University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215A QFT Fall 2016
Assignment 8

Due 11am Tuesday, November 29, 2016

1. **Brain-warmer.** Check that
$$(p \cdot \sigma)(p \cdot \bar{\sigma}) = p^2.$$

2. **Symmetries of the Dirac lagrangian.**

   Find the Noether currents j^μ and j_5^μ associated with the transformations $\Psi \to e^{\mathbf{i}\alpha}\Psi$ and $\Psi \to e^{\mathbf{i}\alpha\gamma^5}\Psi$ of a free Dirac field. Show by explicit calculation that the former is conserved and the latter is conserved if $m = 0$.

3. **Majorana mass.** Show that a *majorana mass* term for a Weyl fermion
$$\mathcal{L}_m = m\psi_R^t \mathbf{i}\sigma^2 \psi_R + h.c. = m\left(\psi_R\right)_\alpha \epsilon^{\alpha\beta}\left(\psi_R\right)_\beta + h.c.$$
is Lorentz invariant, but violates particle number. Figure out what the $+h.c.$ is explicitly. Find the equations of motion. Why isn't $\left(\psi_R\right)_\alpha \epsilon^{\alpha\beta}\left(\psi_R\right)_\beta \stackrel{?}{=} 0$ given the antisymmetry under $\alpha \leftrightarrow \beta$?

4. **Negative-energy solutions of the Dirac equation.** Check that $\Psi(x) = v(p)e^{+\mathbf{i}p\cdot x}$ with
$$v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta^s \\ -\sqrt{p\cdot\bar{\sigma}}\,\eta^s \end{pmatrix}, \quad s = 1, 2$$
solves the Dirac equation if $p^2 = m^2$ and $p^0 > 0$.

   Assuming that η^s comprise an orthonormal basis of 2×2 spinors, check that
$$\sum_{s=1,2} v^s \bar{v}^s = \not{p} - m.$$
Check that $\left(v^s\right)^\dagger(p)v^{s'}(p) = 2\omega_p\delta^{ss'}$. (You might want to choose $\vec{p} = \hat{z}p^3$ and a basis of σ^3 eigenstates to do this.)

5. **Supersymmetry.** Peskin problem 3.5

6. Peskin 3.3 (spinor products)

7. Peskin 3.7a (P,C,T).