Physics 220: Symmetry in Physics

ADMIN: • **OH**: after lectures
OR email / appointment
OR email.

• **WORK** (A) problem sets.
(please do electronically)

(B) brief final paper.

(C) find typos & errors & email me.

mcgreevy.physics.ucsd.edu/f20

• **Zoom**:

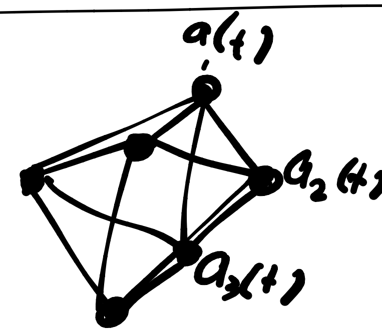
Rough Plan:

- Basic notions
- finite groups & representations
- Lie groups & algebras
- beyond.

Motivating problems:

~~octagon~~:

octahedron:



$$a(t+\Delta t)_i = a(t)_i + \lambda \sum_{\langle ij \rangle} \left(a(t)_j - a(t)_i\right)$$

$$= a(t)_i + \lambda \sum_j H_{ij} \left(a(t)_j - a(t)_i\right)$$

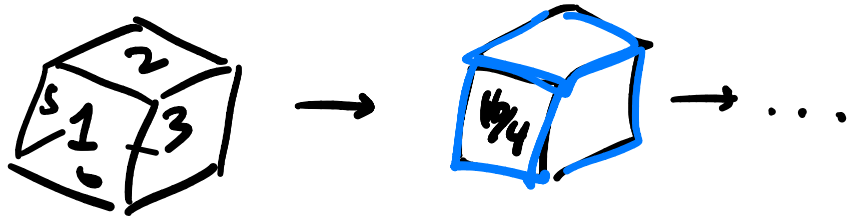
$$H_{ij} \equiv \begin{cases} 1 & \text{if } \langle ij \rangle \text{ is an edge} \\ 0 & \text{else} \end{cases}$$

adjacency matrix

Q: 1) what's $a_i(t \gg \ldots)$

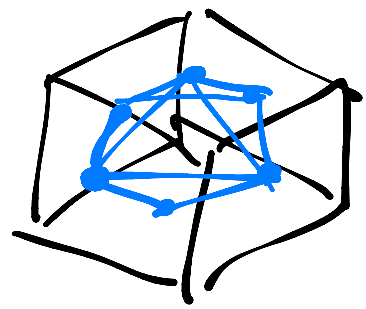
2) how long does it take?

$5+2+6+3=16$



Q: what it's after 30 days?

what error?

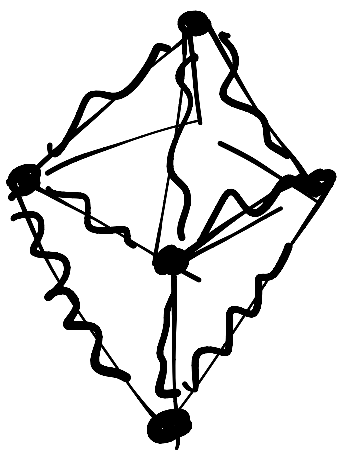


faces $\longleftrightarrow$ vertices

cube octahedron

eg 2

•



Find the normal modes.

'tight binding' model.

• eg: $$H = \sum_{\langle ij \rangle} |i\rangle\langle j| .$$

$$\langle i|H|j\rangle = H_{ij}$$

Q: Find the spectrum.

Symmetry & Topology

Noether's Thm: Continuous Symmetries $\longleftrightarrow$ Conserved quantities.

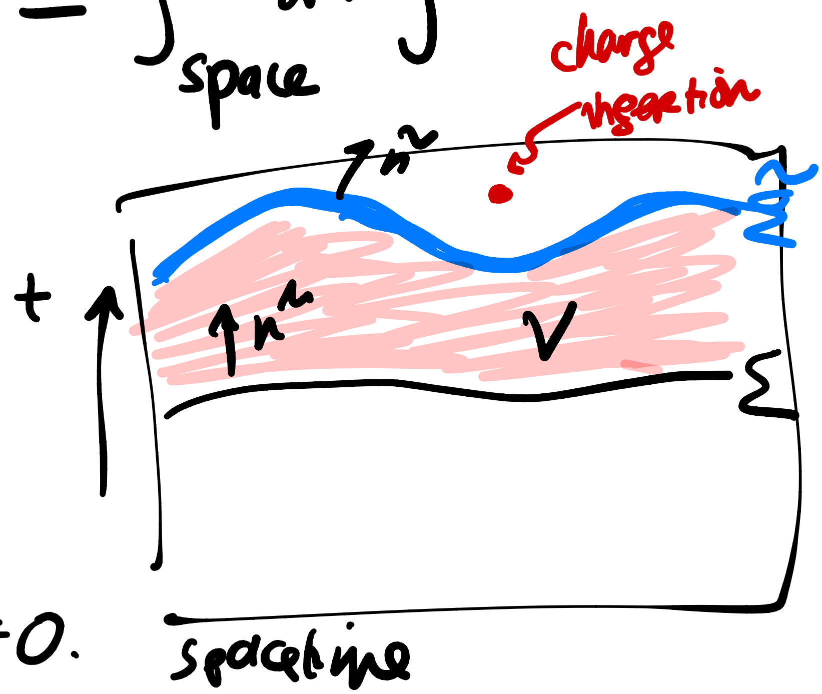
In field theory: dofs are distributed over space.

