University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 217 Fall 2018
Assignment 3

Due 12:30pm Monday, October 22, 2018

1. **Brain-warmer.** Show that the relation $e^{2J'} = \cosh 2J$ can be rewritten as $v' = v^2$ in terms of $v \equiv \tanh J$.

2. **Decimation of 1d Ising model in a field.**

   Consider again a closed (periodic) chain of N classical spins $s_i = \pm 1$ with Hamiltonian

   $$H = -J \sum_i s_i s_{i+1} - h \sum_i s_i + \text{const}, \quad s_{N+1} = s_1$$

   The partition function is $Z(\beta J, \beta h) = \sum_{\{s\}} e^{-\beta H}$; let's measure J, h in units of temperature, *i.e.* set $\beta = 1$.

   Suppose that N is even.

   (a) Decimate the even sites:

   $$\sum_{s_{\text{even}}} e^{-H(s)} \equiv \Delta e^{H_{\text{eff}}(s_{\text{odd}})}.$$

   More explicitly, identify the terms in $H(s)$ that depend on any one even site, $H_2(s)$ and define its contribution to H_{eff} by

   $$\sum_{s_2} e^{-H_2(s)} \equiv \Delta e^{-\Delta H_{\text{eff}}(s_1, s_3)}$$

   Rewriting $H_{\text{eff}}(s_{\text{odd}}) = -J' \sum ss - h' \sum s - \text{const}$ in the usual form, find J', h' and the constant in terms of the microscopic parameters J, h.

   (b) Let $w \equiv \tanh \beta J, v \equiv \tanh \beta h$. Plot some RG trajectories in the v, w plane.

   (c) Find all the fixed points and compute the exponents near each of the fixed points.

3. **High temperature expansion for Ising model.**

   In lecture, we rewrote the partition function of the nearest-neighbor Ising model (on any graph) as a sum over closed loops. Without a magnetic field, the loops were weighted by their length, just like in our discussion of SAWs. If we turn on a magnetic field, how does it change the form of the sum?