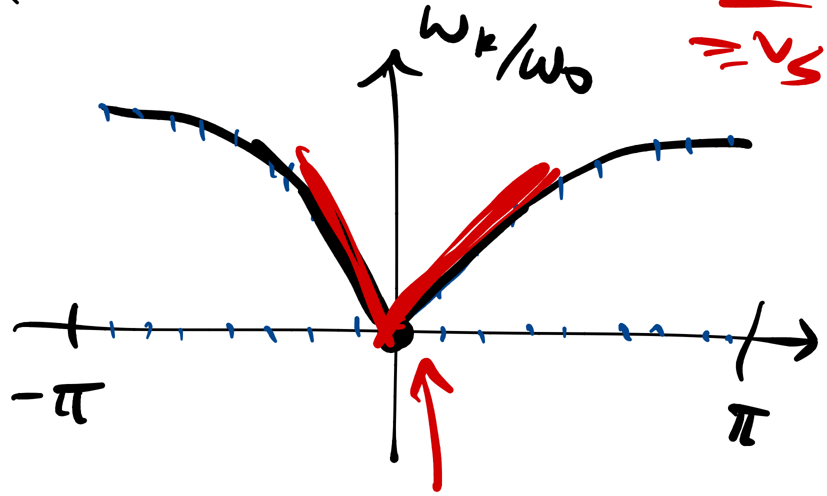
$$\underbrace{\sum_n e^{-i(k+k')na}}_{= N\,\delta_{k+k'}}$$

$$= \sum_k q_k q_{-k} (\underbrace{2 - 2\cos(ka)}_{\overset{\text{trig}}{=}\ 2\sin^2\left(\frac{ka}{2}\right)})$$

$$\Rightarrow H = \sum_k \left(\frac{p_k p_{-k}}{2m} + \frac{1}{2} m \omega_k^2 q_k q_{-k} \right)$$

$$\underline{\omega_k = 2\sqrt{\frac{K}{m}}\sin\frac{|k|a}{2}} \overset{ka\ll 1}{\simeq} \underbrace{\left(\frac{\omega_0 a}{2}\right)}_{=v_s}|k|$$

<u>Spectrum of Normal Modes:</u>



$$\underline{\underline{\omega^2 = v_s^2 k^2}}$$

Lorentz inv't.

<u>sound</u>

wave eqn.

$$\omega_k^2 = \omega_0^2 \sin^2\frac{ka}{2} \overset{ka\ll 1}{=} \frac{\omega_0^2 a^2}{4}k^2 + \underline{\underline{\frac{2\omega_0^2}{3!}\left(\frac{ka}{2}\right)^4 + \ldots}}$$

$$\sin x = x - \frac{x^3}{3!} + \ldots$$

$$\sin^2 x = x^2 - \frac{2}{3!}x^4$$

eom: $0 = \ddot{q} - v^2 \partial_x^2 q - g\, \partial_x^4 q + \dots$

$$H = \sum_k \left(\frac{p_k p_{-k}}{2m} + \frac{m\omega_k^2}{2} q_k q_{-k} \right)$$

$$= \sum_k \hbar\omega_k \left(a^\dagger_k a_k + \frac{1}{2} \right)$$

$a_k^\dagger |0\rangle \equiv |$ one phonon of momentum $\hbar k \rangle$

$$0 = k_\mu k^\mu \equiv k_\mu k_\nu \eta^{\mu\nu}$$

$$k_\mu = (\omega, k)_\mu$$