- EM field $\vec{E}, \vec{B}$
- magnetic material

$$\vec{\Phi}(x) = \left(\text{magnetization near } x \right)$$

$$Q_\Sigma = \int_\Sigma n_\mu j^\mu = \int_{\text{space}} d^d x \, j^0$$

$$Q_{\tilde{\Sigma}} = \int_{\tilde{\Sigma}} \tilde{n}_\mu j^\mu$$



conserved:

$$Q_{\tilde{\Sigma}} - Q_\Sigma = \int_{\substack{\tilde{\Sigma}-\Sigma \\ = \partial V}} n \cdot j \stackrel{\text{stokes}}{=} \int_V \partial_\mu j^\mu = 0.$$

Q is a topological invariant!

symmetries $\longrightarrow$ "topological surface operators".

Converse to Noether: Given $Q = Q^\dagger$

$$\longrightarrow U_Q \equiv e^{i\alpha Q}$$

$$[U_Q, H] = 0.$$

Generalization:

$\exists$ topological operator associated with codim. p surface

$$\int_{X_{D-p}} n_1^{\mu_1} \cdots n_p^{\mu_p} j_{\mu_1 \cdots \mu_p} = \int_{X_{D-p}} {*j}$$

$$\partial^{\mu_1} \underline{j_{\mu_1 \mu_2 \cdots \mu_{p+1}}} = 0.$$

antisymmetric under $\mu_i \mu_j \to \mu_j \mu_i$

p-form symmetry.

Use of groups as top. invariants:

homology groups
Cohomology "
homotopy "
Coborduism ".

useful eg:
- classifying phases of matter
- identify topological defects
- compactifying string theory.

Common situation in field theory:

Given . list of dofs $\{\underline{\vec{\phi}(x)}\}$
. a symmetry group.

<u>Q:</u> how do they interact?

what is $S[\phi] = \int dt\, d^dx\, \mathcal{L}(\phi, \partial_\mu \phi)$

↑ <u>locality</u>

<u>eg:</u> $S_{\underline{Maxwell}}(A) = \int d^3x\, dt\ \frac{1}{2}(E^2 - B^2)$ $\left\{ \begin{array}{l} B = \nabla \times A \\ E = \vec{\nabla} A_0 - \partial_t \vec{A} \end{array} \right.$

eg 2: chunk of material, eg a magnet.

dofs = {local magnetization $\vec{\phi}(x)$}

symmetry: subgroup of spin rotations & spatial rotations ...

Q: what is $\int \mathcal{L}(\phi, \partial\phi \ldots)$? $= S[\phi]$

eg 3: particle physics.

discover particles = quanta of fields ϕ (photon is quantum of A_μ)

symmetries: determined by reactions

$$X + Y \to Z + W$$

what is $S[\text{fields}]$?

other roles of group theory in physics:

- crystalline solids
- stat mech problems

$$Z = \sum_{\text{configs}} e^{-\beta H(\text{config})}$$

- construction of S.M. (gauge theory)
- physics in symmetric spacetimes

  Euclidean space (slow, small)

  Minkowski spacetime (small)

  de Sitter spacetime (big)

Recommendations:

- Zee
- Georgi
- Stone & Goldbart

1.1 Basic Notions

Def: A group G is a set w/ a product

$$\cdot : G \times G \longrightarrow G$$
$$(g_1, g_2) \longmapsto g_2 \cdot g_1$$

such that

1) (associative) $(g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 \cdot g_3)$
2) (identity) $\exists\, e$ s.t. $e \cdot g = g = g \cdot e$ $\forall g \in G$.
3) (inverses) $\forall g$ $\exists\, g^{-1}$ s.t.

$$g^{-1} g = e = g g^{-1}.$$

Convention: Doing g_1 then g_2 is $g_2 \cdot g_1$ (time goes to the left)

If $g_1 g_2 = g_2 g_1$ $\forall g_{1,2}$ G is abelian else non-abelian.

Order of the group $|G|$ = # of elements.

can be finite or ∞.

∞: discrete, continuous

compact, noncompact.

(G is compact if $\sum_{g\in G} 1$ makes sense.)

Multiplication table:

$\mathbb{Z}_4$	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

$i^2=j^2=k^2$

$ij=k$

Q_8	1	-1	i	-i	j	-j	k	-k
1	1	-1	i	-i	j	-j	k	-k
-1	-1	1	-i	i	-j	j	-k	k
i	i	-i	-1	1	k	-k	-j	j
-i	-i	i	1	-1	-k	k	j	-j
j	j	-j	-k	k	-1	1	i	-i
-j	-j	j	k	-k	1	-1	-i	i
k	k	-k	j	-j	-k	k	-1	1
-k	-k	k	-j	j	k	-k	1	-1

nonabelian.

<u>Sudoku rule</u> : $ax = bx \Rightarrow a = b$.

$(\text{BHS})x^{-1}$

$\Rightarrow$ each element appears exactly once in each row & col of the mult. table.

<u>1.2. Examples of groups & where to find them.</u>

<u>Equivalence of groups</u>:

A map $f: G \to H$ which preserves the product is a <u>group homomorphism</u>

$$f(g_1)f(g_2) = f(g_1 g_2).$$

Two groups are equivalent if $\exists$ a group homomorphism which is 1-1 and onto (bijection)