University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215A QFT Fall 2021
Assignment 9

Due 11:59pm Wednesday, November 24, 2021 (but a day or two late is fine)

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1. **Brain warmer.** Can there ever be a resonance in a t-channel diagram? Let me break the question down a bit:

   (a) Consider a $2 \leftarrow 2$ scattering process where all the particles have the same mass. Let p_1, p_2 be the momenta of the particles in the initial state. Prove that the Mandelstam variables $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$ cannot be positive when the particles are on-shell $p_i^2 = m^2$.

   (b) Bonus problem: What happens if the particles have different masses? It may be worth distinguishing two cases:
   (a) when the collision is *elastic*, so that the particles retain their identity and therefore $m_1 = m_3$ and $m_2 = m_4$.
   (b) the fully general case where m_i are all different.

2. **Decay of a scalar particle.**

   Consider the following Lagrangian, involving two real scalar fields Φ and ϕ:

   $$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \Phi \partial^\mu \Phi - M^2 \Phi^2 + \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2\right) - \mu \Phi \phi^2.$$

   The last term is an interaction that allows a Φ particle to decay into two ϕs, if the kinematics allow it. Calculate the lifetime of the Φ particle to lowest order in μ. In this problem you can set $d = 3$. What is the condition on the masses for a finite lifetime?

3. **Scalar particle scattering cross-sections.**

   What is the leading-order differential cross-section $\frac{d\sigma}{d\Omega}$ for $2 \to 2$ snucleon-snucleon scattering in $d = 3$ space dimensions in the center-of-mass frame?

   What is the total cross section in the limit that the snucleons are massless?