University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 220 Symmetries Fall 2020
Assignment 1

Due 12:30pm Monday, October 12, 2020

- Homework will be handed in electronically. Please do not hand in photographs of hand-written work. The preferred option is to typeset your homework. It is easy to do and you need to do it anyway as a practicing scientist. A LaTeX template file with some relevant examples is provided here. If you need help getting set up or have any other questions please email me.
- To hand in your homework, please submit a pdf file through the course's Canvas website, under the assignment labelled hw01. Please put the filename in the format:

  `220-hw01-YourLastName-YourFirstName.pdf`

Thanks in advance for following these guidelines. Please ask me by email if you have any trouble.

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1. **Brain-warmer.** Consider the group made from the set $\{2, 4, 6, 8\}$ under multiplication modulo 10.

   (a) Write the multiplication table and check the sudoku rule. What is the identity?

   (b) Is it equivalent to $\{1, 2, 3, 4\}$ under multiplication modulo 5? If not, why not?

2. **A more economical definition of group.** Show that we can weaken two of the conditions defining a group without changing the outcome, as follows:

   (2') A left identity $\mathbb{1}$ exists, such that for any element g, $\mathbb{1}g = g$.

   (3') For any element g, a left inverse f exists: $fg = \mathbb{1}$.

   Show that these two conditions imply that the left identity is also the right identity and the left inverse is also the right inverse.

3. **The group with three elements.** Write down the multiplication table for the group with three elements; show that it is uniquely fixed by the definition. Is it abelian?

4. **Groups with four elements.** Write down the multiplication table for all groups with four elements. Are they all abelian?

5. **Conjugacy classes of S_n.**

   (a) Write the following elements of S_n in cycle notation:

   $$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}.$$

   (b) Check that the cycle structure is preserved by conjugation, *e.g.* for $\pi^{-1}\sigma\pi$, $\pi^{-1}\rho\pi$.

   (c) Bonus problem: give a proof that this always works.

6. **Quaternions.** [I'm belatedly punting this problem to hw 2.]

   Decompose the quaternion group Q_8 into conjugacy classes.